

Spin-fluctuations, theory and planned application to nanostructures

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Samir Lounis, Manuel dos Santos Dias, Stefan Blügel

Outline

- Motivation
- Introduction: localized vs itinerant electron magnets
- Spin-fluctuations
 - Phenomenological approach
 - Gaussian statistics
 - Zero-point spin-fluctuations
- Goal of the project: calculation of spin-fluctuations in nanostructures
- Conclusions

Motivation

- How well does DFT (+LDA) describe the ground state magnetic properties?
 - ✓ Fair description of several systems such as 3-d ferromagnets, Fe, Co; magnetic compounds of transition metals FeNi, NiCr, CoMn,; ...
 - x Contradiction with experiments for weak itinerant electron magnets:

Magnetic moments:

	DFT	Experiments
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ZrZn ₂	0.5	0.2

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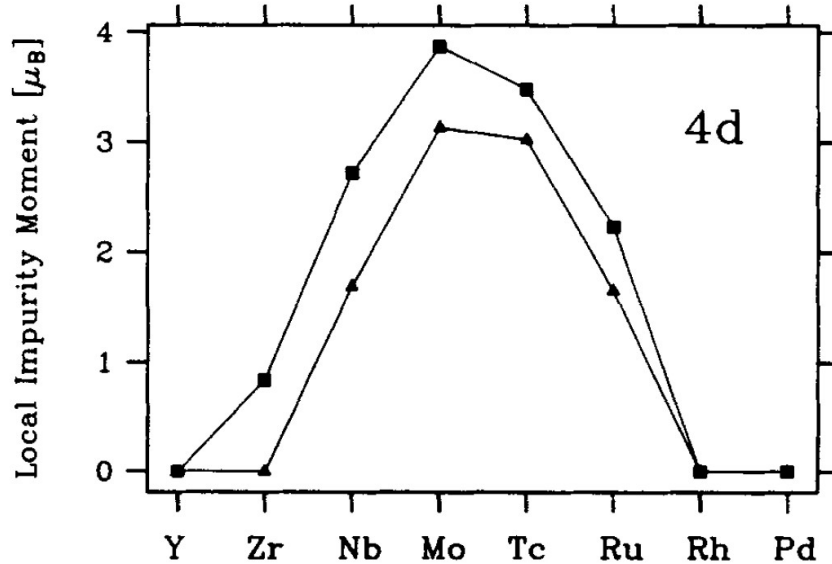
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- The DFT (+LDA) description of magnetism is at a mean field level, and **neglects fluctuations**: tendency to overestimate magnetism
- Evidence for large effects of **spin-fluctuations**:
 - Measurements of spin-fluctuations of the **order of Bohr magneton** in Y_{0.93}Sc_{0.07}Mn₂ [Shiga et al, JPSP **57**, 3141 (1988)]
 - Spin-fluctuations in Ni₃Ga destabilize the ferromagnetic ground state predicted by DFT [Aguayo et al, PRL **92**, 147201 (2004)]
 - Spin-fluctuations in Ni₃Al can correct the magnetic moment predicted by DFT [Ortenzi et al, PRB **86**, 064437 (2012)]

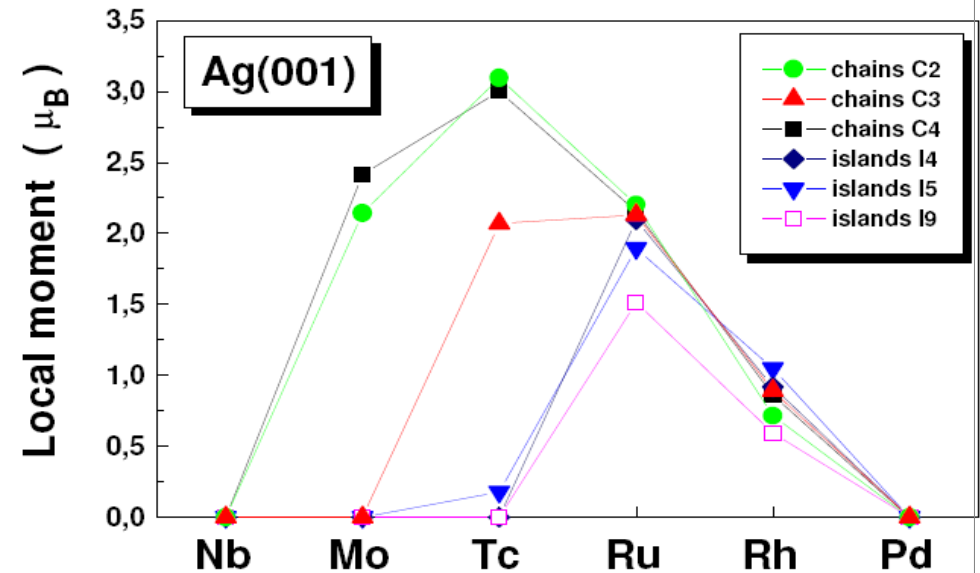
Motivation: nanostructures

Impurities on Ag(100)



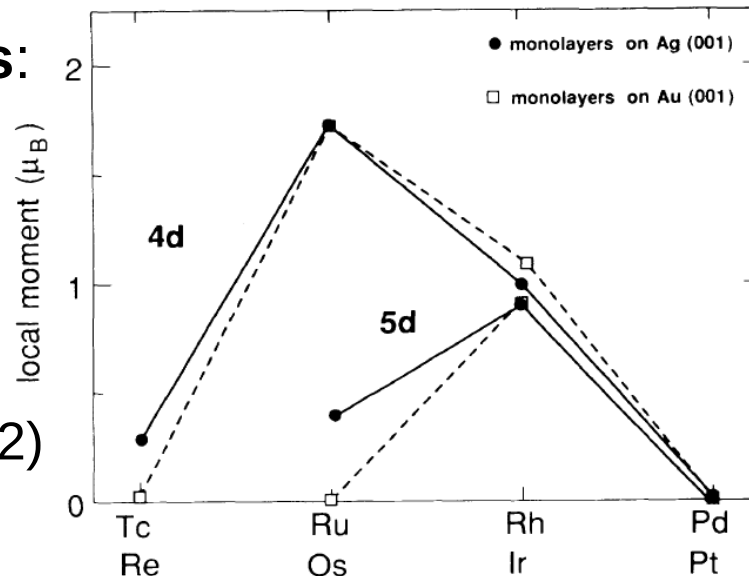
Lang, Stepanyuk, Wildberger, Zeller and Dederichs, SSC **92** 755 (1994)

Chains



Wildberger, Stepanyuk, Lang, Zeller and Dederichs, PRL **75** 509 (1995)

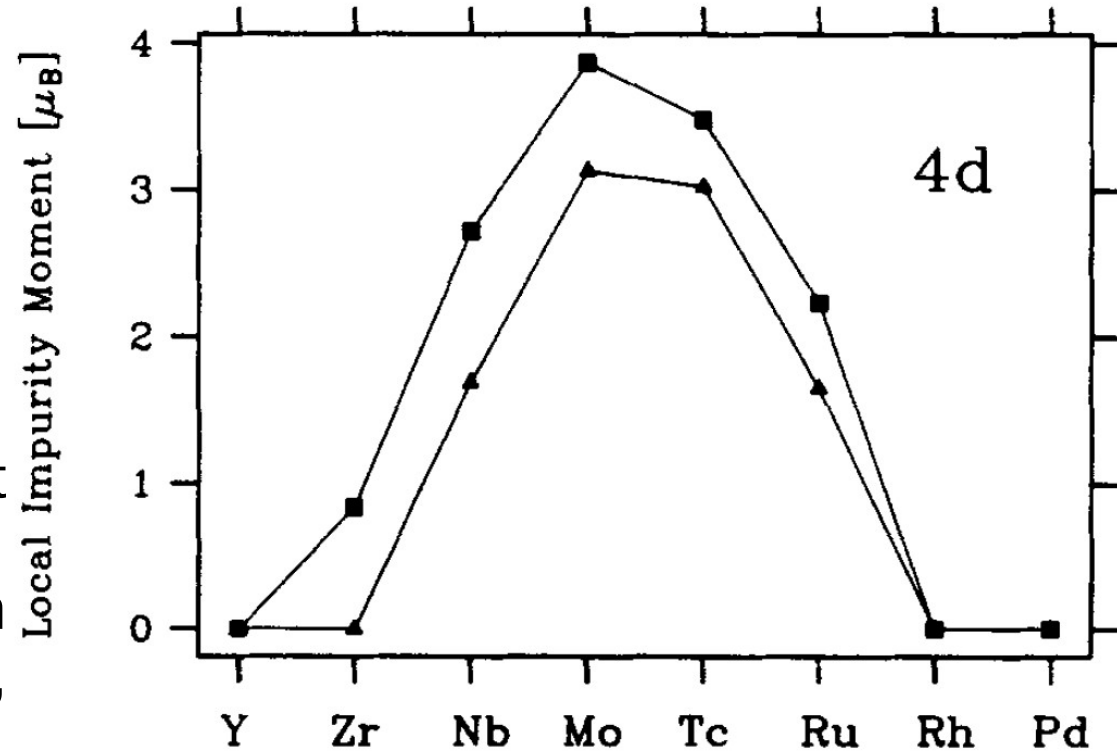
Monolayers:



Blügel, PRL **68** 851 (1992)

Motivation: nanostructures

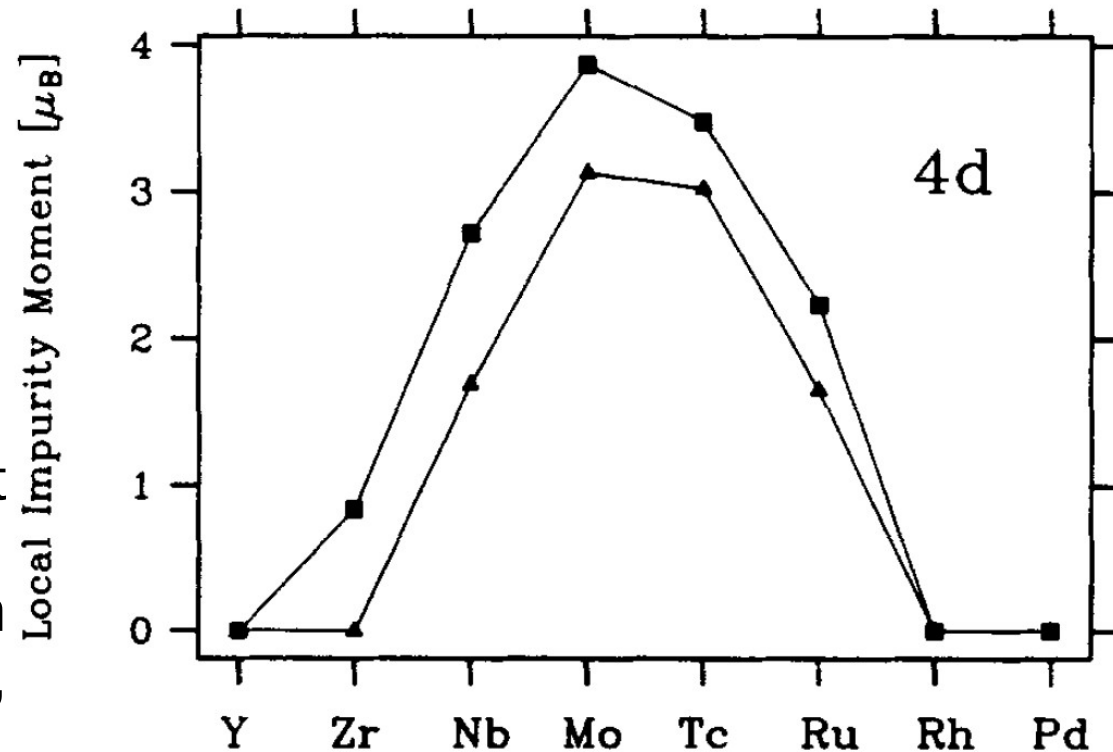
- Nanostructures on non-magnetic metallic substrates
- DFT predicts large local moments (Stoner criterion almost always fulfilled)
- Experimentally, no local moment has been found in these systems [see, for instance, Ru and Rh in Ag(100), Honolka et al, PRB **76**, 144412 (2007)]



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Can spin-fluctuations correct the local magnetic moments predicted by DFT?

Srivastaka et al, JPPM **18**, 9463 (2006)

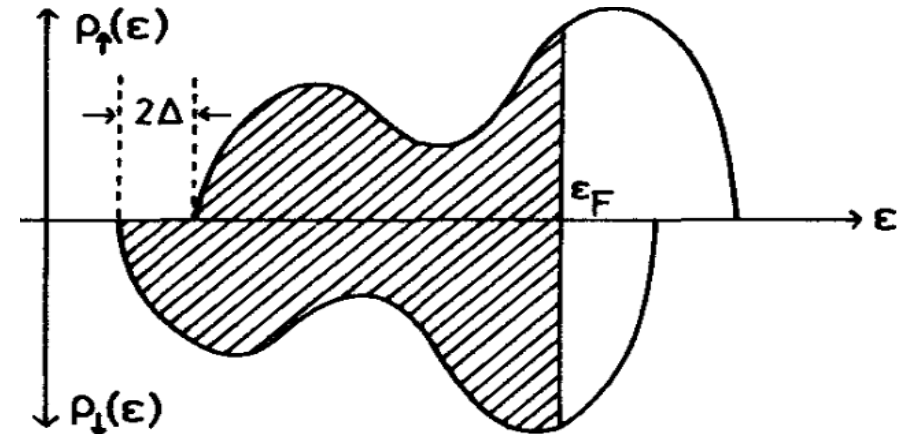
Introduction: opposite views of magnetism

Localized electrons



- Localized in real space
- Magnetic moment: **integer** value (Hund's rule)
- Examples: magnetic insulators, rare earth metals...

Itinerant electrons



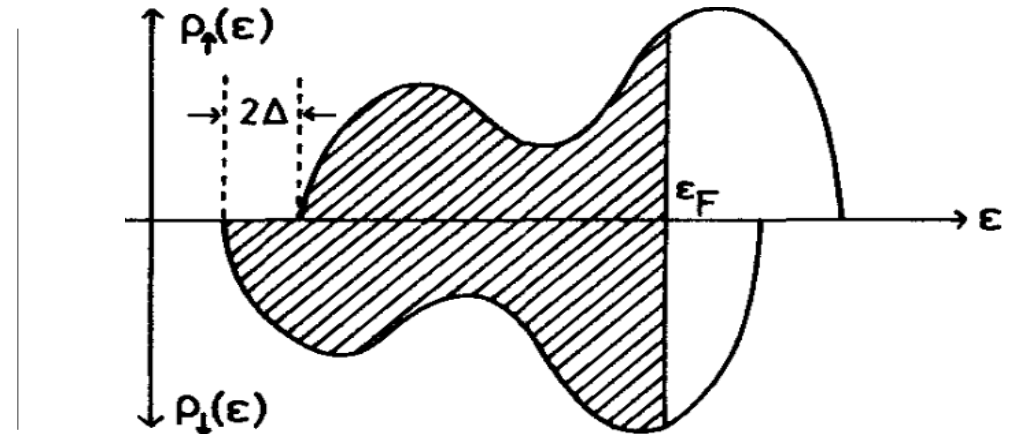
- Localized in momentum space
- Magnetic moment: **rational** value
- Examples: transition metals, weak ferromagnetic compounds...

Introduction: opposite views of magnetism

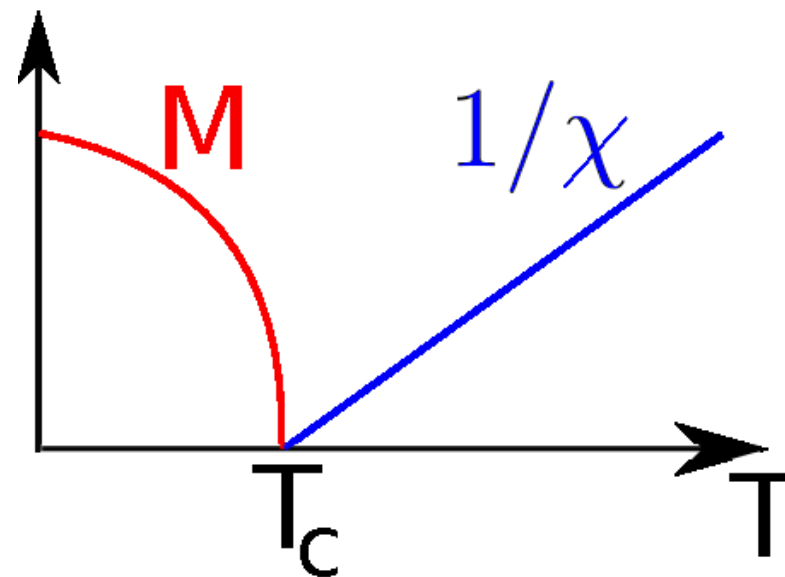
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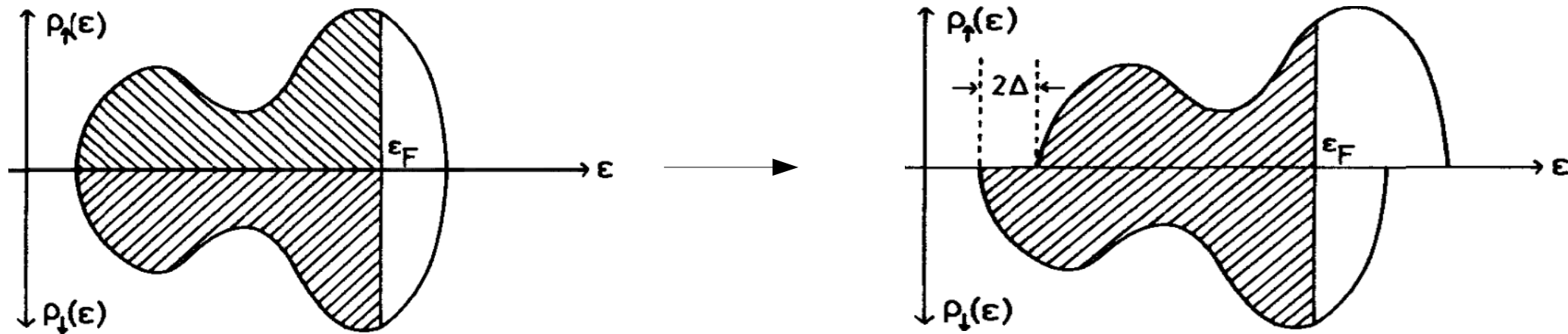
Itinerant electrons



- General common feature: temperature dependence of magnetic moment and inverse susceptibility

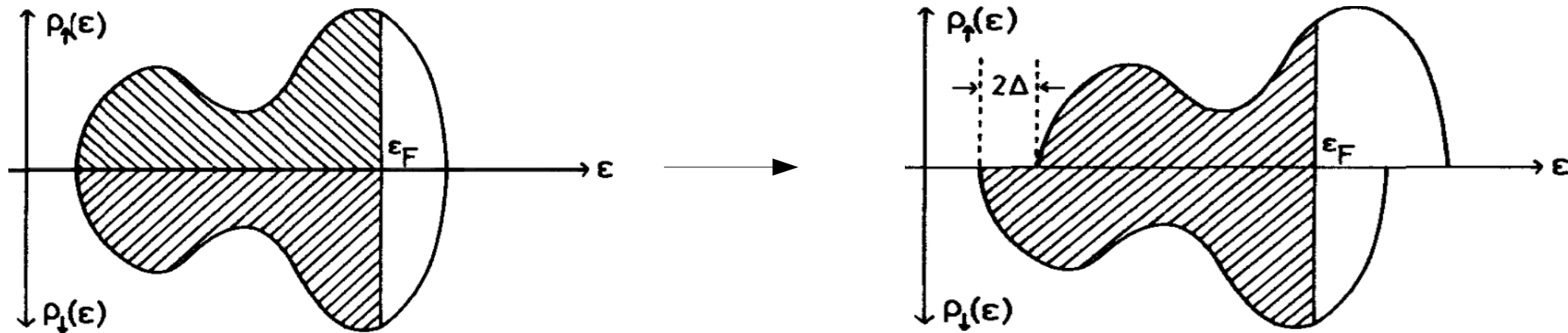


Stoner model



- Effects of exchange are treated within a molecular field term: $\mathbf{H} = \pm\Delta\hat{\mathbf{z}}$
- Magnetic susceptibility $\chi = \frac{\chi_0}{1 - I_s\rho(\epsilon_F)}$
- Criterion for magnetism provided: $I_s\rho(\epsilon_F) > 1$

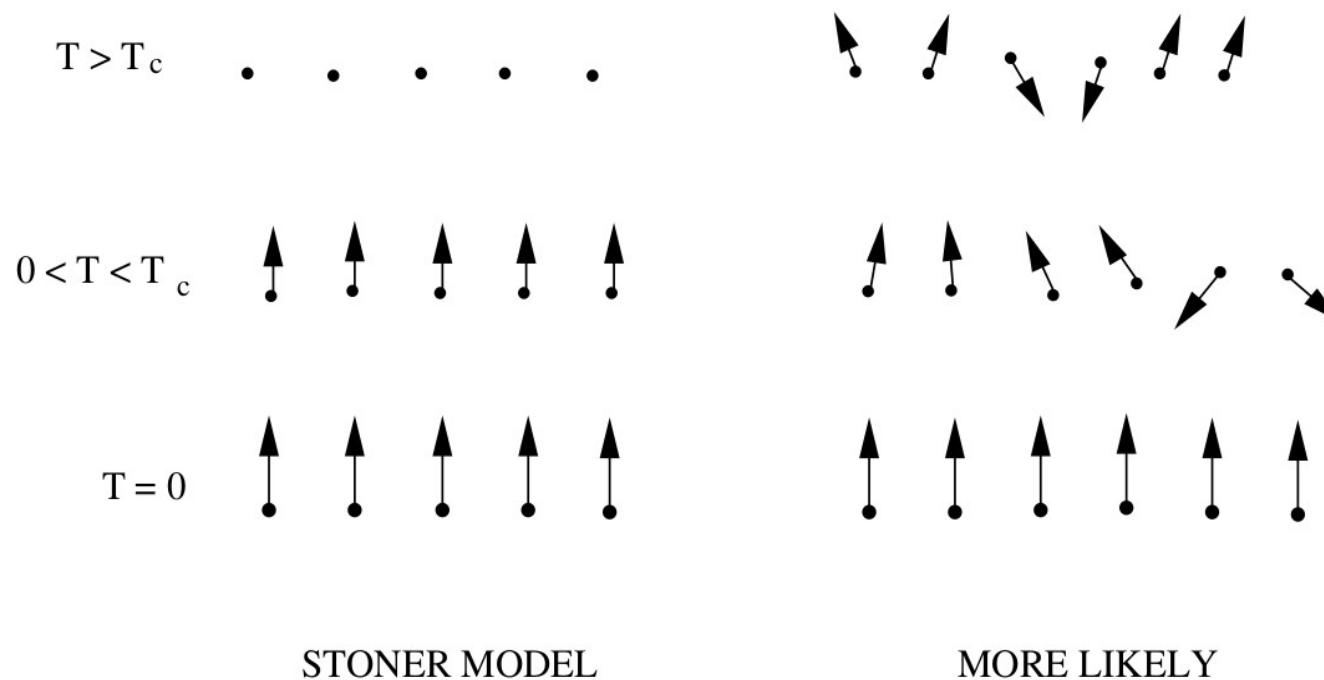
Stoner model



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$$\chi = \frac{\chi_0}{1 - I_s \rho(\epsilon_F)}$$
- Criterion for magnetism provided: $I_s \rho(\epsilon_F) > 1$
- Main drawback of Stoner theory: extension to **finite temperatures**
 - Temperature enters only through Fermi occupation factors: too weak temperature dependence
 - Wrong analytic behavior of temperature dependent variables as compared to experiments; in particular, **Curie-Weiss law not recovered**

Spin fluctuations

- Moriya and Kawabata, JPSJ **34** 639 (1973); Moriya, JMMM **14** 1 (1979)
- Murata and Doniach, PRL **29** 285 (1972)
- Shimizu, RPP **44** 329 (1981)
- Lonzarich and Taillefer, JPCCM **18** 4339 (1985)
- Mohn and Wohlfart, JPFMP **17** 2421 (1986)
- Takahashi, JPSJ **55** 3553 (1986)
- Solontsov and Wagner, PRB **51** 12410 (1994)
- ...



Spin fluctuations

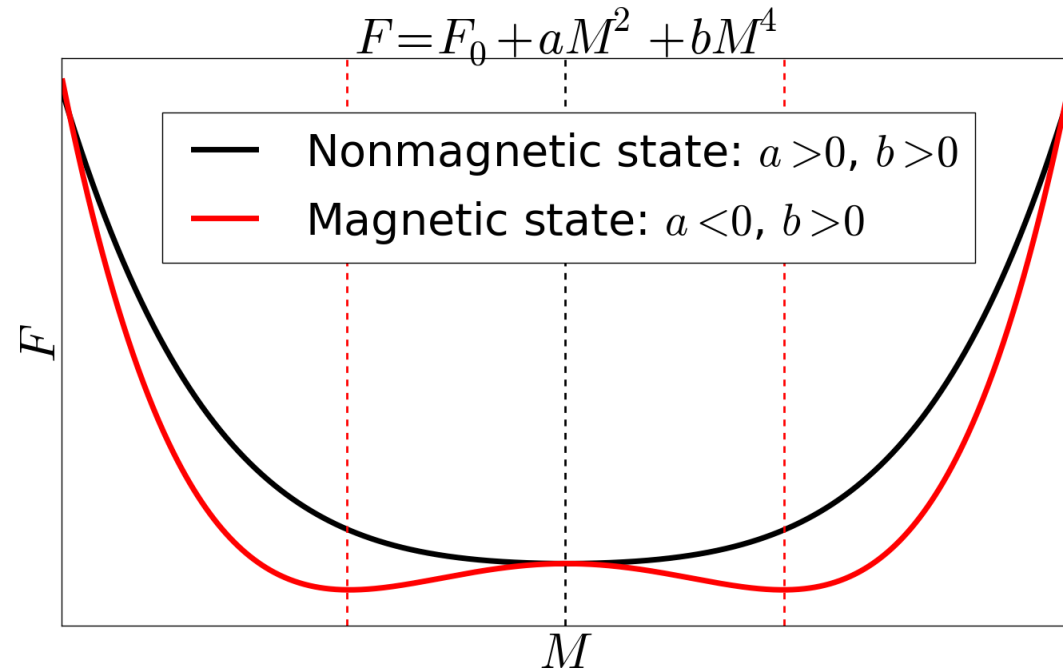
Phenomenological approach

- Phenomenological Landau-Ginzburg method: expansion of free energy with magnetization as the order parameter

$$F = F_0 + aM^2 + bM^4 + \dots$$

With $a \propto \chi^{-1} \propto 1 - I_s \rho(\epsilon_F)$

Stoner criterion, $a < 0 \Leftrightarrow I_s \rho(\epsilon_F) > 1$



Spin fluctuations

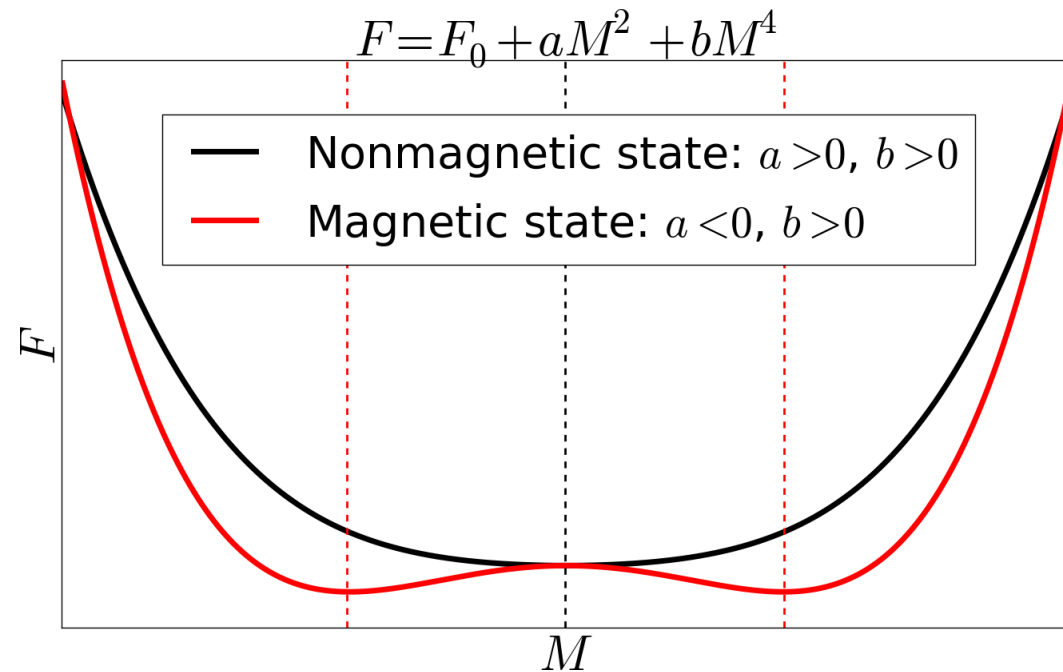
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- Including spin-fluctuations (\mathbf{m}); from scalar to vectorial quantities

$$M^{2n} \rightarrow \left\langle \left(\mathbf{M} + \sum_{i=1}^3 \mathbf{m}_i \right)^{2n} \right\rangle_{\bar{a}}$$

$$F = F_0 + M^2 \overbrace{\left(a + b(6\langle m_{\perp}^2 \rangle + 4\langle m_{\parallel}^2 \rangle) \right)}^{\bar{a}} + M^4 (b + \mathcal{O}(cM^6)) + \dots$$

- Spin-fluctuations affect the (main) coefficient responsible for the magnetic order; $\bar{a} = a + b(6\langle m_{\perp}^2 \rangle + 4\langle m_{\parallel}^2 \rangle) \longrightarrow$ push towards a nonmagnetic state

Spin fluctuations

Gaussian statistics

- Gibbs-Bogoliubov (Peierls-Feynman) inequality:

$$F \leq F_0 + \langle H - H_0 \rangle_0$$

Spin fluctuations

Gaussian statistics

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$$H = \int d\mathbf{r} \left(E \left(\mathbf{M} + \sum_{i=1}^3 \mathbf{m}_i(\mathbf{r}) \right) + C \sum_{i,j} (\nabla_j m_i(\mathbf{r}))^2 \right)$$

$$H_0 = \sum_{i=1}^3 \int d\mathbf{r} d\mathbf{r}' \Omega_i(\mathbf{r} - \mathbf{r}') \mathbf{m}_i(\mathbf{r}) \mathbf{m}_i(\mathbf{r}')$$

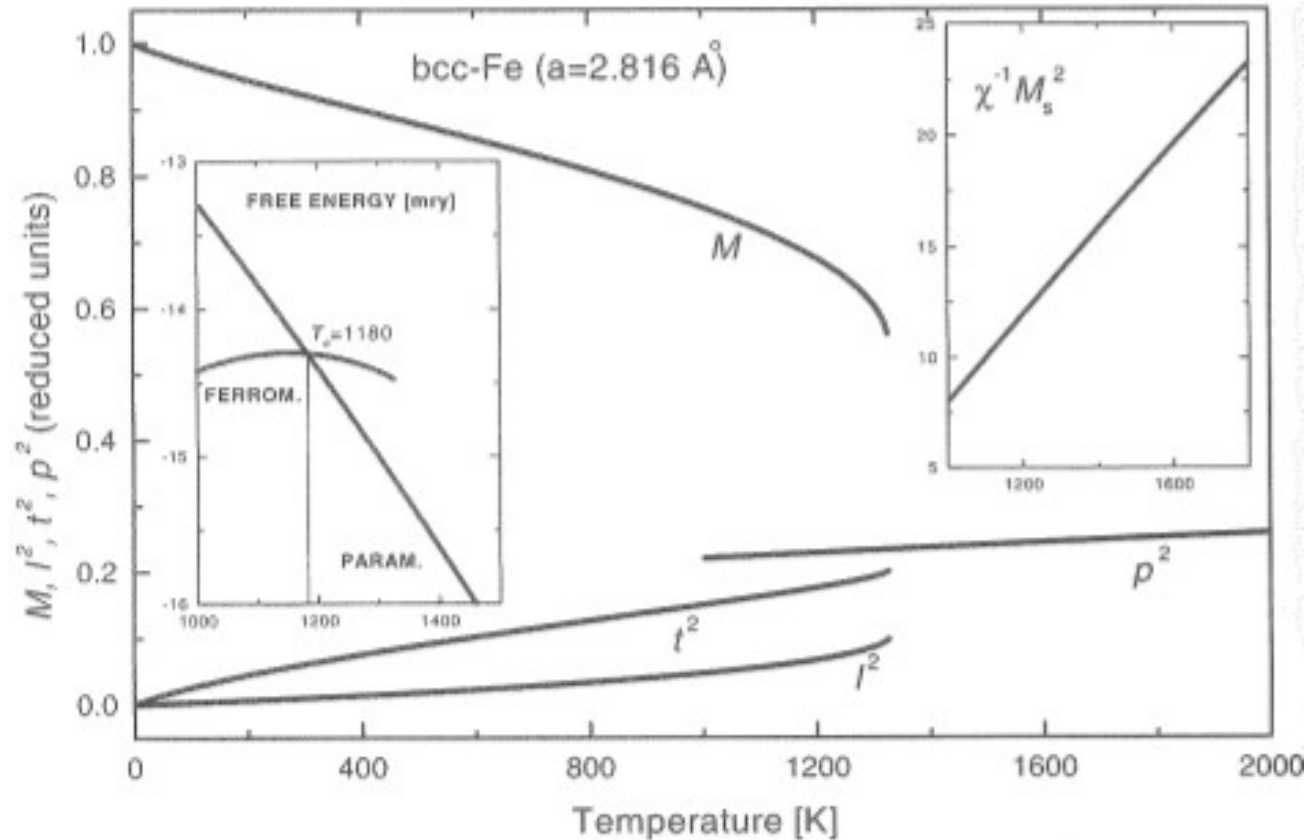
$\Omega_i(\mathbf{r} - \mathbf{r}')$ are variational parameters to be determined from $\frac{\delta F}{\delta \Omega_i} = 0$

- The theory leads to a set of equations to be solved self-consistently to determine (numerically) quantities such as bulk moment, spin-fluctuations or inverse susceptibility at a given temperature T:

$$\langle \mathbf{m}_i^2 \rangle \propto \frac{k_B T}{C} (1 - G(\mathbf{m}_i^2, \mathbf{M}^2, T))$$

Spin fluctuations

Gaussian statistics

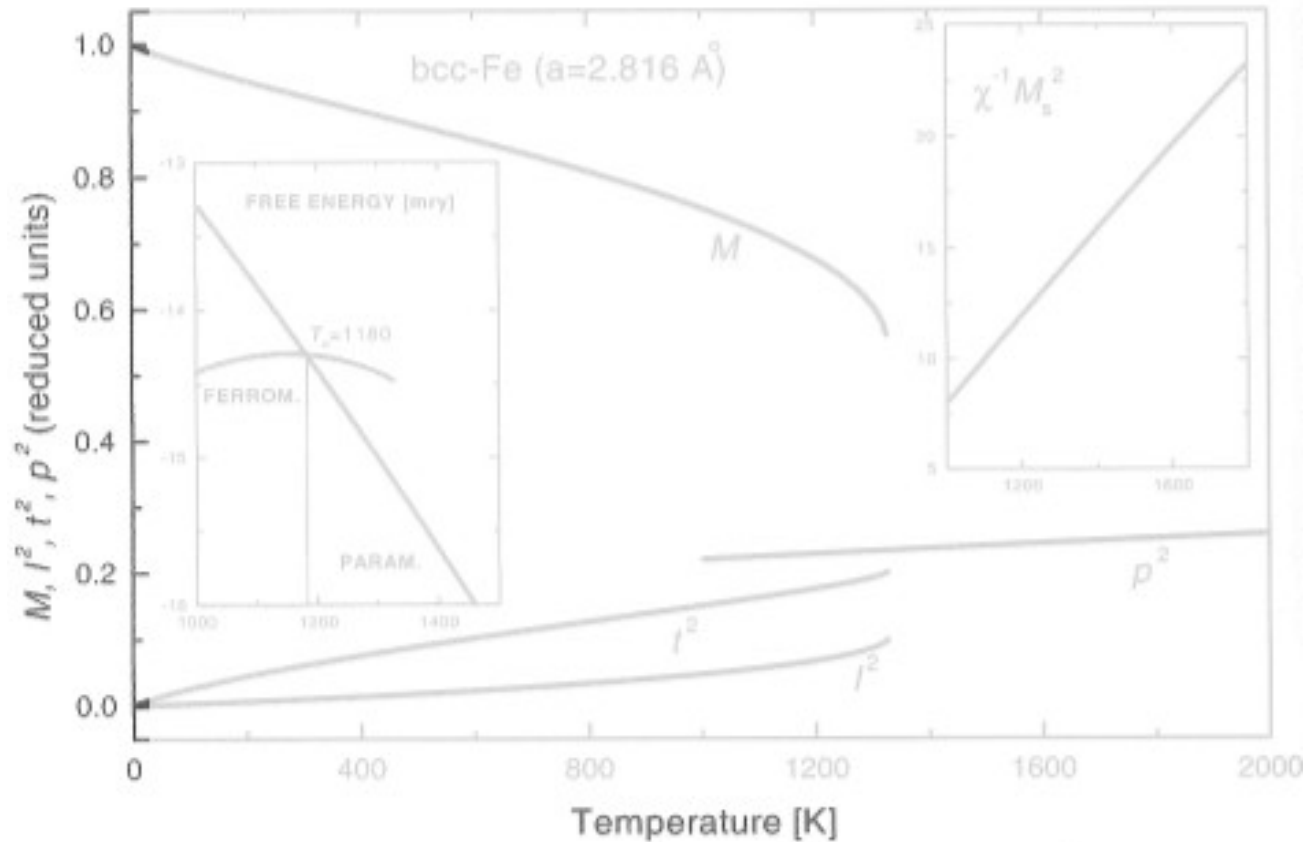


Kübler, *Theory of Itinerant Electron Magnetism*

- Renormalization of expansion coefficients of free energy by spin-fluctuations
- ✓ Finite temperatures: deviations of Curie temperature and susceptibility within 15% as compared to experiments: spin-fluctuations seem to cover the essential physics of magnetic coupling
- ✓ Curie-Weiss law of itinerant electron magnets
- ✗ Inappropriate discontinuous change of the spontaneous magnetization at T_c
- ✗ Neglects spin-fluctuations at $T=0$

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- **Fluctuation-dissipation theorem:**

$$\langle m_i^2(\mathbf{q}) \rangle = \frac{1}{2\pi} \int_0^\infty d\omega (1 + 2n(\omega, T)) \text{Im}\chi_i(\mathbf{q}, \omega)$$

with the Bose occupation factor $n(\omega, T) = (\exp(\omega/T) - 1)^{-1}$

- **Zero-point (ZP) spin-fluctuations:**

$$\langle m_i^2(\mathbf{q}) \rangle_{\text{ZP}} = \frac{1}{2\pi} \int_0^\infty d\omega \text{Im}\chi_i(\mathbf{q}, \omega)$$

- Experimental evidence for large **zero-point** spin-fluctuations, $\langle m_i^2(\mathbf{q}) \rangle_{ZP}^{1/2} \sim \mu_B$, [Shiga etal, JPSP **57**, 3141 (1988); Ziebeck etal, PRB **31**, 5884 (1982)]
- [Aguayo etal, PRL **92**, 147201 (2004)] Renormalization of the Stoner criterion by zero-point spin-fluctuations; calculation of approximate susceptibility within DFT+LDA

$$\chi_0(\mathbf{q}, \omega) = \rho(\epsilon_F) - Aq^2 + iB\omega/q$$

$$\bar{a} = a + 10b\langle m^2 \rangle > 0 \Rightarrow \tilde{I}_s = I_s - 10b\langle m^2 \rangle$$

- Correct the magnetic properties of DFT+LDA at T=0 adjusting the exchange-correlation potential taking into account ZP spin-fluctuations [Ortenzi etal, PRB **86**, 064437 (2012)]

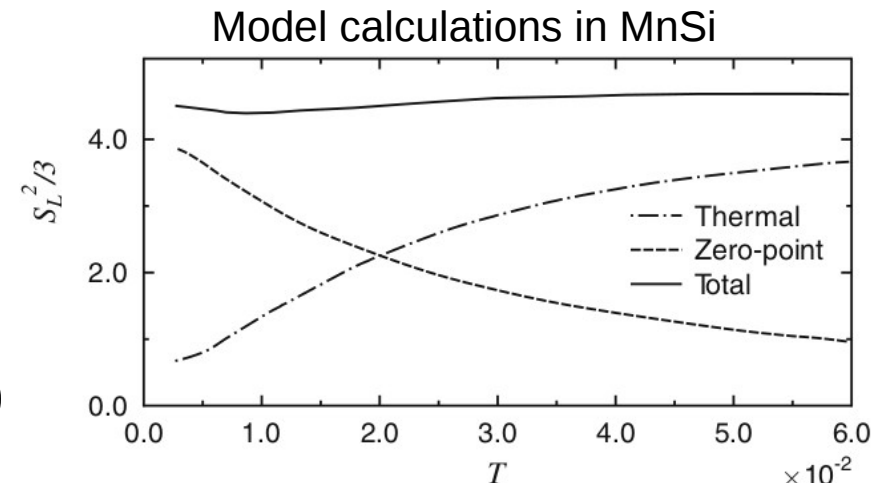
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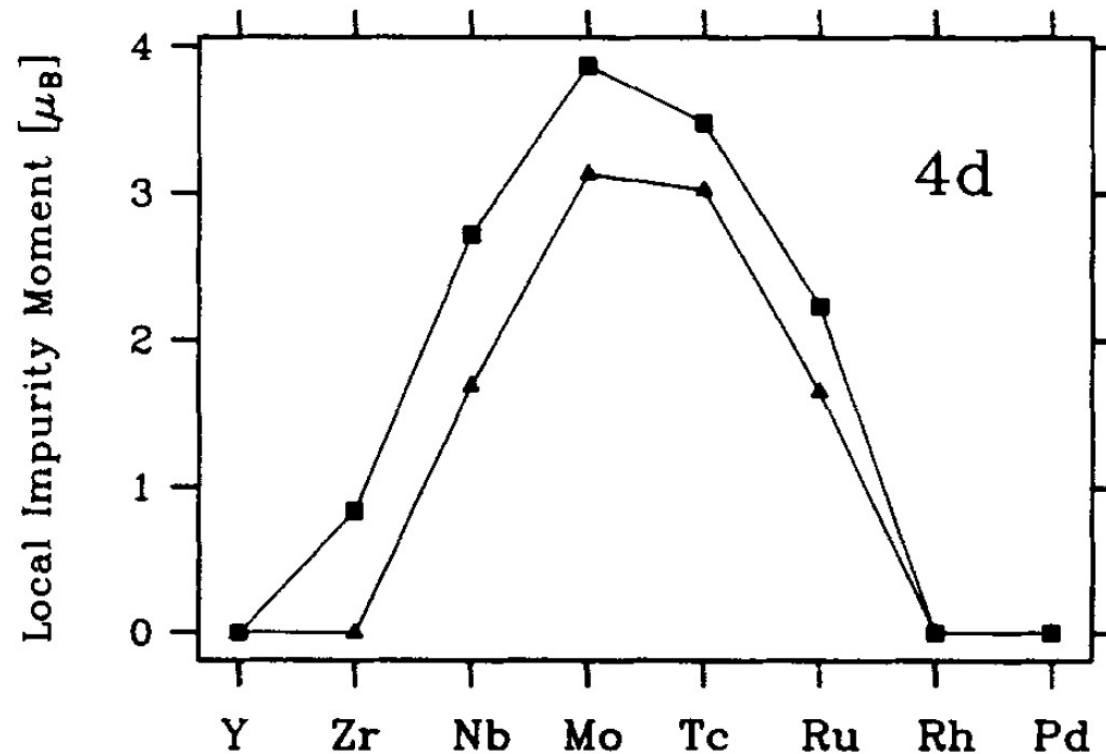
- Effect of ZP spin-fluctuations at finite T?
See review by:
Takahashi, JPCM **13** 6323 (2001)
also:
Solontsov & Wagner, PRB **51** 12410 (1994)



Takahashi & Moriya JPSP **54** 1592 (1985)

Goal of the project: nanostructures

Can **zero-point** spin-fluctuations correct the magnetic state predicted by DFT?



Lang, Stepanyuk, Wildberger, Zeller
and Dederichs, SSC **92 755** (1994)

Can **zero-point** spin-fluctuations correct the magnetic state predicted by DFT?

Goal: calculate the zero-point spin-fluctuation from *ab-initio*

- Method: Korringa-Kohn-Rostoker Green function
- Real-space approach for describing the impurity

$$G(\mathbf{r}, \mathbf{r}'; z) = G_0(\mathbf{r}, \mathbf{r}'; z) + \int d\mathbf{r}'' G_0(\mathbf{r}, \mathbf{r}''; z) \Delta V(\mathbf{r}'') G(\mathbf{r}'', \mathbf{r}'; z)$$

$G_0(\mathbf{r}, \mathbf{r}'; z)$: Green function of the **unperturbed** system

$G(\mathbf{r}, \mathbf{r}'; z)$: Green function of the **perturbed** system (impurity)

$\Delta V(\mathbf{r})$: change in the potential induced by the perturbation

- Access to dynamical magnetic susceptibility: $\chi = \chi_0 (1 - K_{xc} \chi_0)^{-1}$

$$\chi_0^{ij}(\mathbf{r}, \mathbf{r}'; \omega) = -\frac{1}{\pi} \int dz f(z) \left[G_{ij}^{\downarrow}(\mathbf{r}, \mathbf{r}'; z + \omega) \text{Im} G_{ij}^{\uparrow}(\mathbf{r}, \mathbf{r}'; z) \right. \\ \left. + G_{ij}^{\downarrow}(\mathbf{r}, \mathbf{r}'; z) \text{Im} G_{ji}^{-\uparrow}(\mathbf{r}, \mathbf{r}'; z - \omega) \right]$$

Details in Lounis, Costa, Muniz, Mills, PRL **105** 187205 (2010)

Lounis, Costa, Muniz, Mills, PRB **83** 035109 (2011)

- Technical problem: **frequency integration**

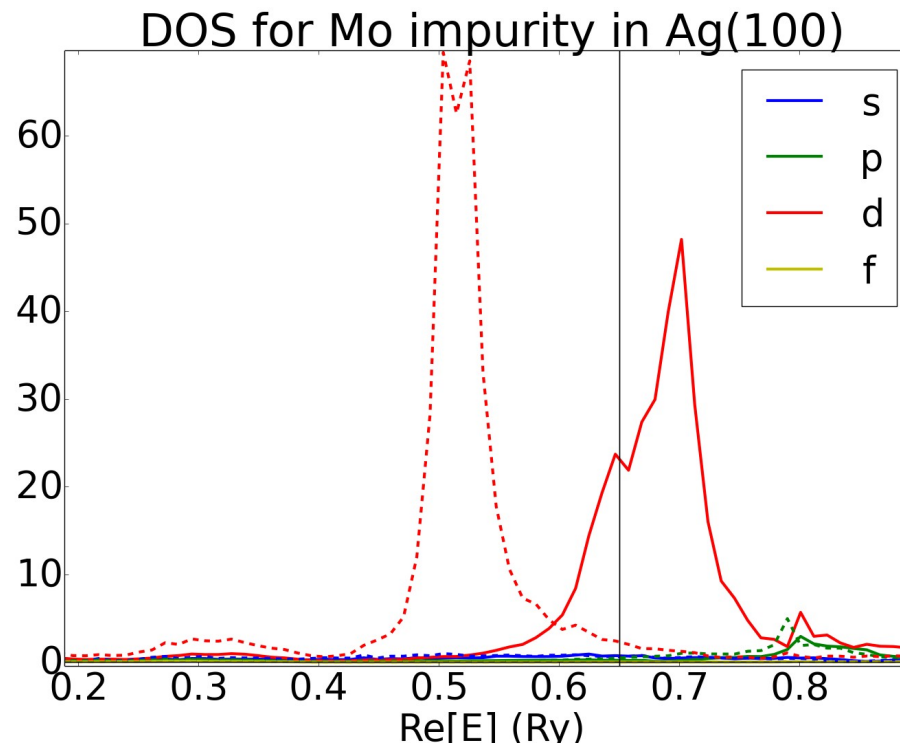
$$\langle m_i^2 \rangle_{\text{ZP}} = \int d\mathbf{r} \langle m_i^2(\mathbf{r}) \rangle_{\text{ZP}} = \frac{1}{2\pi} \int_0^\infty d\omega \int d\mathbf{r} \text{Im} \chi_i(\mathbf{r}, \mathbf{r}; \omega)$$

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- Frequency integration along the real axis: computationally very heavy
Possible solution: integrate in complex plane, but...
 - Currently, the method gives access to the susceptibility only along the real axis
 - Green functions are not analytical on the same side of the complex plane

Final remarks

- What is the magnitude of longitudinal spin-fluctuations? (coupling to the charge density)
- Are spin-fluctuations independent of temperature, as suggested by Takahashi?
- Impact of spin-orbit interaction
- Impact of dimensionality: surfaces, thin-films

Thank you

DOS for 3d adatoms on Cu(111)

