Colloidal Hydrodynamics and Interfacial Effects

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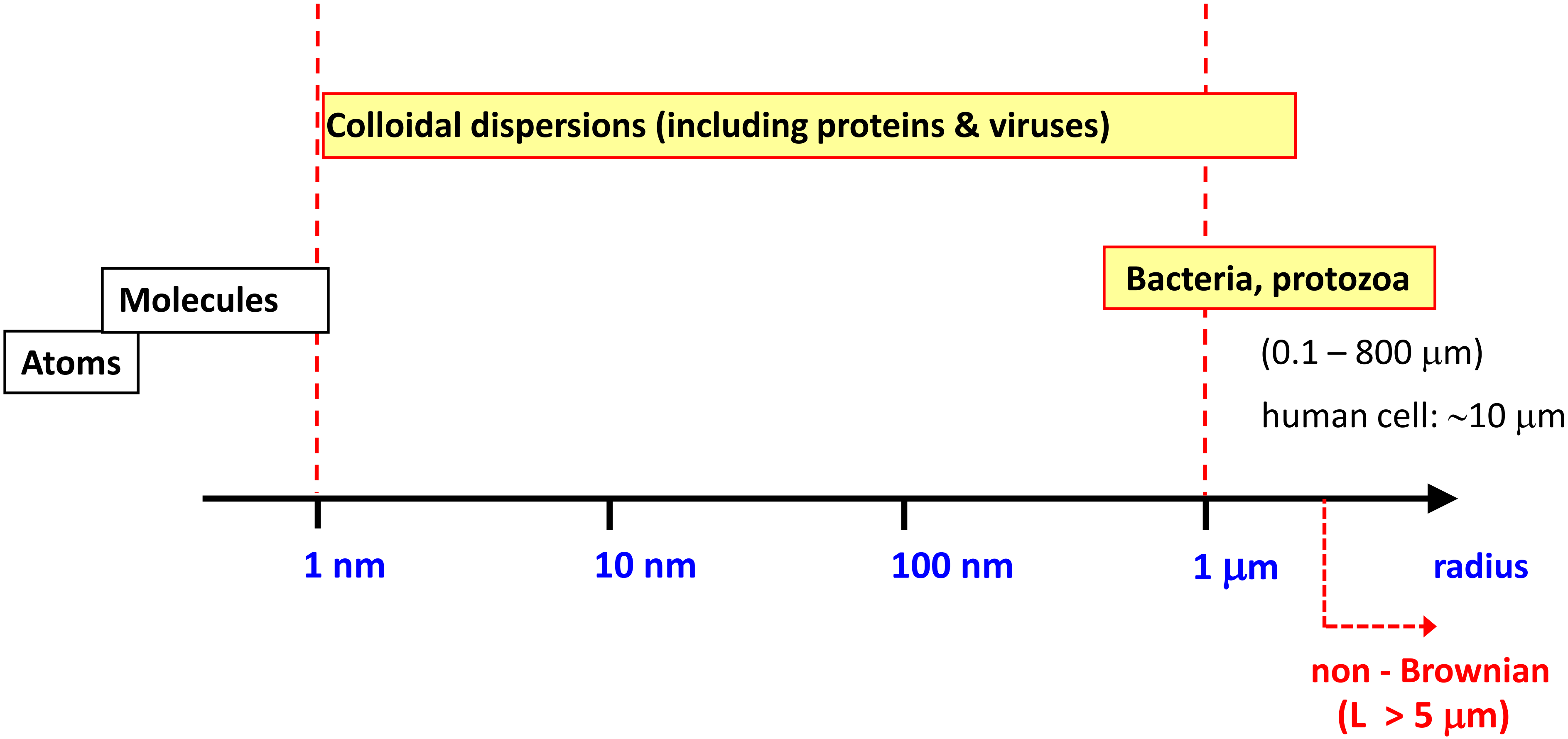
Content

1. Inertia-free dynamics in microworld
2. Hydrodynamic interactions
3. Particle near an interface
4. Active swimmers near a surface

1. Inertia-free dynamics in microworld

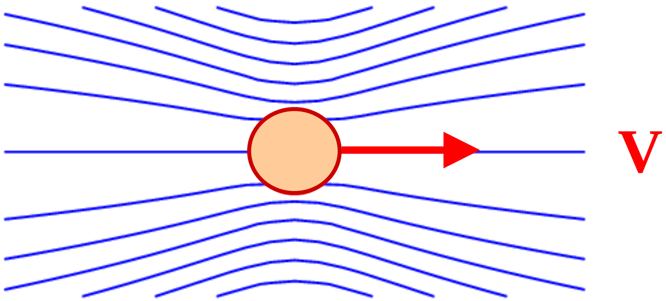
Length scales of colloids and micro-organisms



- Colloids, proteins, bacteria, single-celled eukaryotes share **friction-dominated** hydrodynamics

Low-Reynolds-number fluid flow

- Inertial forces tiny as compared to friction forces: **Reynolds - # << 1**



$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho_f L V}{\eta_0}$$

in water:

wale:	10^8
human swimmer:	10000
colloid particle:	0.0001
bacteria /cells:	0.00001

$$\nabla \cdot \mathbf{u}(\mathbf{r}) = 0$$

incompressibility

$$-\nabla p(\mathbf{r}) + \eta_0 \nabla^2 \mathbf{u}(\mathbf{r}) + \mathbf{f}(\mathbf{r}) = 0$$

linear Stokes equation

inertia-free force balance

Likewise: inertia – free microparticle dynamics

$$M \frac{d\langle V\rangle(t)}{dt} \approx -\zeta_0 \langle V\rangle(t)$$

$$\Delta t >> \tau_B = \frac{M}{\zeta_0} \approx 10^{-5}\ \text{s}$$

$$\Delta x >> V_0 \tau_B \sim L \times Re \ll 1$$

"stopping distance"

$$V_0 \approx 30 - 50\ \mu m / s$$

Rhodospirillum bacteria (length $L \approx 5\ \mu m$)

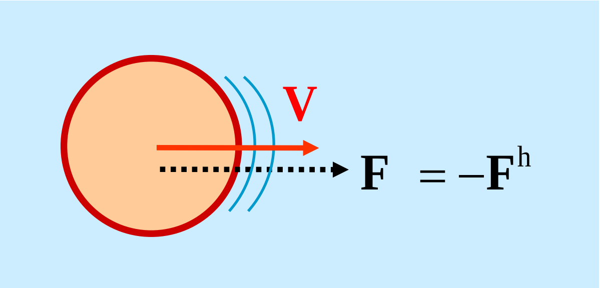
- Inertia free motion on coarse-grained colloidal time- and length scales
- Bacteria need no breakes

1. Implication: linear particle force – velocity relation

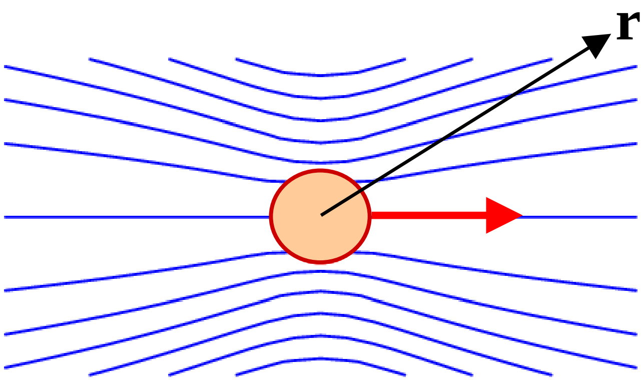
- Forced sphere with no-slip BC

$$\mathbf{V} = -\mu_0^t \, \mathbf{F}^h \qquad \mu_0^t = \frac{1}{6\pi\eta_0 a}$$

translational mobility



„No force – no motion“ (Aristotles)



$$\mathbf{u}(\mathbf{r}) \sim 1/r$$

long-range decay
(momentum conservation)

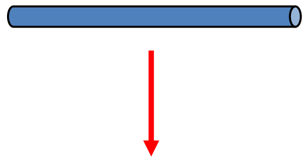
$$\mathbf{u} \times d\mathbf{r} = \mathbf{0} \quad \Rightarrow$$

streamlines parallel to local fluid velocity

2. Implication: Linearity

$$\mathbf{V}_{||} = \mu_{||}\mathbf{F}_{||}$$

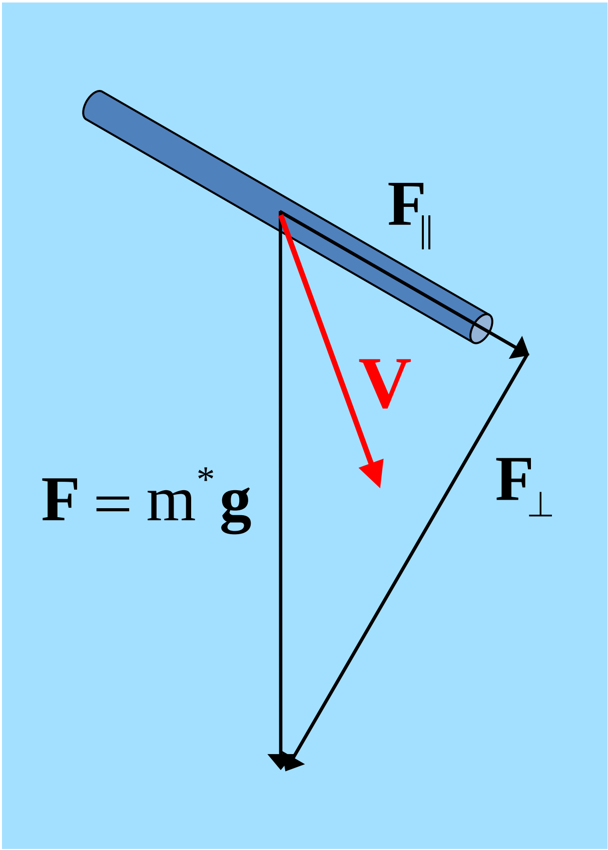
$$\mathbf{V}_{\perp} = \mu_{\perp}\mathbf{F}_{\perp}$$



$$\mu_{\perp} \approx \frac{1}{2}\mu_{||}$$

(transl. mobilities for L >> D)

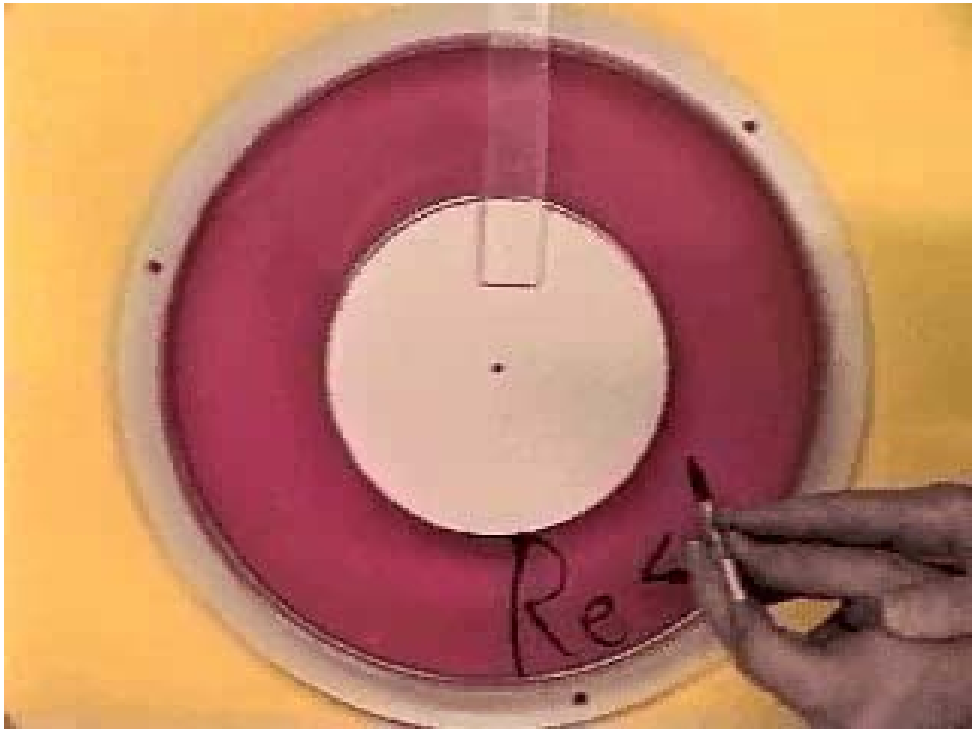
$$\mathbf{V} = \mathbf{V}_{||} + \mathbf{V}_{\perp}$$



Experiment: needle in syrup, no rotation

- Linear superposition of particle velocities and flow fields allowed

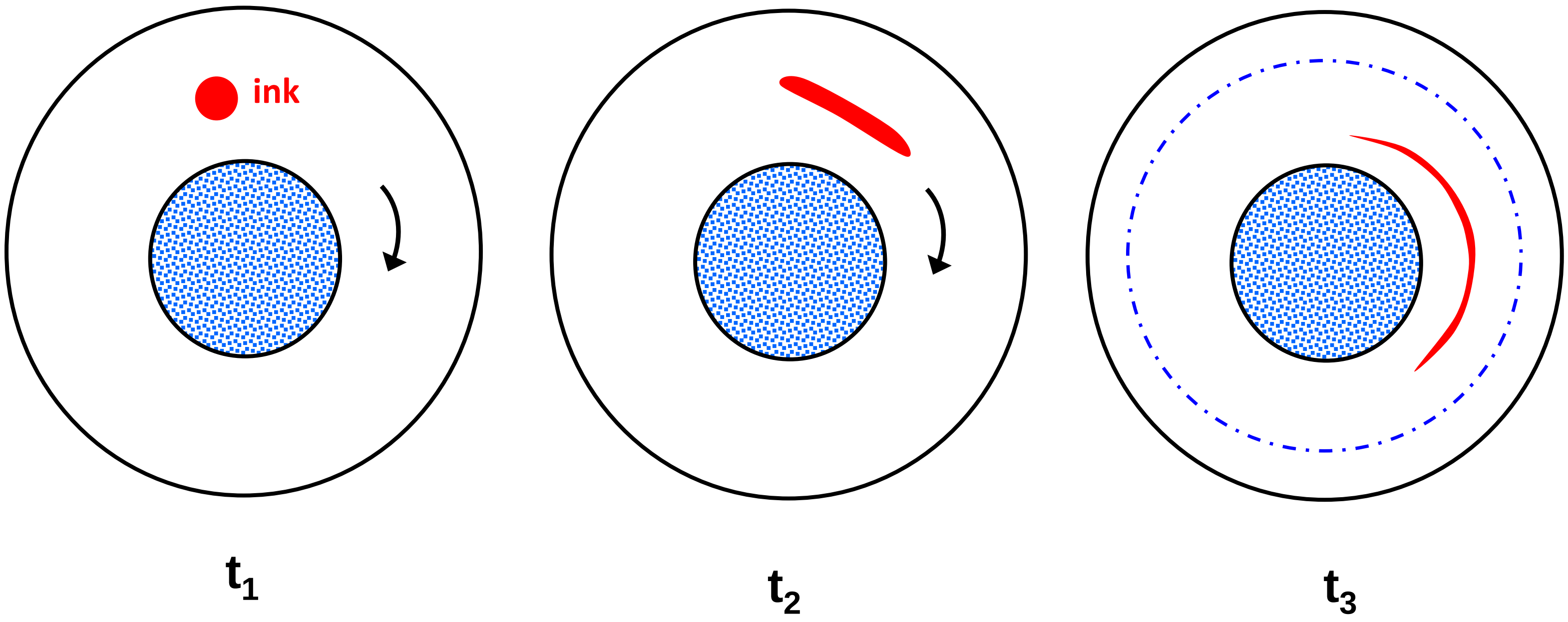
3. Implication: Kinematic reversibility



G.I. Taylor, Cambridge
https://web.mit.edu/hml/ncfmf.html

- Flow determined by instantaneous boundary configuration and velocities
- Force reversal and application history retracing :

  Fluid elements retrace motion along unchanged streamlines



- **Ink (colloid) particle motion across circular streamlines induced by:**

  - **Brownian motion**

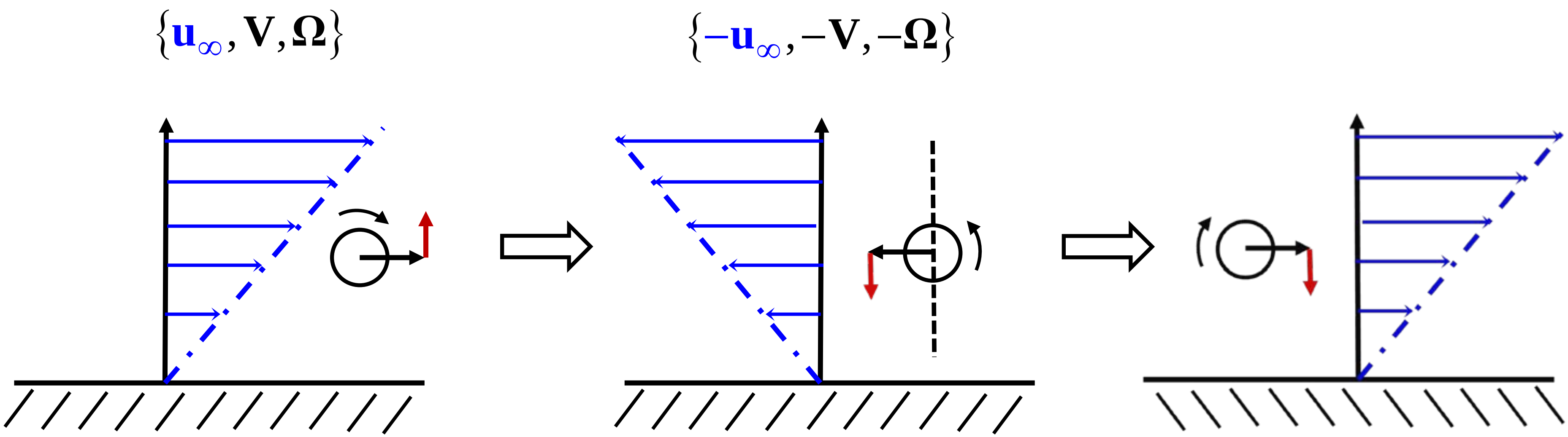
  - Inertia effects (Re ~ 1)

  - Many - particle HI in concentrated systems (chaotic trajectories)

  - Reversibility - breaking direct interactions

D. Pine et al., Nature **438** (2005)
J. Gollub and D. Pine, Physics Today, August 2006

Kinematic reversibility + symmetry = useful tool

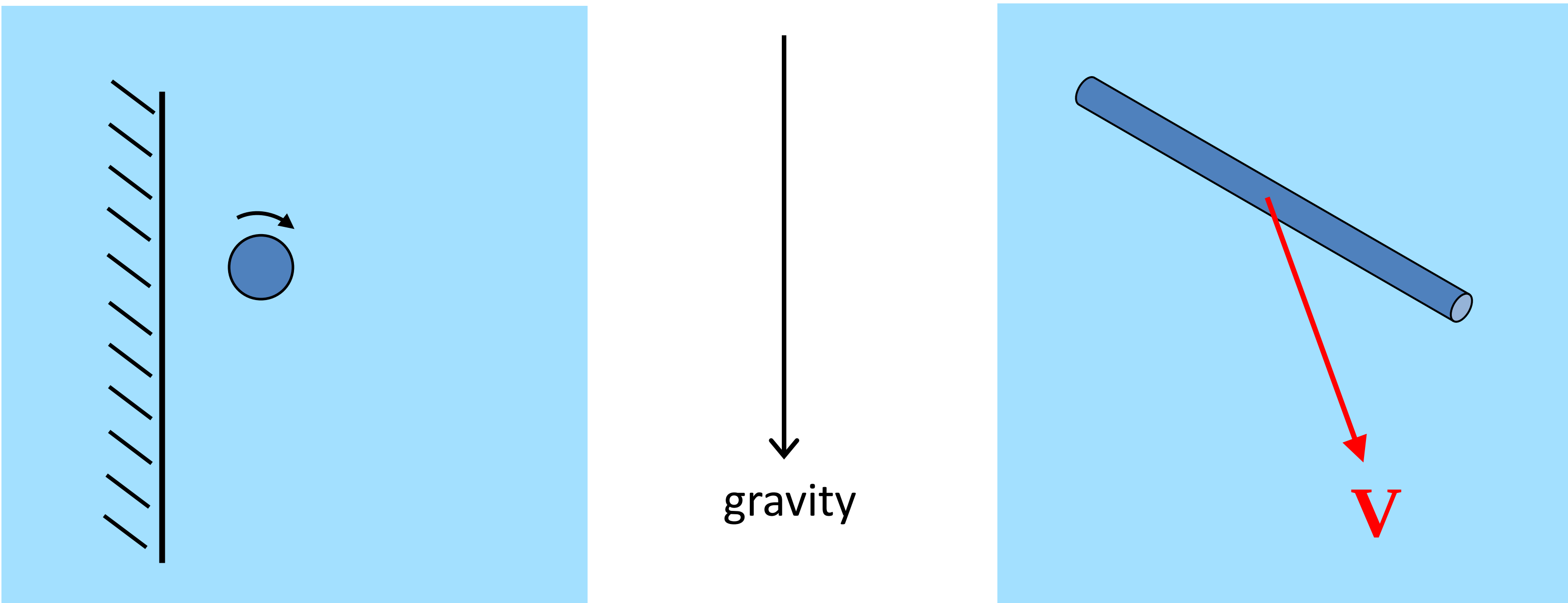


$$\Delta \mathbf{V} = 0$$

- Lift force arises here only if non- zero inertial contributions: d**u**/dt $\neq$ 0
- $\Delta \mathbf{V} \neq 0$ also for:

  Non-spherical rigid or flexible particle (polymer, droplet), flexible wall

Two homework problems w/o math



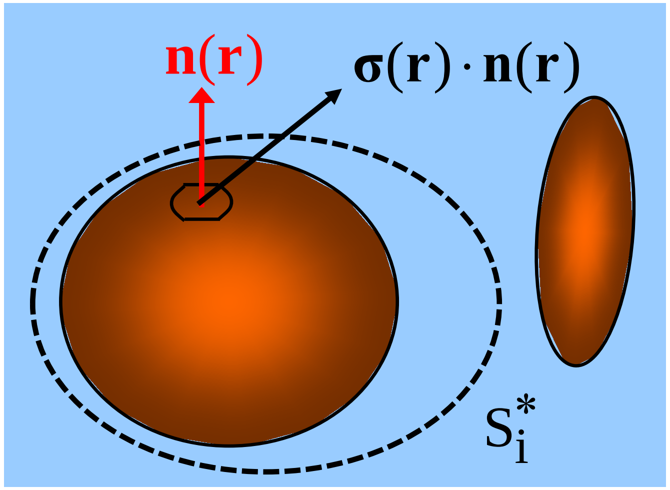
- Is rod re-orienting itself while settling with a sidewise velocity component?
- Is settling sphere moving towards / away from rigid wall? Is it rotating?

Particle forces balance in microworld

$$M_i \frac{d}{dt} \mathbf{V}_i = \mathbf{0} = \mathbf{F}_i^h + \mathbf{F}_i^{ext} + \mathbf{F}_i^{int} + \mathbf{F}_i^B$$

$$\mathbf{F}_i^h = \int_{S_i^*} dS \, \underbrace{\boldsymbol{\sigma}(\mathbf{r}; X) \cdot \mathbf{n}(\mathbf{r})}$$

fluid force / area on sphere surface element dS



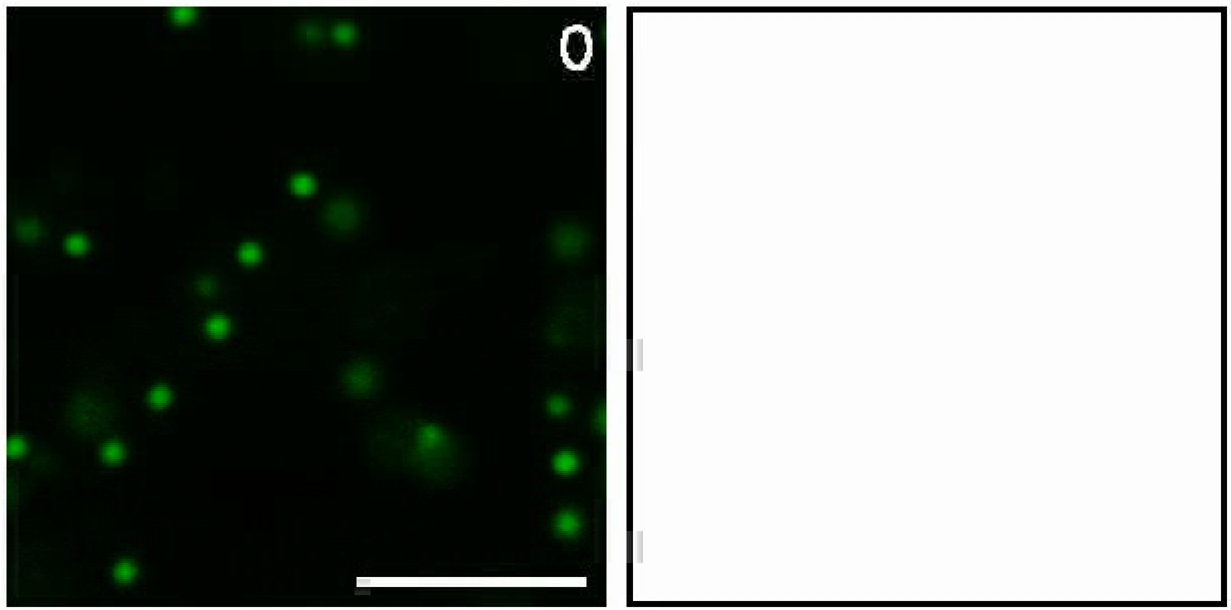
$$\sigma_{\alpha\beta}(\mathbf{r}) = -p(\mathbf{r})\delta_{\alpha\beta} + \eta_0\left[\partial_\alpha u_\beta(\mathbf{r}) + \partial_\beta u_\alpha(\mathbf{r})\right]$$

- **u**, p from Stokes Eqs. & BC

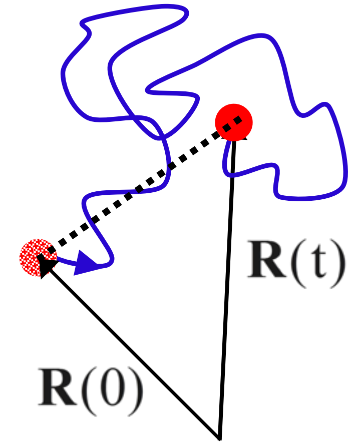
- Hydrodynamic drag force determined by fluid boundary configuration and velocities plus BCs
- Direct Interactions and Brownian forces break reversibility

Brownian motion

Movie: E.R. Weeks, Austin



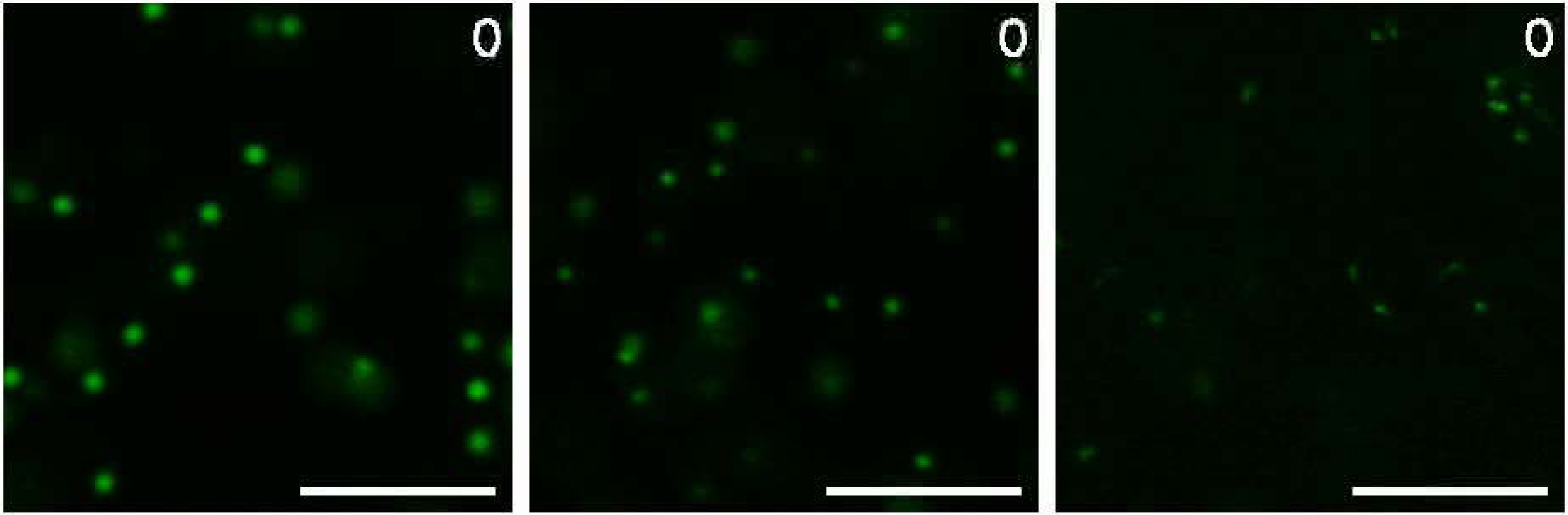
← 10 µm →



$$\left\langle [\mathbf{R}(t) - \mathbf{R}(0)]^2 \right\rangle = 6\, D_0^t\, t$$

Mean-squared displacement

Big particles E.R. Weeks, Austin **Small particles**



a = 1.5 μm **1.0 μm** **0.5 μm**

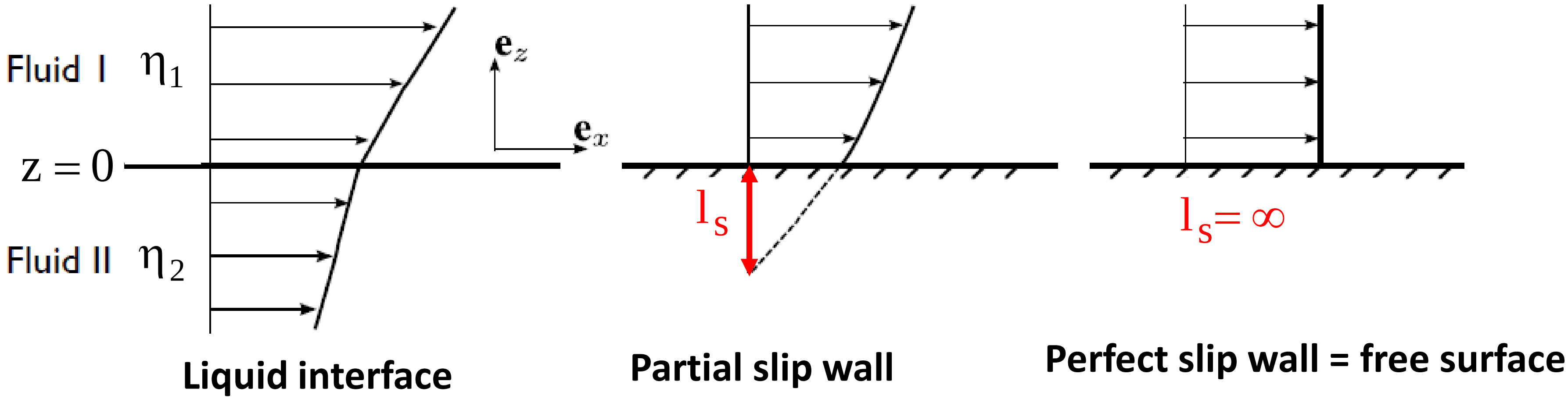
Stokes - Einstein - Sutherland:

$$D_0^t = \frac{kT}{6\pi\eta_0 \mathbf{a}}$$

$$\tau_a = \frac{a^2}{D_0^t} \propto a^3$$

- Brownian motion negligible (too slow) if a > 5 μm

Surface- and interfacial boundary conditions



- Liquid – liquid interface:

$$\eta_1 \frac{du_{x,y}^{(1)}}{dz} = \eta_2 \frac{du_{x,y}^{(2)}}{dz}$$

$$u_z^{(1)} = u_z^{(2)} = 0$$

$$u_{x,y}^{(1)} = u_{x,y}^{(2)}$$

- Rigid partial-slip surface:

$$u_{x,y} = l_s \frac{du_{x,y}}{dz}$$

$$u_z = 0$$

Sedimentation of a sphere: no-slip and perfect slip (gas bubble)

- No-slip sphere: outside flow

$$\mathbf{u}(\mathbf{r}) = \left\{ \frac{1}{r}(\mathbf{1}+\hat{\mathbf{r}}\hat{\mathbf{r}}) - \left(\frac{a^2}{3}\right)\frac{1}{r^3}(3\hat{\mathbf{r}}\hat{\mathbf{r}}-\mathbf{1}) \right\} \cdot \frac{\mathbf{F}}{8\pi\eta_0}$$

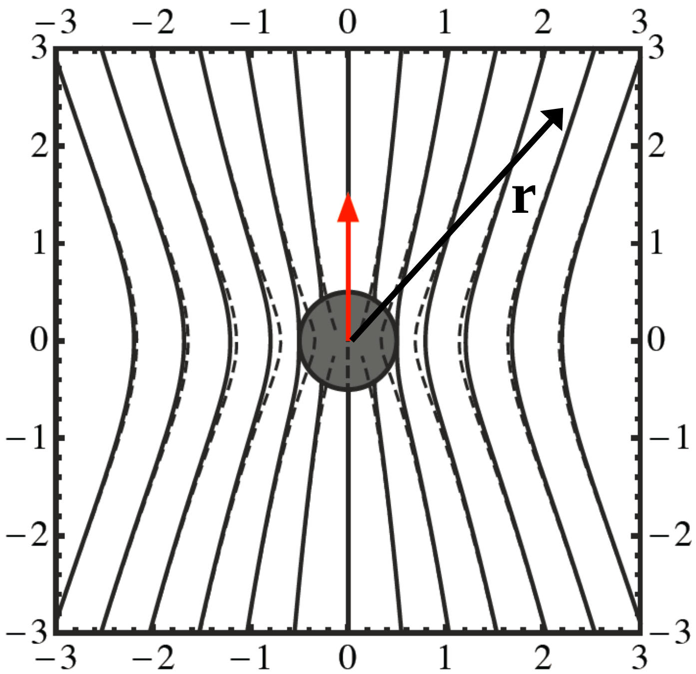
- Forced point particle = **Stokeslet flow:**

$$\mathbf{u}_{St}(\mathbf{r}) = \frac{1}{8\pi\eta_0}\left\{ \frac{1}{r}(\mathbf{1}+\hat{\mathbf{r}}\hat{\mathbf{r}}) \right\} \cdot \mathbf{F} = \mathbf{T}(\mathbf{r}) \cdot \mathbf{F}$$

Oseen tensor

- Gas bubble: outside flow:

$$\mathbf{u}_{gas}(\mathbf{r}) = \left\{ \frac{1}{r}(\mathbf{1}+\hat{\mathbf{r}}\hat{\mathbf{r}}) \right\} \cdot \frac{\mathbf{F}}{4\pi\eta_0}, \quad (r > a)$$

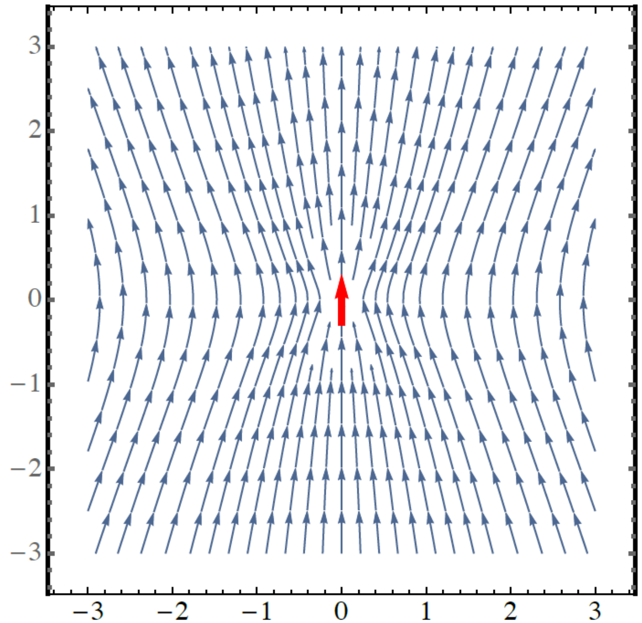


$$\mathbf{V} = \frac{\mathbf{F}}{6\pi\eta_0 a} = \mu_0^t(\text{stick})\mathbf{F}$$

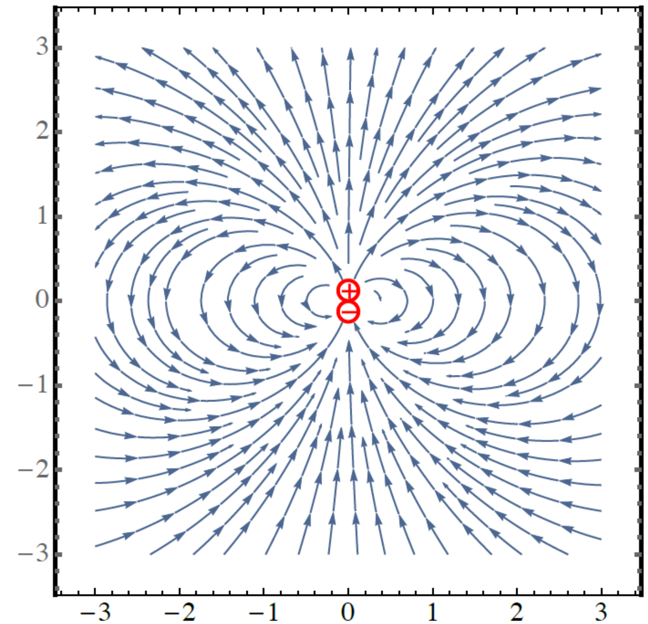
$$\mathbf{V}_{gas} = \frac{\mathbf{F}}{4\pi\eta_0 a} = \mu_0^t(\text{slip})\mathbf{F}$$

➢ Flow around no-slip sphere is superposition of **Stokeslet** and **Source doublet**

$$8\pi\eta_0 \mathbf{u}(\mathbf{r}) = \mathrm{F}\cdot\underbrace{\left(\frac{1}{r}(\mathbf{1}+\hat{\mathbf{r}}\hat{\mathbf{r}})\cdot\mathbf{e}\right)}_{\text{Stokeslet}} - \frac{a^2 F}{3}\cdot\underbrace{\left(\frac{1}{r^3}(3\hat{\mathbf{r}}\hat{\mathbf{r}}-\mathbf{1})\cdot\mathbf{e}\right)}_{\text{Source doublet}}$$



$O(r^{-1})$



$O(r^{-3})$

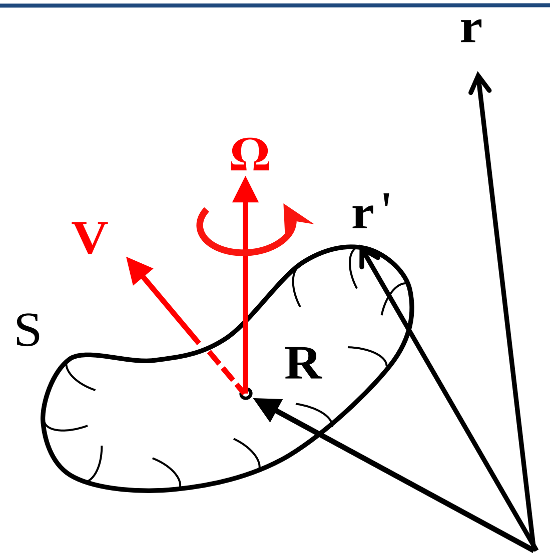
- Arbitrary outside particle flows can be **superposed by Stokeslet** and higher – order **singular elemental flow solutions** (certain derivatives of Stokeslet)

Application: Boundary layer method

- Rigid particle of arbitrary shape with **no - slip** BC:

$$\mathbf{u}(\mathbf{r}) = -\int_S dS' \, \underbrace{\mathbf{T}(\mathbf{r}-\mathbf{r}')\cdot[\boldsymbol{\sigma}\cdot\mathbf{n}](\mathbf{r}')}$$

Stokeslet superposition



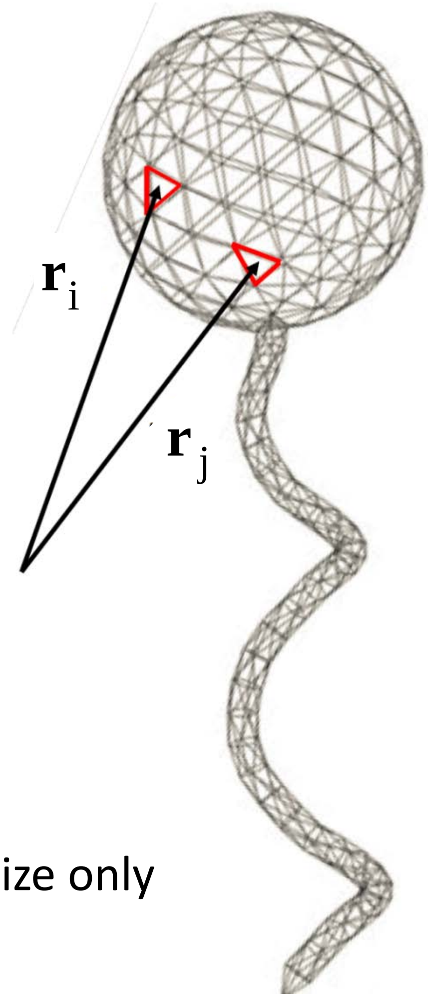
- Inserting BC gives surface integral equation for stress:

$$\mathbf{V} + \boldsymbol{\Omega}\times(\mathbf{r}-\mathbf{R}) = -\int_S dS' \, \mathbf{T}(\mathbf{r}-\mathbf{r}')\cdot[\boldsymbol{\sigma}\cdot\mathbf{n}](\mathbf{r}') \qquad (\mathbf{r}\in S)$$

$$\{\mathbf{V},\boldsymbol{\Omega}\} \Rightarrow \{[\boldsymbol{\sigma}\cdot\mathbf{n}]\} \Rightarrow \{\mathbf{F}^h,\mathbf{T}^h,\mathbf{u}\}$$ (Mobility problem)

- Particle with complex surface: Triangularization in N triangles

$$\mathbf{V}+\boldsymbol{\Omega}\times(\mathbf{r}_i-\mathbf{R})=-\sum_{j=1}^{N}\underbrace{\mathbf{T}(\mathbf{r}_i-\mathbf{r}_j)}_{3N\times 3N\text{ inversion}}\cdot[\boldsymbol{\sigma}\cdot\mathbf{n}](\mathbf{r}_j)$$



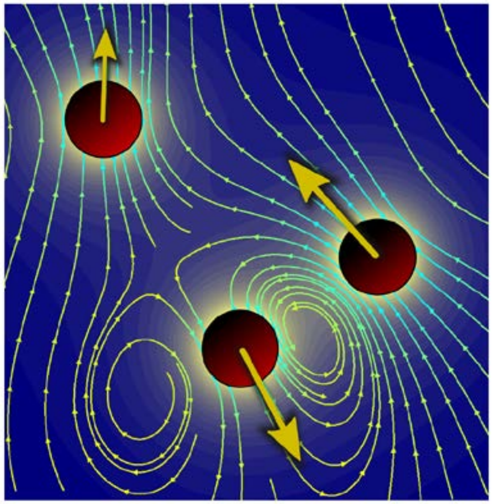
$$\begin{pmatrix}\mathbf{V}\\ \boldsymbol{\Omega}\end{pmatrix}=-\underbrace{\begin{pmatrix}\boldsymbol{\mu}^{tt} & \boldsymbol{\mu}^{tr}\\ \boldsymbol{\mu}^{rt} & \boldsymbol{\mu}^{rr}\end{pmatrix}}\cdot\begin{pmatrix}\mathbf{F}^{h}\\ \mathbf{T}^{h}\end{pmatrix}$$

Mobility matrix:

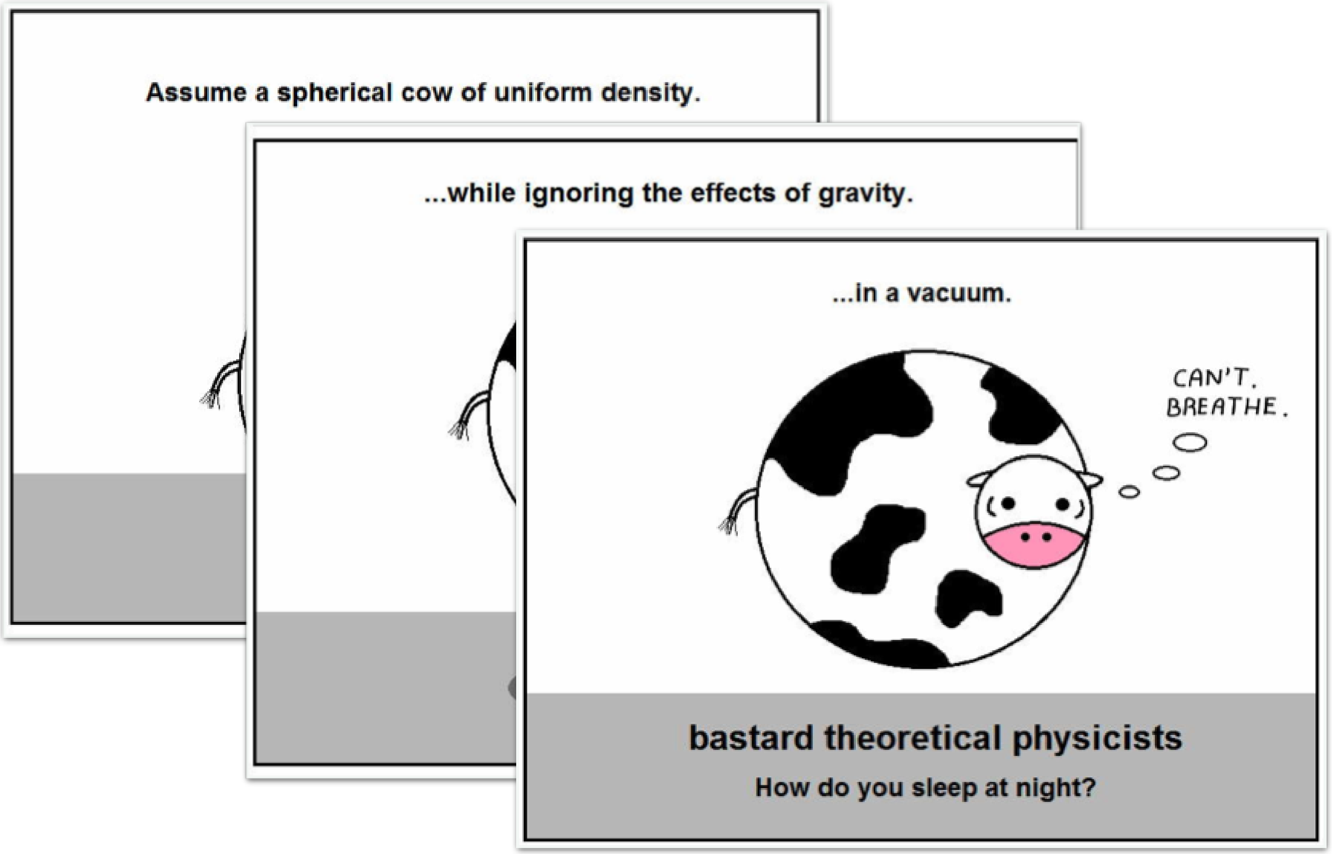
Translation and rotation are coupled

- Mobility matrix depends on particle shape, orientation and size only
- Key quantity in theory and simulations
- **What about many „particles“ and their hydrodynamic motion coupling?**

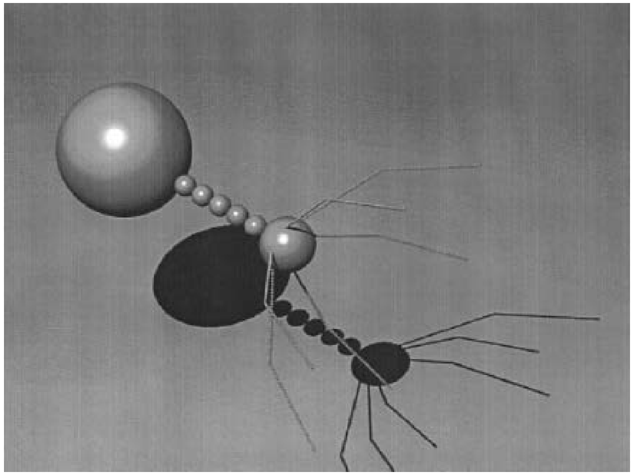
2. Hydrodynamic interactions



Too complicated? Make it spherical!

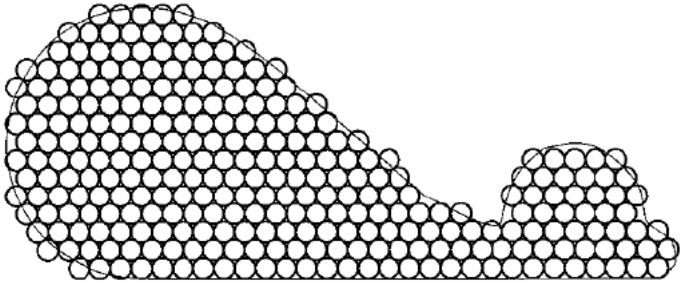


Bead modeling of complex – shaped particles



T2 – bacteriophage model

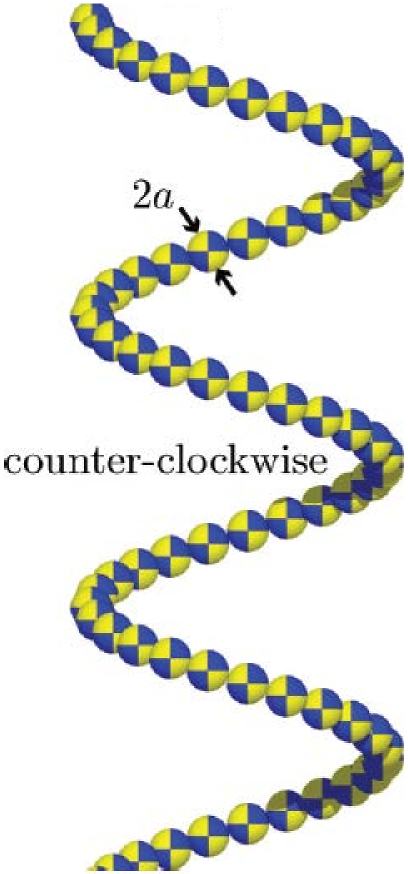
(V. Bloomfield, Biopolymers 5 (1967))



B. Carrasco & **J. Garcia de la Torre**,

Biophysical J. **75** (1999)

HYDROPRO packages



Stokesian Dynamics simulation

J.W. Swan et al.

Phys. Fluids **23** (2011)

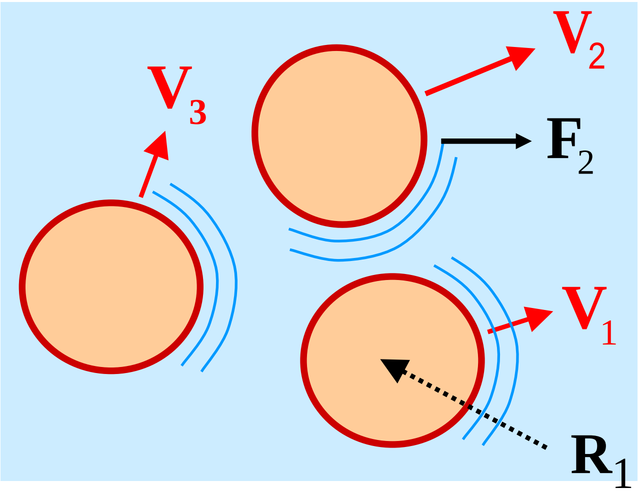
N freely rotating spheres

$$\mathbf{V}_\mathbf{i} = \sum_{\mathbf{j}=1}^{N} \boldsymbol{\mu}_{ij}^{tt}(X) \cdot \mathbf{F}_\mathbf{j}$$

3×3 mobility tensors

$$V = \mu^{tt}(X) \cdot F$$

mobility problem



- **Mobility matrix depends on configuration X**
- **Long-ranged ~ 1/r**
- **Many – body character**
- **Positive definiteness is crucial →**

Approximations

$$X = \{\mathbf{R}_1, \ldots, \mathbf{R}_N\}$$

$$\mathbf{V} = (\mathbf{V}_1, \ldots, \mathbf{V}_N)$$

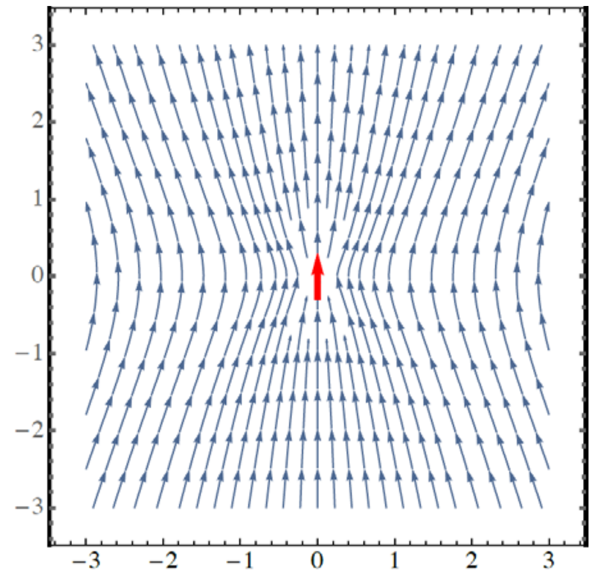
$$\mu^{tt} = \begin{pmatrix} \boldsymbol{\mu}_{11}^{tt} & \cdots & \boldsymbol{\mu}_{1N}^{tt} \\ \vdots & & \vdots \\ \boldsymbol{\mu}_{N1}^{tt} & \cdots & \boldsymbol{\mu}_{NN}^{tt} \end{pmatrix}$$

3N x 3N matrix

Mobility approximations

- **Point particles approximation:**

$$\mathbf{V}_i = \mu_i^t \mathbf{F}_i + \mathbf{u}_{inc}^{(i)}(\mathbf{r}) = \mu_i^t \mathbf{F}_i + \sum_{j \neq i} \mathbf{T}(\mathbf{R}_i - \mathbf{R}_j) \cdot \mathbf{F}_j$$



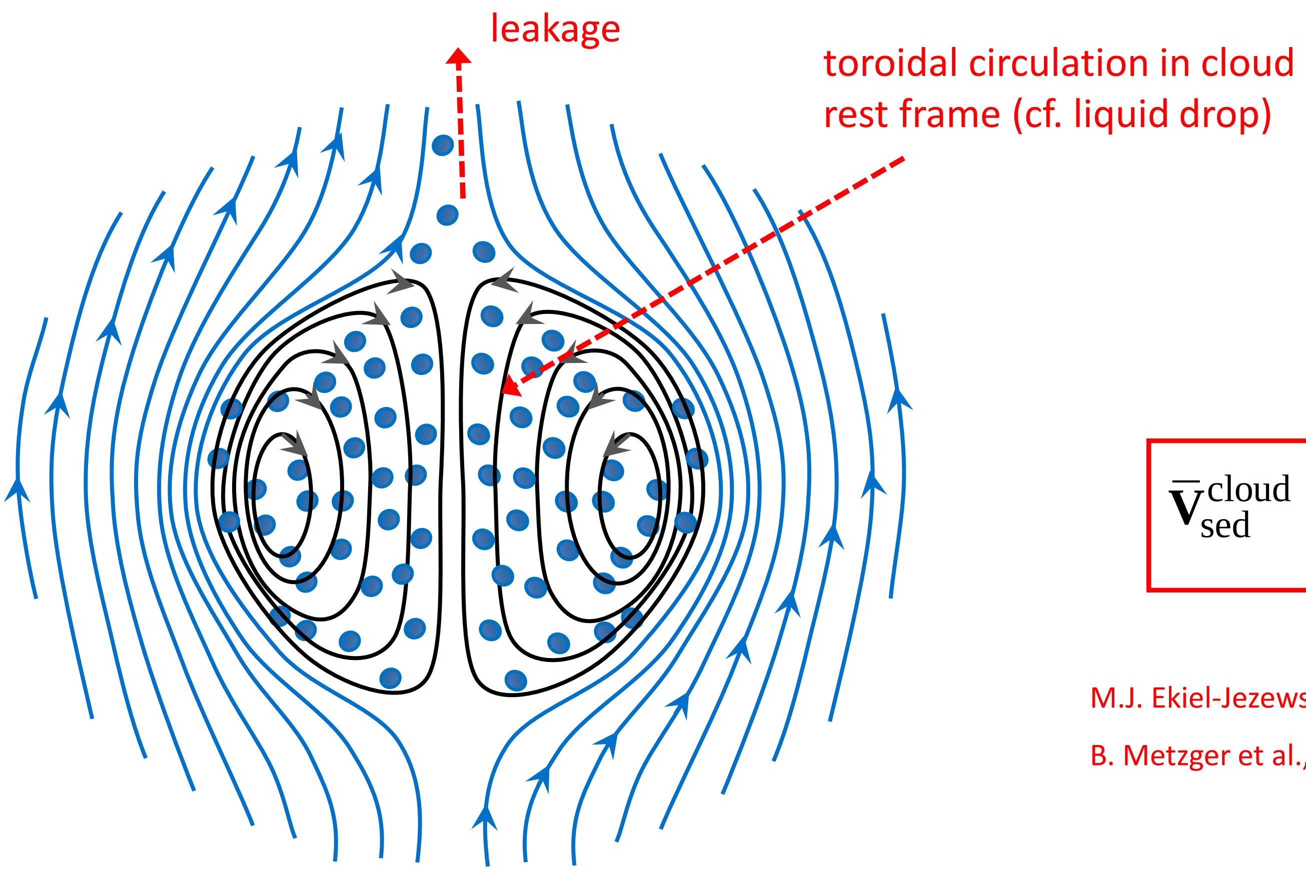
  – Reflects far – distance behavior only
  – Can violate positve definiteness for close particles

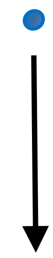
- **Rotne – Prager approximation:**
  – Far – distance only, but positive definiteness preserved (see notes)

- **Full hydrodynamic interactions (e.g. HYDROMULTIPOLE code)**

  → **numerically expensive**

Sedimentation of spherical cloud of non - Brownian beads





$$\mathbf{V}_{\text{sed}}^{0} = \mu_0^{t}\,\mathbf{F}$$

$$\bar{\mathbf{V}}_{\text{sed}}^{\text{cloud}} \approx \mathbf{V}_{\text{sed}}^{0} + \frac{N-1}{5\pi\eta_0 R}\,\mathbf{F}$$

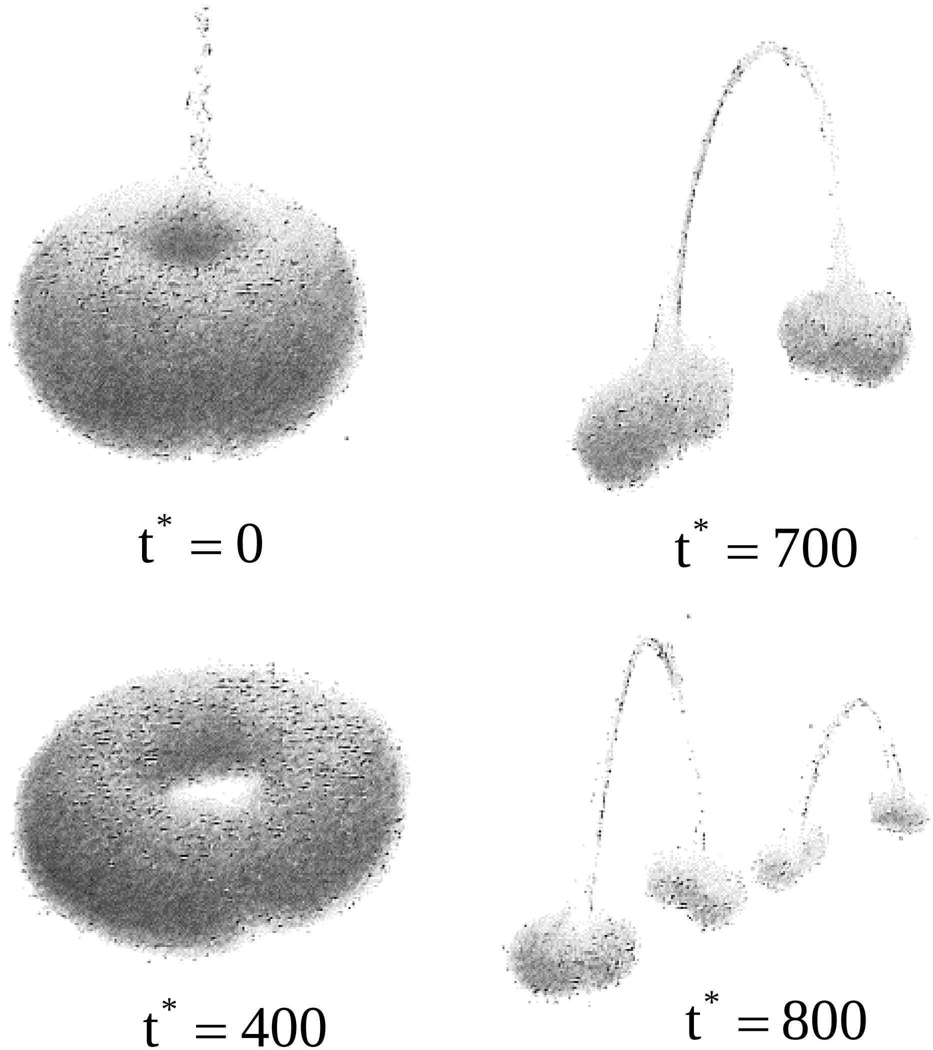
M.J. Ekiel-Jezewska, Phys. Fluids **18** (2006),

B. Metzger et al., J. Fluid Mech. **580,** 238 (2007)

- Cloud sediments faster than single bead (point model overestimates velocity)
- Instability for large N and large settling times

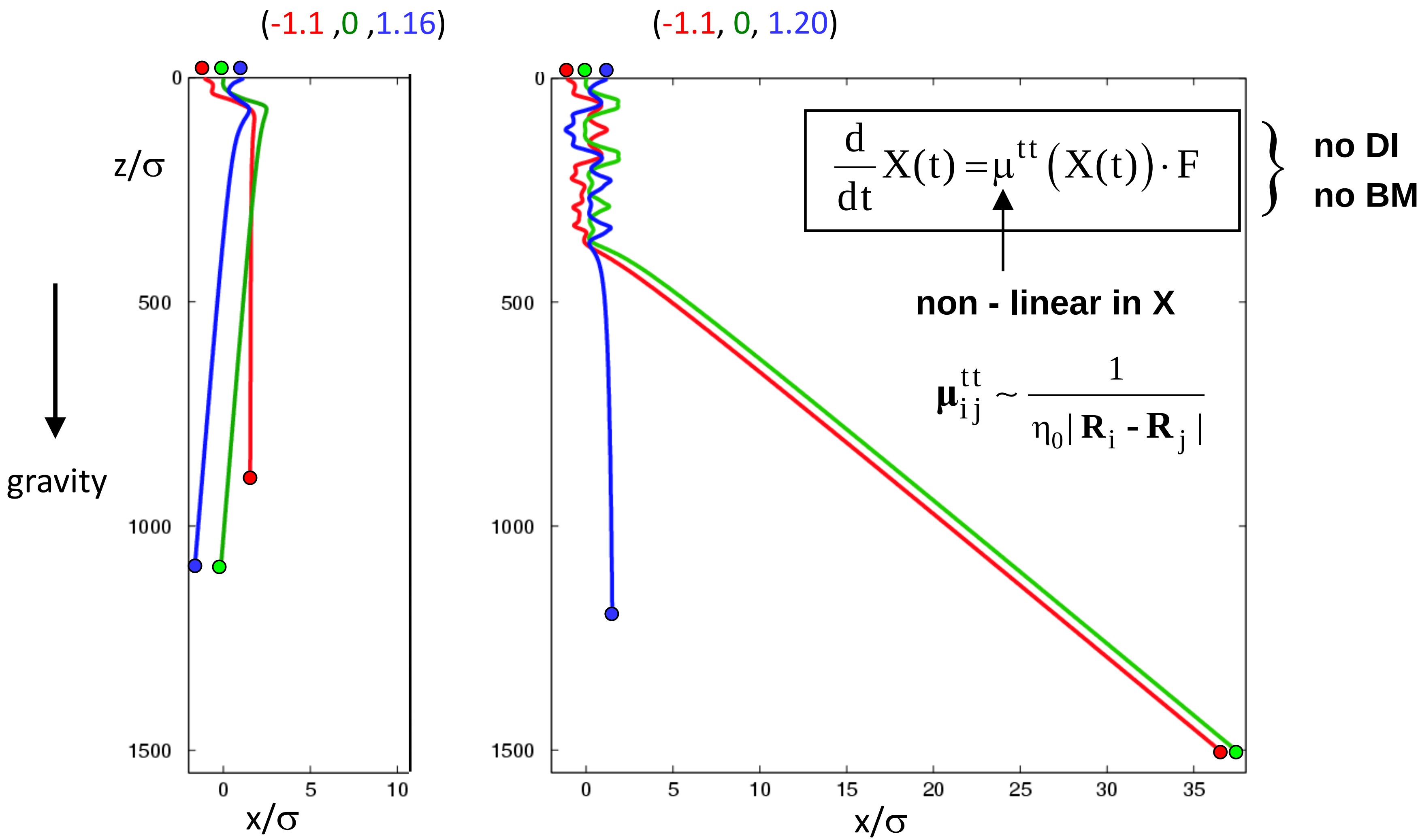
- Evolution: spherical cloud → torus → breakup in two clouds → …

$$t^* = t / \tau_R^{sed}$$



E. Guazzelli and J.F. Morris, *A Physical Introduction to Suspension Dynamics*, Cambridge (2012)

Reason: Chaotic fluctuations due to many - body HI

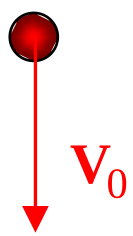


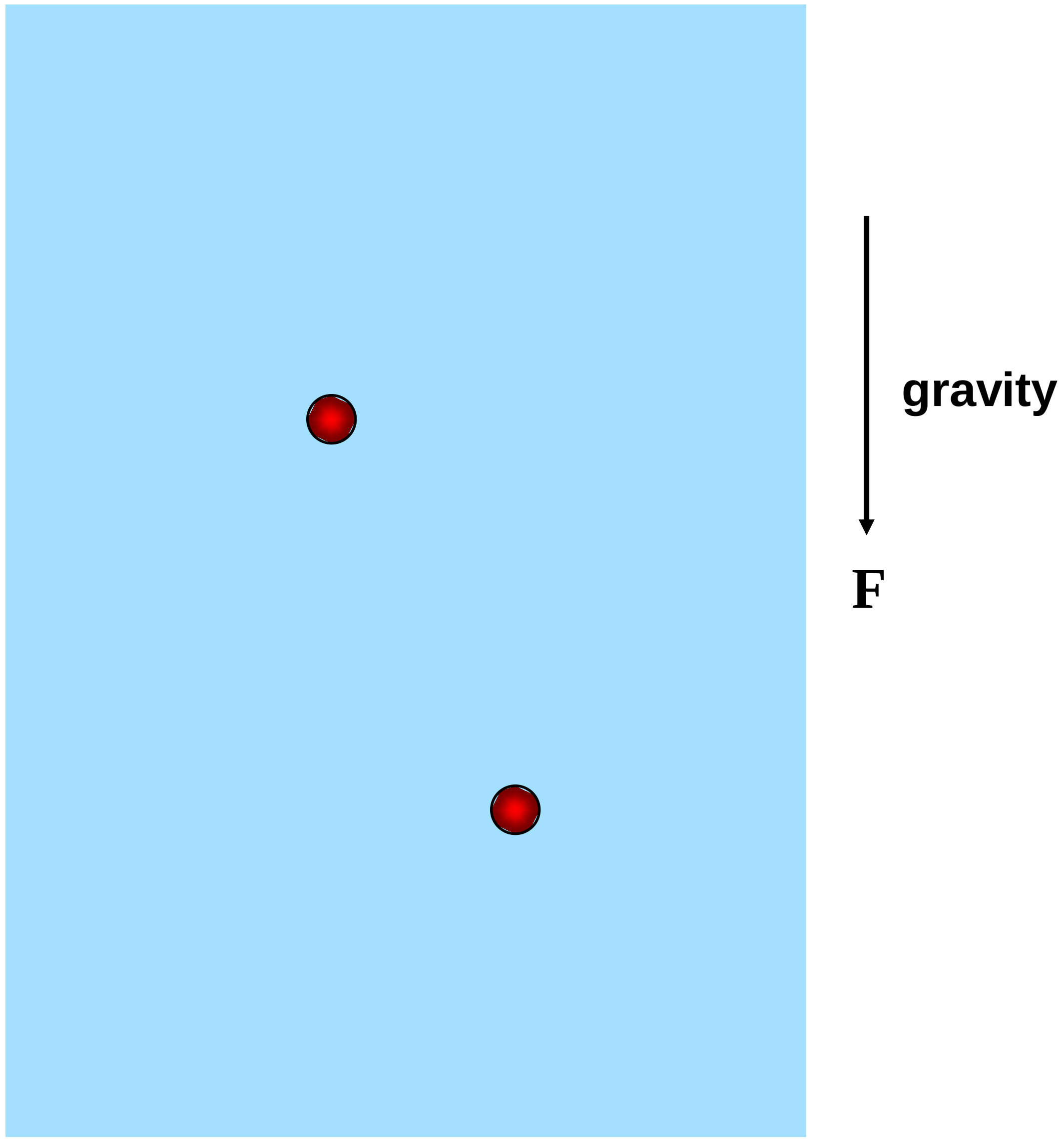
- Sensitive dependence on initial configuration for N > 2 $\rightarrow$ **deterministic chaotic trajectories**

Courtesy: M. Ekiel-Jezewska & E. Wajnryb, Phys. Rev. E **83**, 067301 (2011)

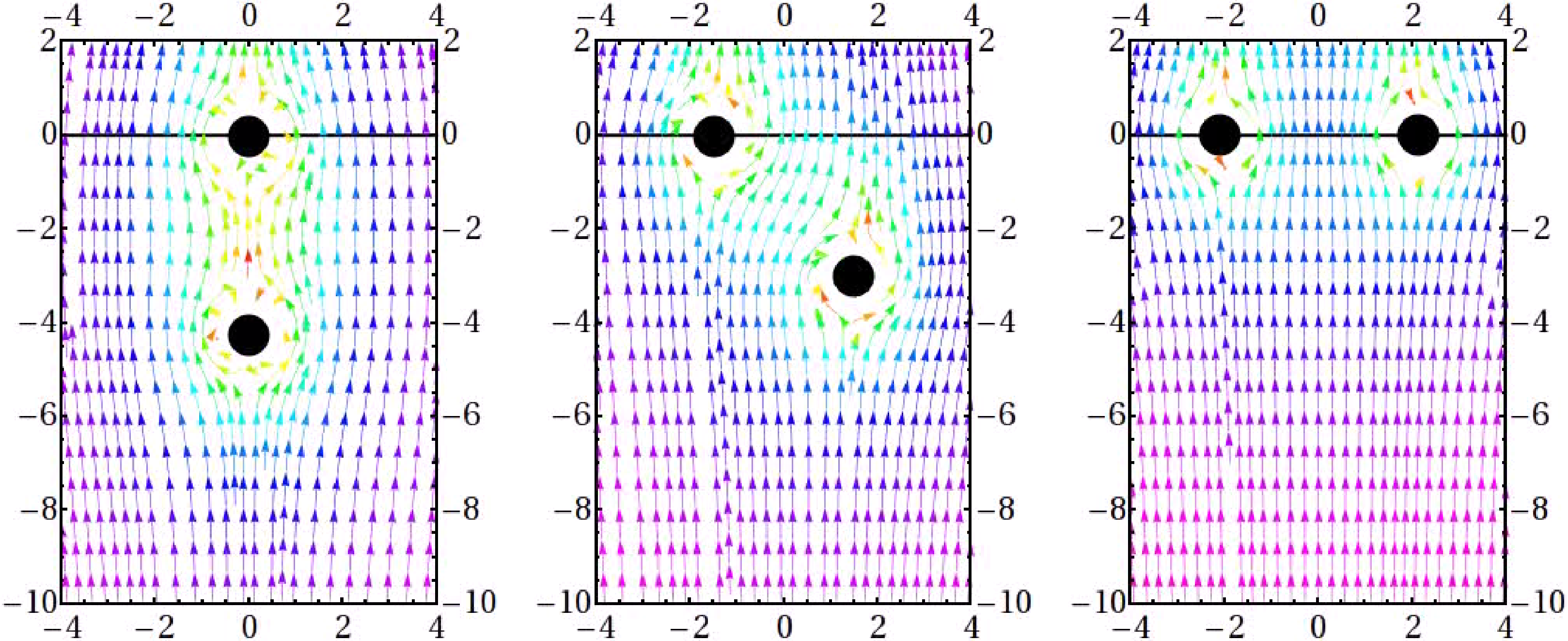
To relax: just two non - Brownian microspheres sedimenting

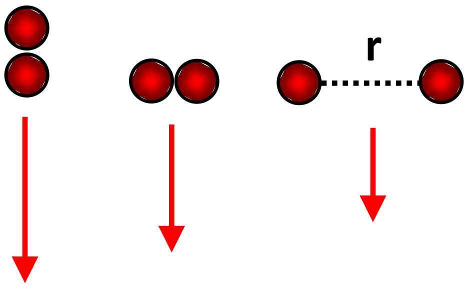
$$\mathbf{V}_0 = \mu_0^t \mathbf{F}$$





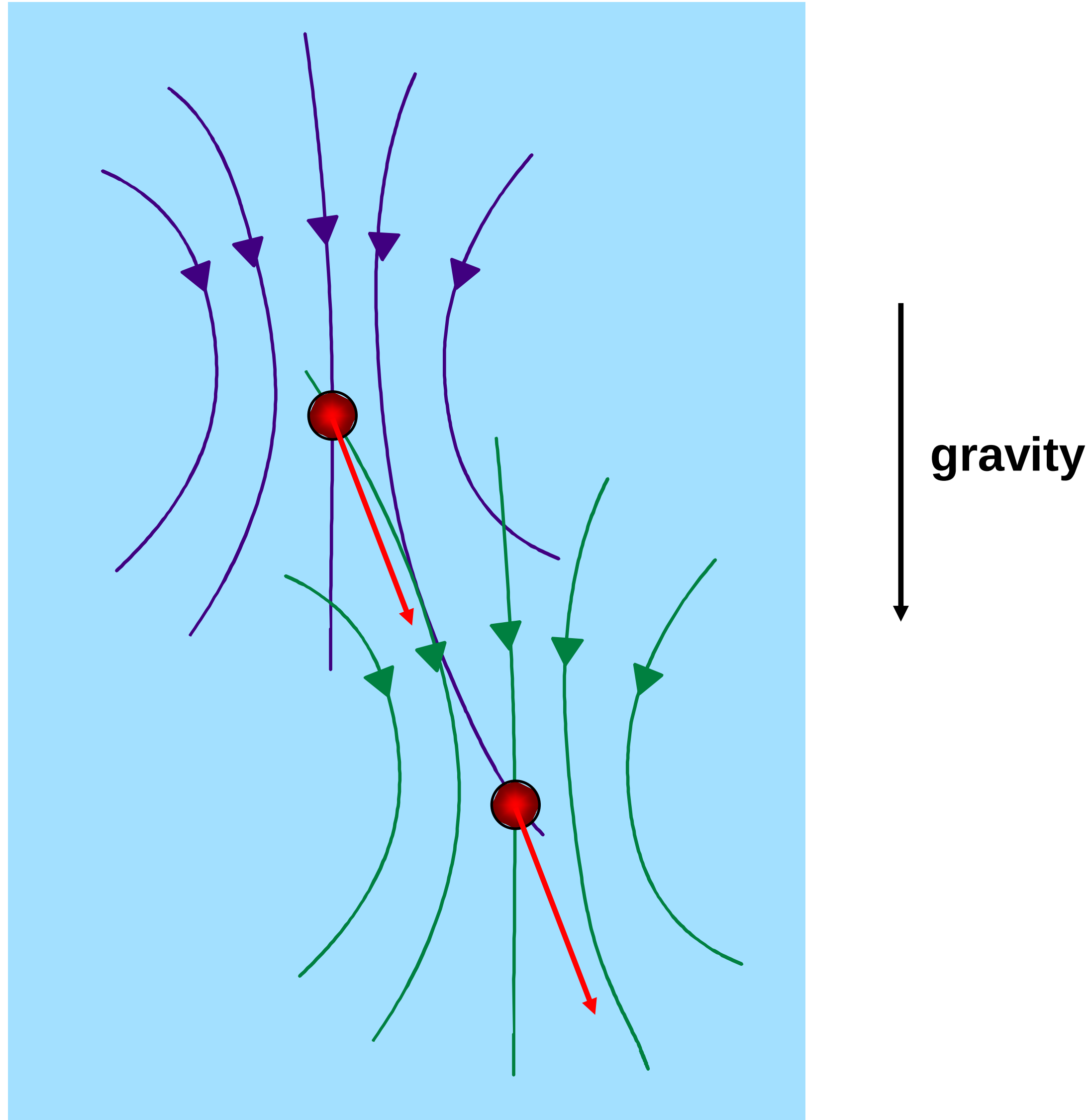
The sedimentation race: bet which pair settles fastest, and how ?



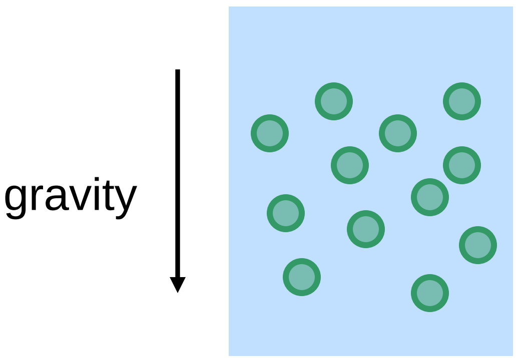


constant sep. vector **r**

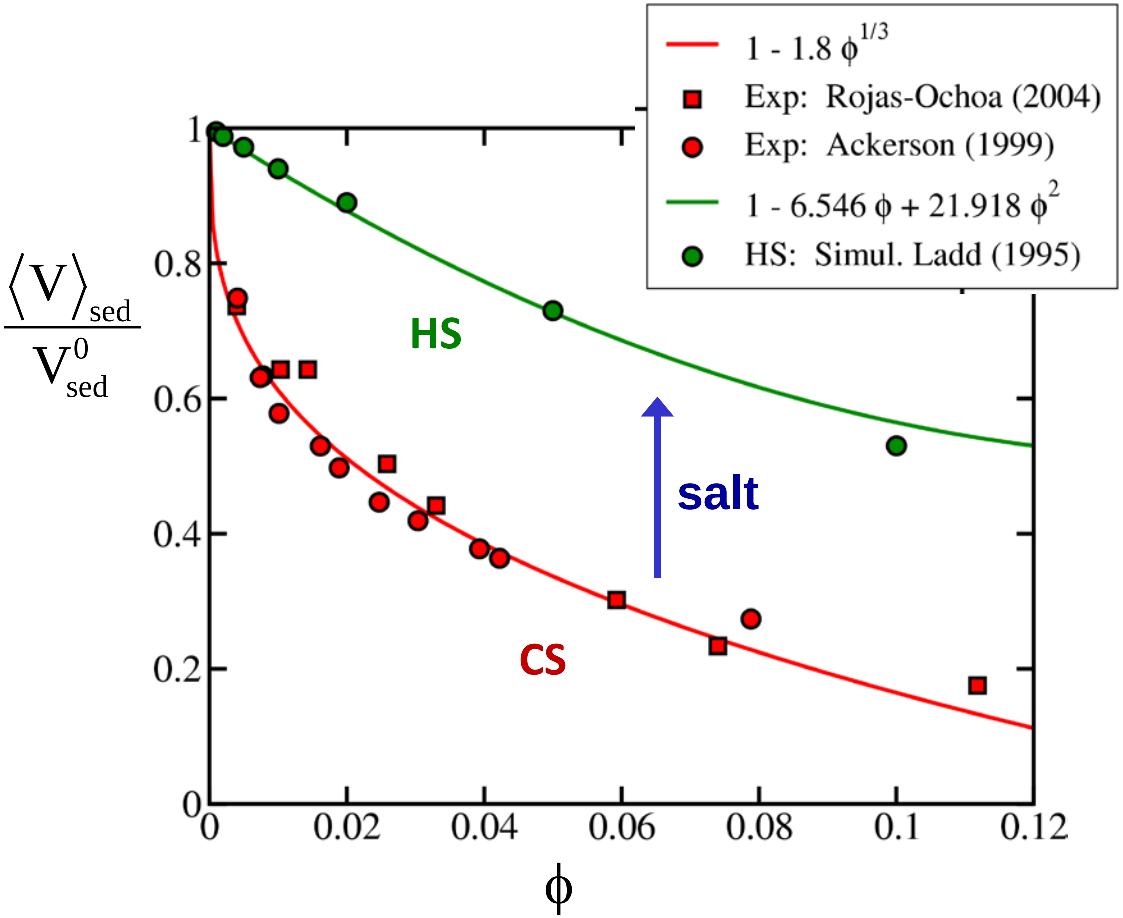
$$V_{sed}^{pair} > V_{sed}^{0} = \mu_0^t \, \mathbf{F}$$



Sedimentation in homogeneous suspension



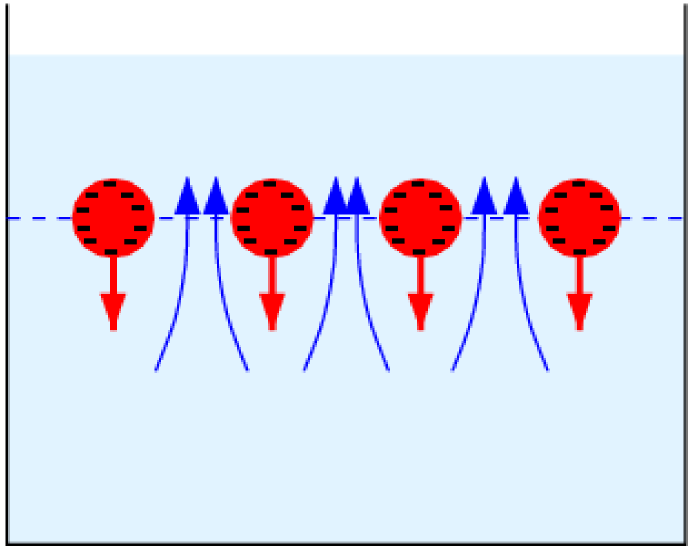
$$\langle V \rangle_{sed} / V^0_{sed} \approx 1 - 1.8\phi^{1/3}$$

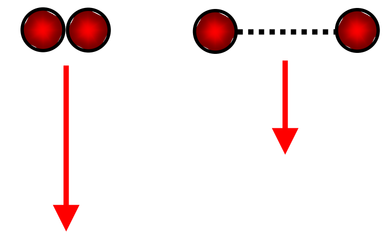


- Suspension sedimentation velocity smaller than that of isolated particle
- Slower sedimentation of charged clay particles **(river - delta)**

Banchio & Nägele, JCP **127**, 2008

Plausibility argument

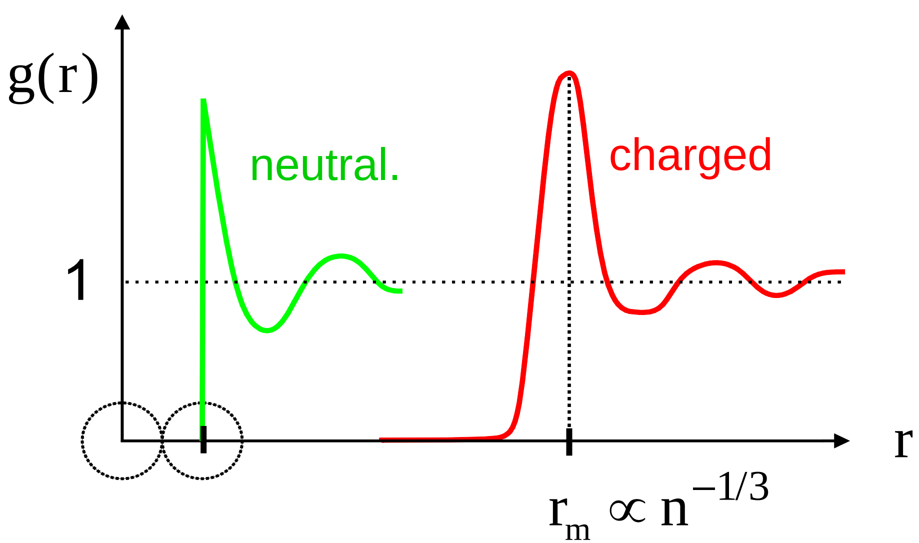




$$\nabla \langle p \rangle_{sa} = n\,\mathbf{F}$$

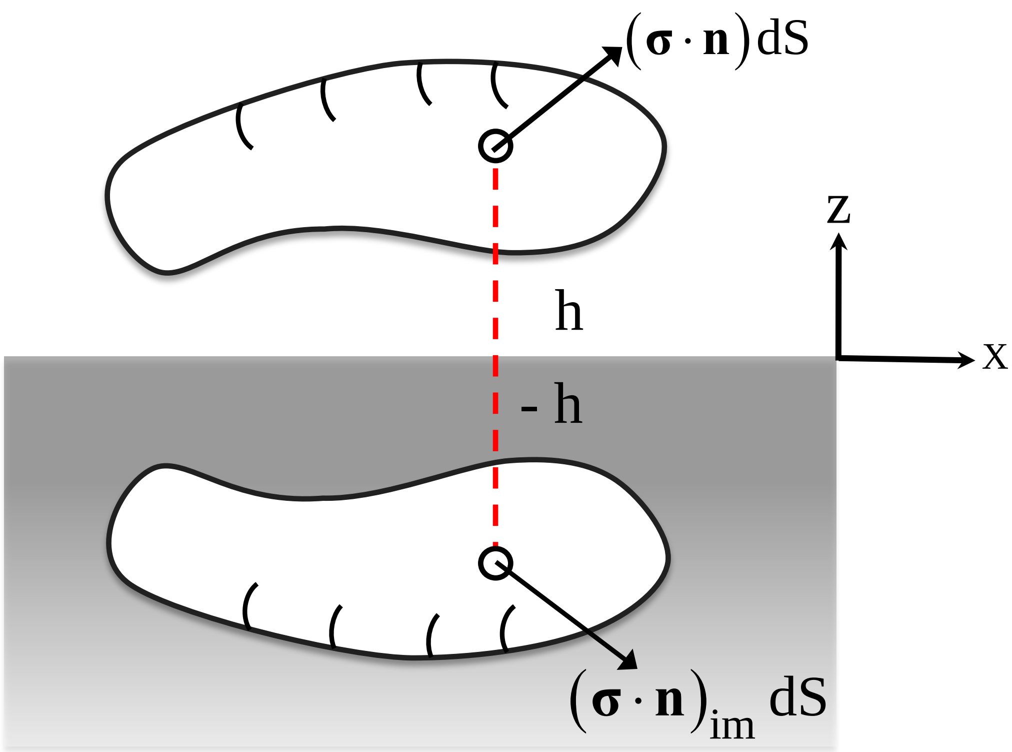
- Pressure gradient drives mean fluid backflow
- **Bottom wall important**

- Increased friction with backflowing fluid



3. Particle near an interface

Influence of free planar surface: method of images



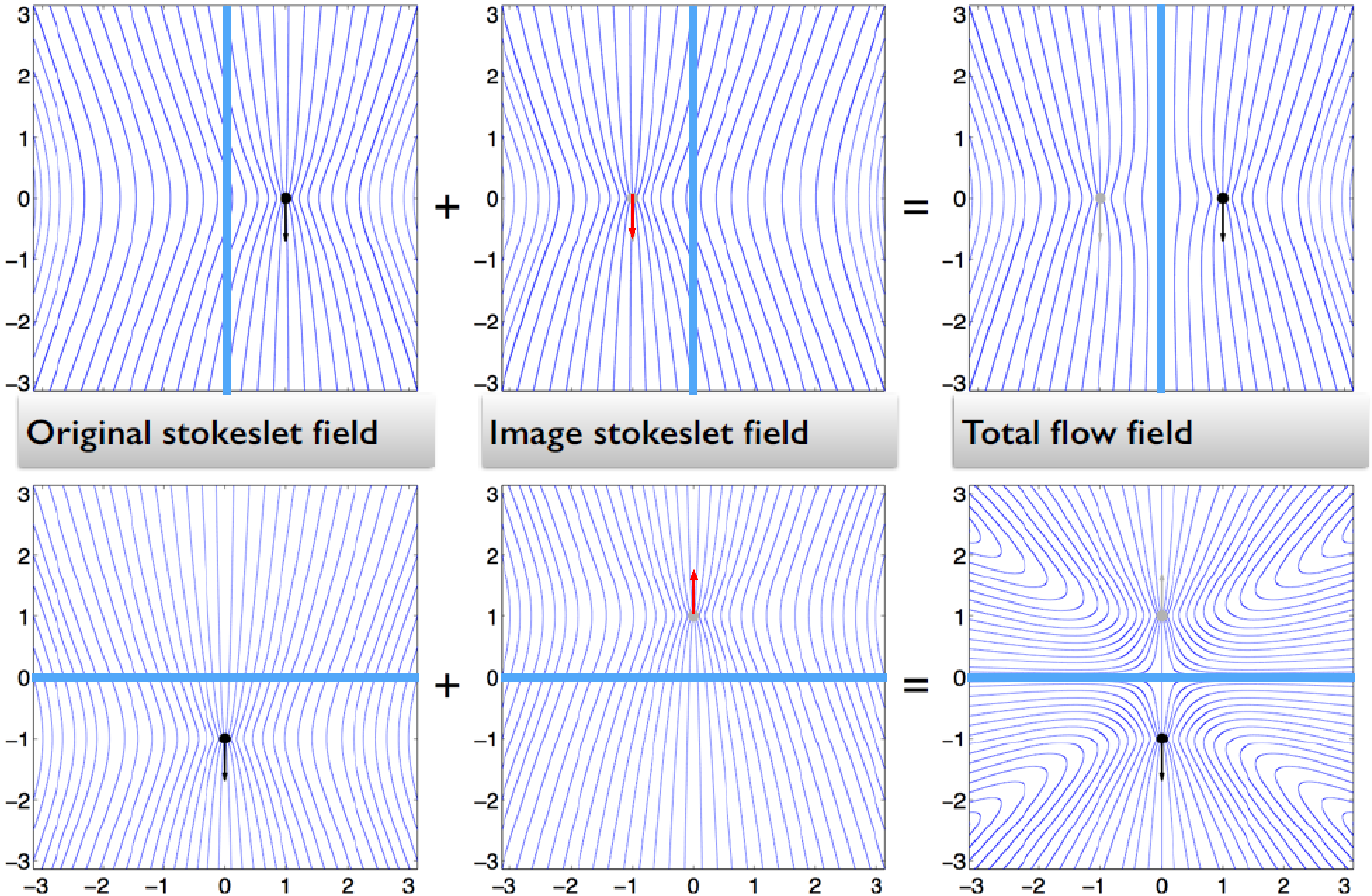
$$\frac{du_{x,y}(z=0)}{dz}=0$$

$$u_z(z=0)=0$$

- Surface superposition of image Stokeslets

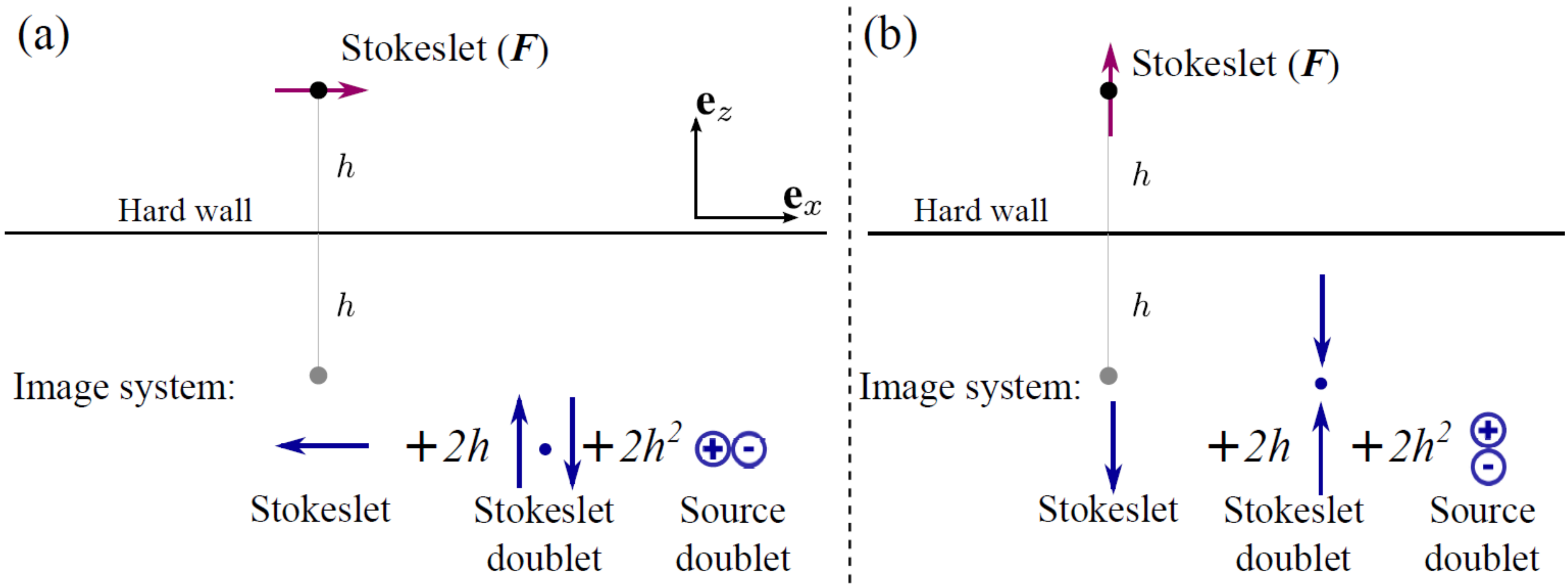
$$\mathbf{u}(\mathbf{r}) = -\int_S dS'\,\mathbf{T}(\mathbf{r}-\mathbf{r}')\cdot[\boldsymbol{\sigma}\cdot\mathbf{n}](\mathbf{r}') - \int_{S_{im}} dS'\,\mathbf{T}(\mathbf{r}-\mathbf{r}')\cdot[\boldsymbol{\sigma}\cdot\mathbf{n}]_{im}(\mathbf{r}')$$

Point particle (Stokeslet) near a free surface



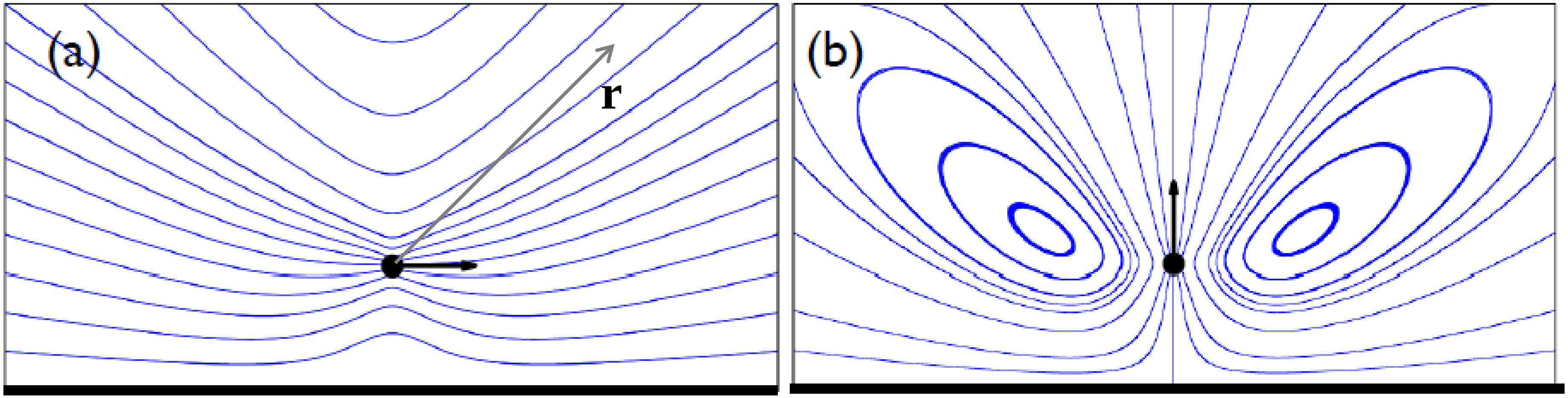
M. Ekiel-Jeżewska, R. Boniecki, ML

Stokeslet near a no-slip wall (I)



- To enforce zero tangential slip: more complex image system required
- Faster than O(1/r) decay for $z \rightarrow \infty$: momentum taken out from system (**screening**)

Stokeslet near a no-slip wall (II)



$$\mathbf{u}(\mathbf{r}) \sim O(r^{-2}), \quad z \to +\infty \qquad\qquad \mathbf{u}(\mathbf{r}) \sim O(r^{-3})$$

- No-slip particle near a no-slip wall:

$$\mathbf{u}(\mathbf{r}) = -\int_S dS' \, \mathbf{T}(\mathbf{r}-\mathbf{r}') \cdot [\boldsymbol{\sigma}\cdot\mathbf{n}](\mathbf{r}') \; - \int_{S_{im}} dS' \, \mathbf{T}_{im}(\mathbf{r}-\mathbf{r}') \cdot [\boldsymbol{\sigma}\cdot\mathbf{n}]_{im}(\mathbf{r}')$$

Hydrodynamic screening

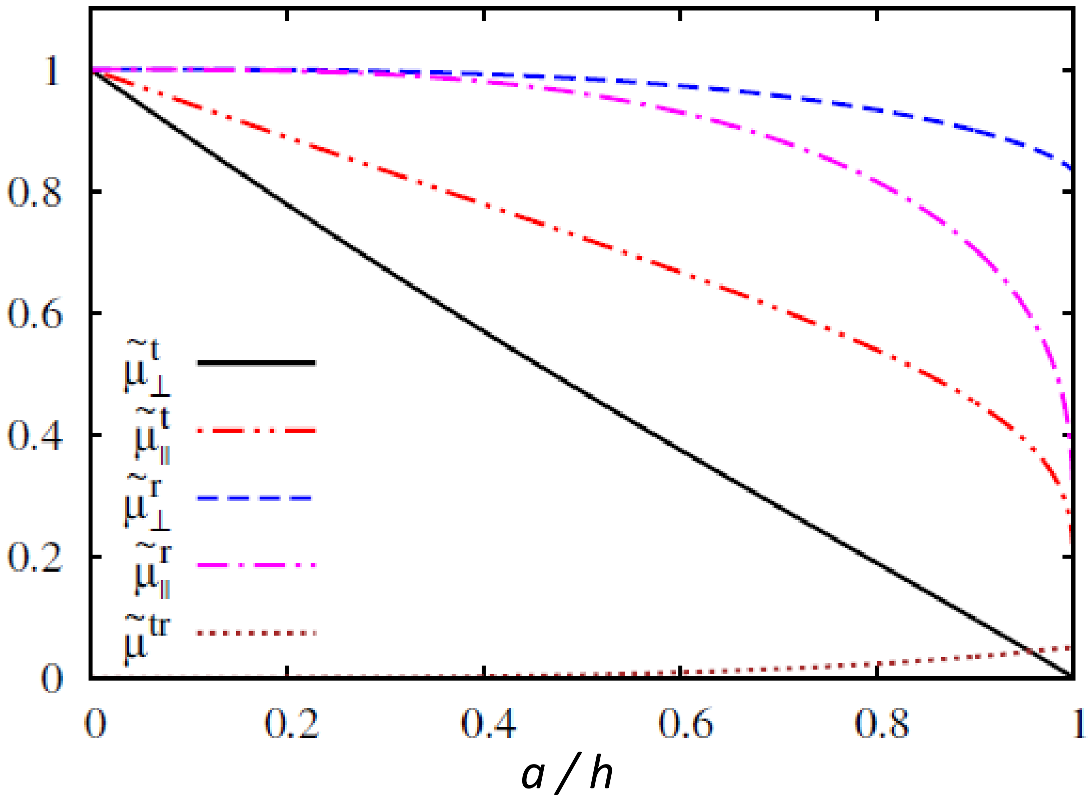
- Long-range forced flow decay can be screened by fixed obstacles in the flow
- Examples: flow due to a point force parallel to walls / perpendicular to axis:

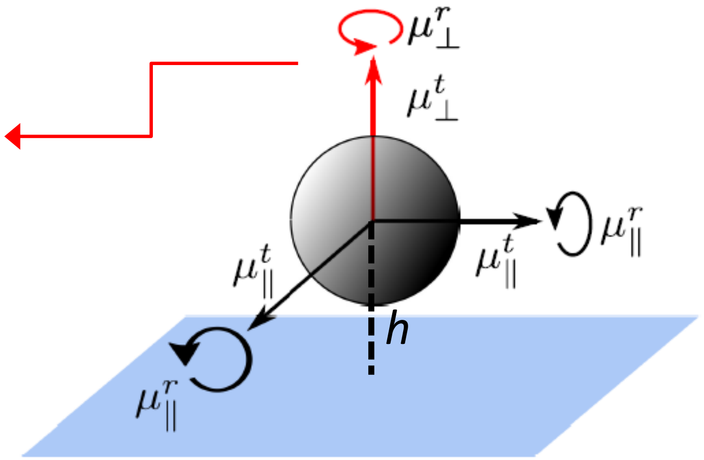
Obstacles	**Flow asymptotics**
1. cylindrical pipe	$\sim e^{-\alpha r}$
2. two parallel walls	$\sim 1/r^2$
3. electrophoresis	$\sim 1/r^3$

A.J. Banchio, G. Nägele et al., Phys. Rev. Lett. **96** (2006)

H. Diamant, Israel Journal of Chemistry **47** (2007)

Anisotropic mobilities of sphere close to wall





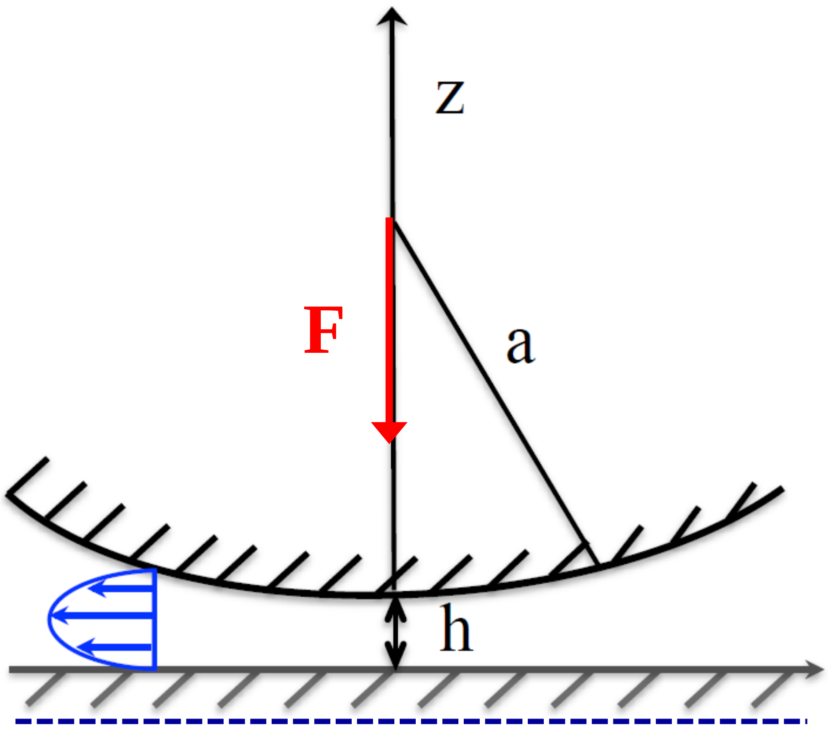
Perkins & Jones, Physica A 189 (1992)

Cichocki & Jones, Physica A 258 (1998)

Experimental verification: Holmqvist, Dhont, Lang, JCP 128 (2007)

- Only rotational mobility for vertical axis non-zero at contact → **Lubrication**

Lubrication: squeezing motion of no-slip sphere towards no-slip wall



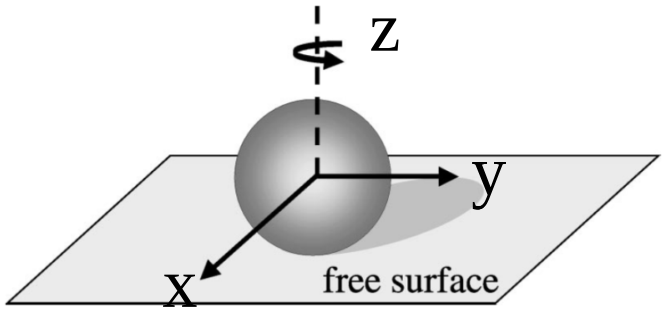
$$l_{\text{rough}} < h < 0.01a$$

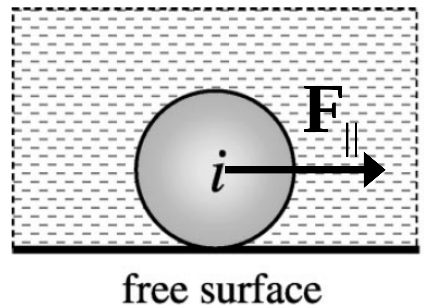
$$\frac{dh}{dt} = \mu_{\text{rel}}(h)F \approx -\frac{\mu_0^t |F|}{a} h$$

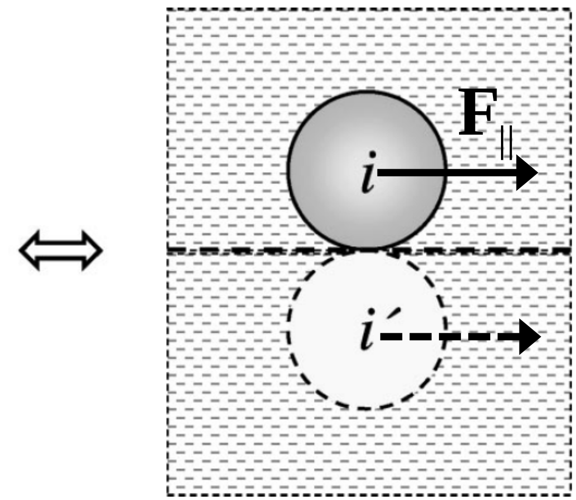
$$h(t) \approx h_0 \exp\left\{-\left(\frac{\mu_0^t |F|}{a}\right)t\right\}$$

- Strong pressure gradient inside gap determines dynamics (viscous stress negligible)
- **Finite contact time** due to: surface roughness, van der Waals attraction, …

In-touch motion along a planar free interface







$$\Omega_\perp = \mu^r_\perp \mathbf{T}_\perp$$

$$V_{||} = \mu^t_{||} \mathbf{F}_{||}$$

$$\mu^r_\perp / \mu^r_0 \approx 1.109$$

$$\mu^t_{||} / \mu^t_0 \approx 1.380$$

- Experimental realization: Particles at water – air interface of hanging drop
- **Extension to Q2D system of interacting microspheres:**

Cichocki , Ekiel-Jezwska, **G. N.**, Wajnryb, JCP **121** (2004)

Apparent like - charge attraction of particles near boundary

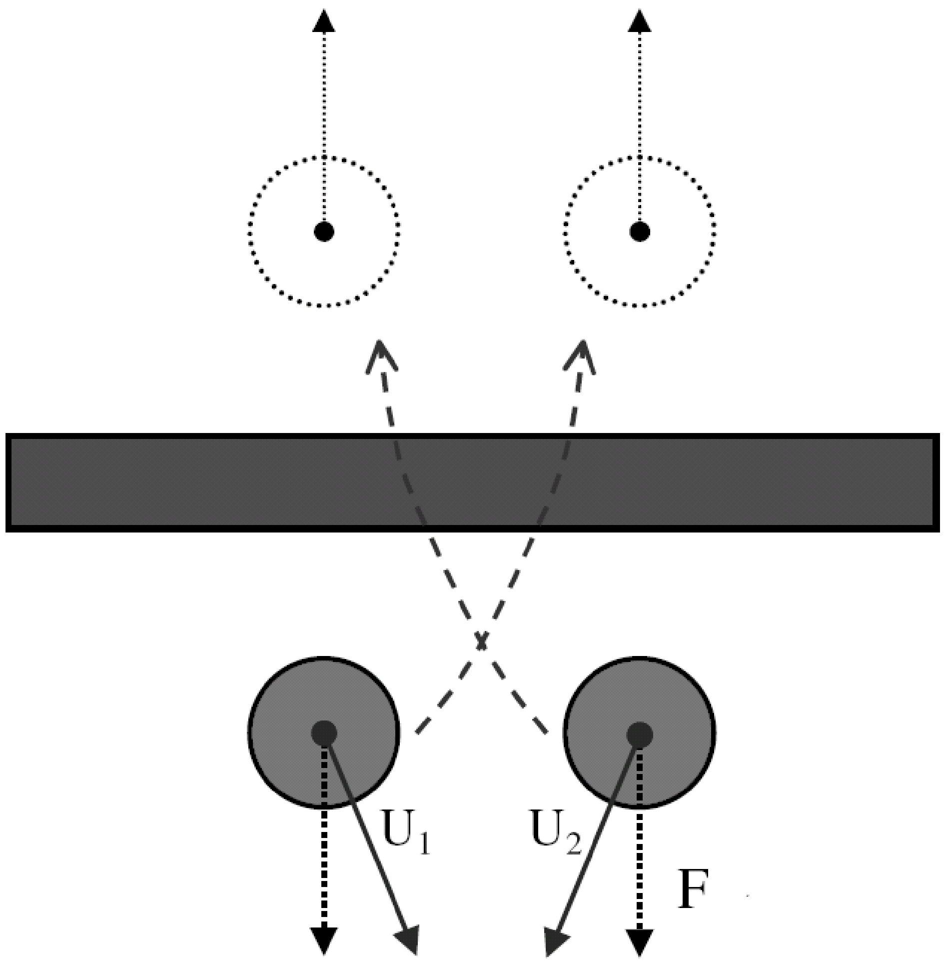


figure taken from:
Squires & Brenner, PRL (2000)

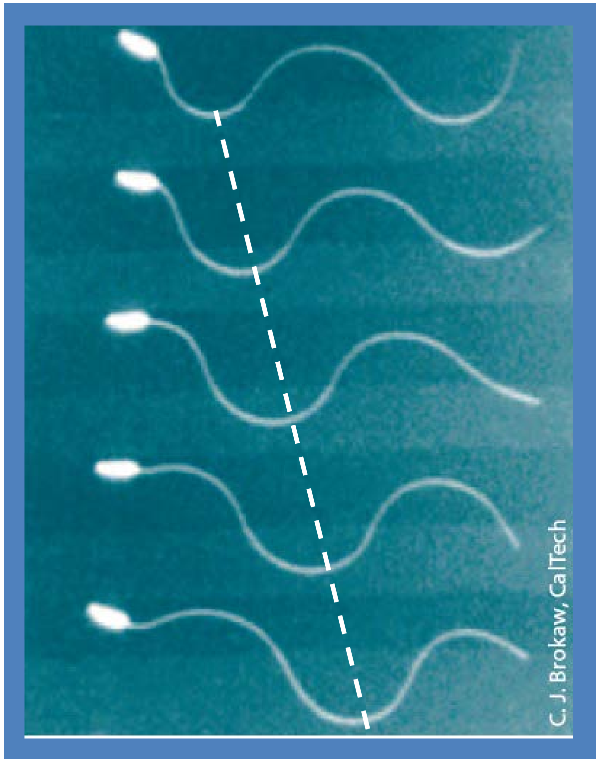
charged glass wall or
liquid - air interface

F → - F ?

- Attractive wall: apparent repulsion

4. Active swimmers near a surface

- sea urchin sperm

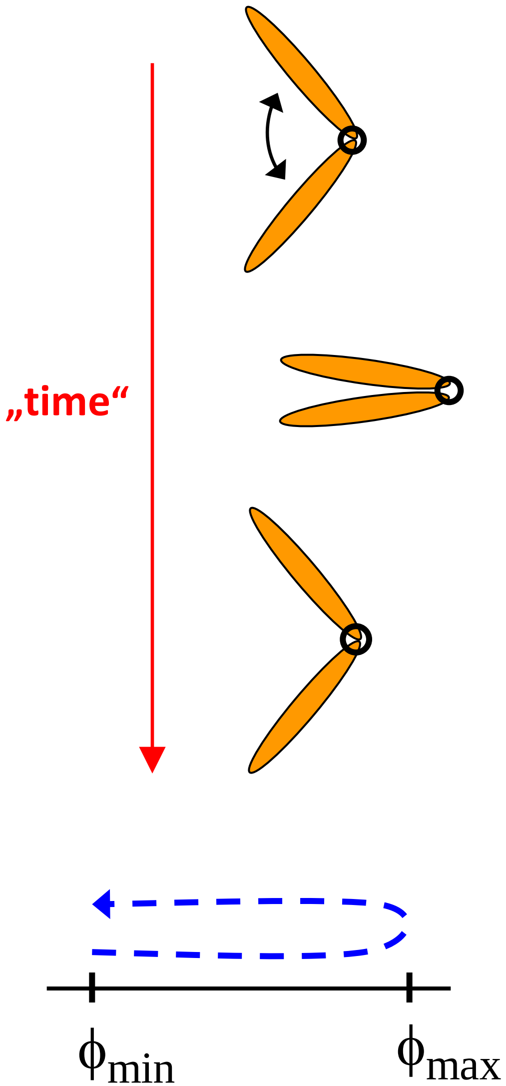


wave-like flagellum motion

- bacterium

rotating rigid flagellum

Purcell's scallop theorem for autonomous microswimmers



- **Internal forces and torques only:**

$$\mathbf{F}^{h} = 0 = -\mathbf{F}$$

$$\mathbf{T}^{h} = 0 = -\mathbf{T}$$

- **Purcell's microscallop theorem:**

  For net displacement after one shape cycle:

  - Non - reciprocal sequence of body deformations:
  - At least 2 - parametric deformations
  - Skew – symmetric flagellar rotation, e.g.

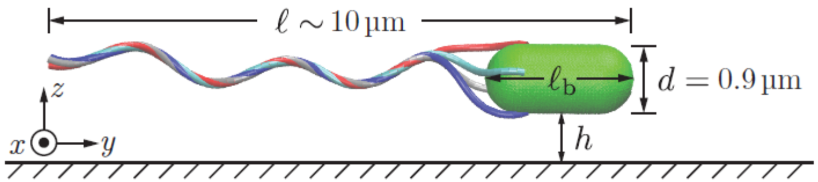
- **Additionally required: Friction asymmetry**

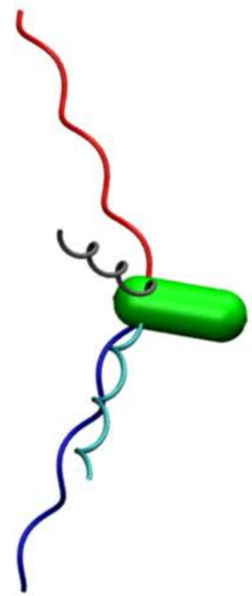
E.M. Purcell, *Life at low Reynold's number*, Am. J. Phys. **45**, 3 (1977)

Re << 1: microrganism in water or macroscopic swimmer in glycerin

G.I. Taylor, Cambridge
https://web.mit.edu/hml/ncfmf.html

MPC simulation of swimming **E. coli** in bulk (full HI and Brownian motion included)



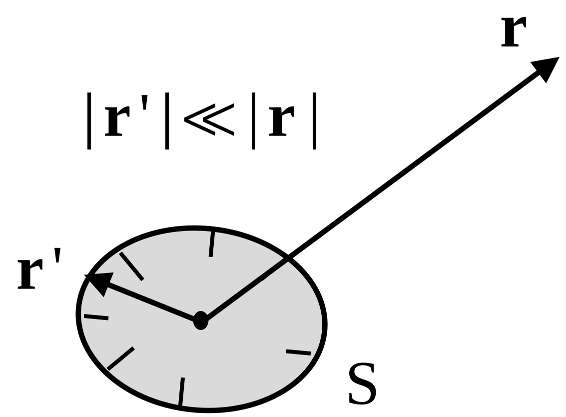


Courtesy: R. G. Winkler, J. Hu and G. Gompper, FZ Juelich, work submitted (2014)

Far – distance flow field around a no - slip particle

- Expand around inside point:

$$\mathbf{u}(\mathbf{r}) = -\int_S dS' \left[\mathbf{T}(\mathbf{r}) - \mathbf{r}'\cdot\nabla\mathbf{T}(\mathbf{r}) - \dots\right]\cdot\left[\boldsymbol{\sigma}\cdot\mathbf{n}\right](\mathbf{r}')$$



- Split in symmetric and anti-symmetric parts:

$$\mathbf{u}(\mathbf{r}) \approx -\mathbf{T}(\mathbf{r})\cdot\mathbf{F}^h + \frac{1}{8\pi\eta_0 r^2}\hat{\mathbf{r}}\times\mathbf{T}^h - \frac{1}{8\pi\eta_0 r^2}\left(\hat{\mathbf{r}}\mathbf{1} - 3\hat{\mathbf{r}}\hat{\mathbf{r}}\hat{\mathbf{r}}\right):\mathbf{S}^h + O(r^{-3})$$

- Freely mobile particle (force- and torque-free): $\mathbf{F}^h = 0 = \mathbf{T}^h$ **autonomous microswimmer**

- Creates O(r^{-2}) flow disturbance by its **symmetric force dipole**

$$\mathbf{S}^h = \frac{1}{2}\int_S dS'\left[\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{r}')\mathbf{r}' + \boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{r}') - \frac{2}{3}\mathbf{1}\,\mathrm{Tr}\left(\mathbf{r}'\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{r}')\right)\right]$$ **surface moment tensor**

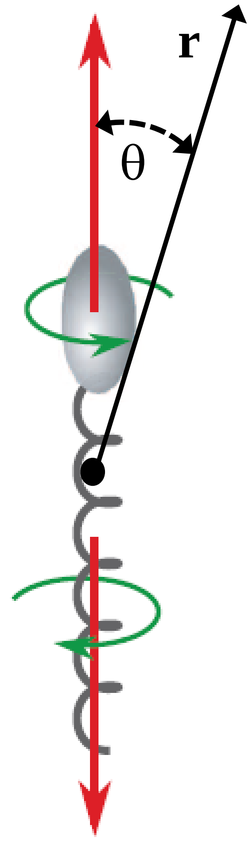
Symmetric force dipole (pusher: p > 0)

$$\mathbf{u}(\mathbf{r}) = \left[\mathbf{T}_0(\mathbf{r} - d\,\mathbf{e}_z) - \mathbf{T}_0(\mathbf{r} + d\,\mathbf{e}_z)\right] \cdot F\,\mathbf{e}_z$$

$$\mathbf{S}^h = p\left(\mathbf{e}_z\,\mathbf{e}_z - \frac{1}{3}\mathbf{1}\right)$$

$$p = 2Fd \sim \eta\,|V|\,L^2$$

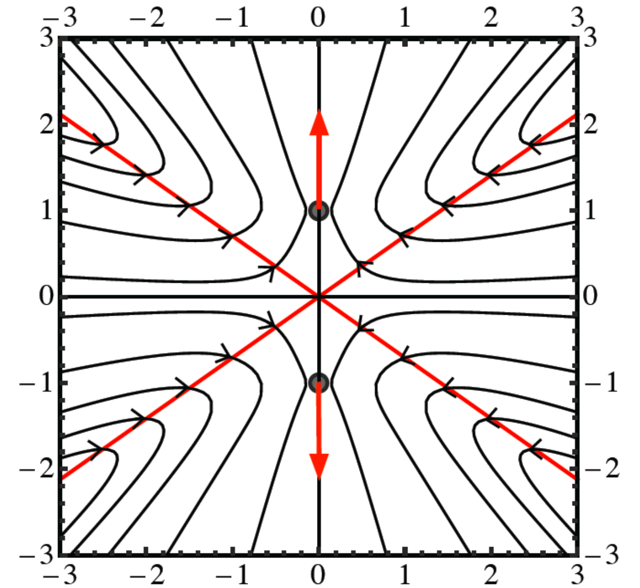
dipole moment

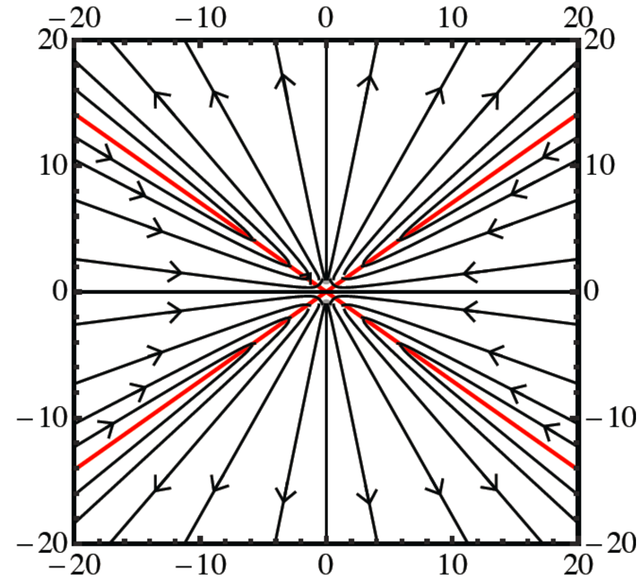


- Far – field flow : $\cos\theta = \mathbf{e}_z \cdot \hat{\mathbf{r}}$

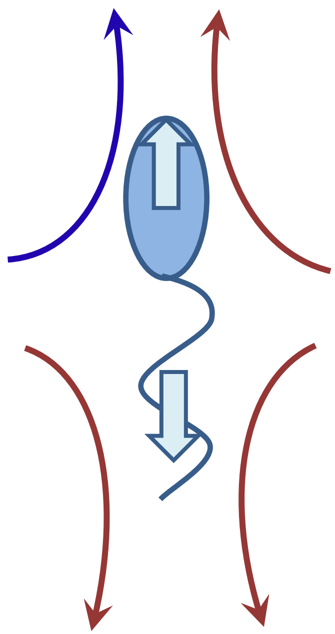
$$\mathbf{u}_D(\mathbf{r}) \sim \frac{p}{8\pi\eta_0\,r^2}\left[3\cos^2\theta - 1\right]\hat{\mathbf{r}}$$

- **Swimmer - created flow has form of symmetric force dipole for distances > L**





- **Pusher: p > 0**

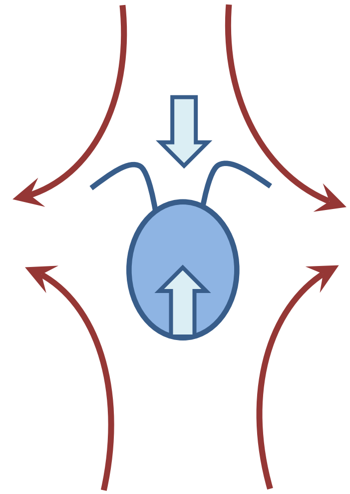


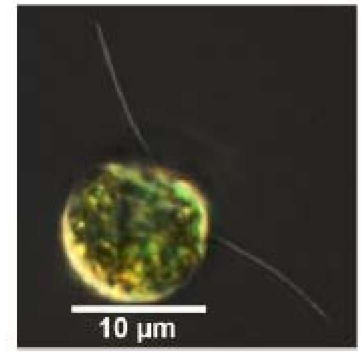
– E. Coli, **salmonella**, sperm, …

– Propelling part at rear

– Tend to attract each other.

- **Puller: p < 0**





Production stroke

– **Algae Chlamydomonas**, …

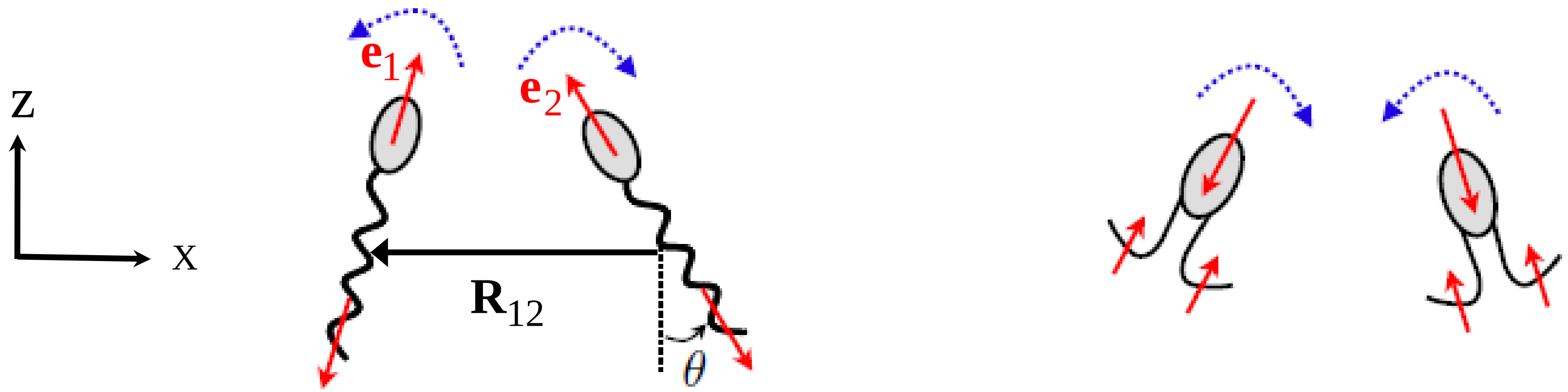
– Propelling part on head side

– Tend to repel each other („asocial“).

Hydrodynamic interaction of dipole swimmers

$$\mathbf{u}_1^{adv}(\mathbf{R}_1) = \mathbf{u}_D(\mathbf{R}_1 - \mathbf{R}_2; \mathbf{e}_2) = \frac{p}{8\pi\eta(R_{12})^2}\left[3\left(\hat{\mathbf{R}}_{12}\cdot\mathbf{e}_2\right)^2 - 1\right]\hat{\mathbf{R}}_{12}$$

$$\mathbf{u}_2^{adv}(\mathbf{R}_2) = \mathbf{u}_D(\mathbf{R}_2 - \mathbf{R}_1; \mathbf{e}_1)$$



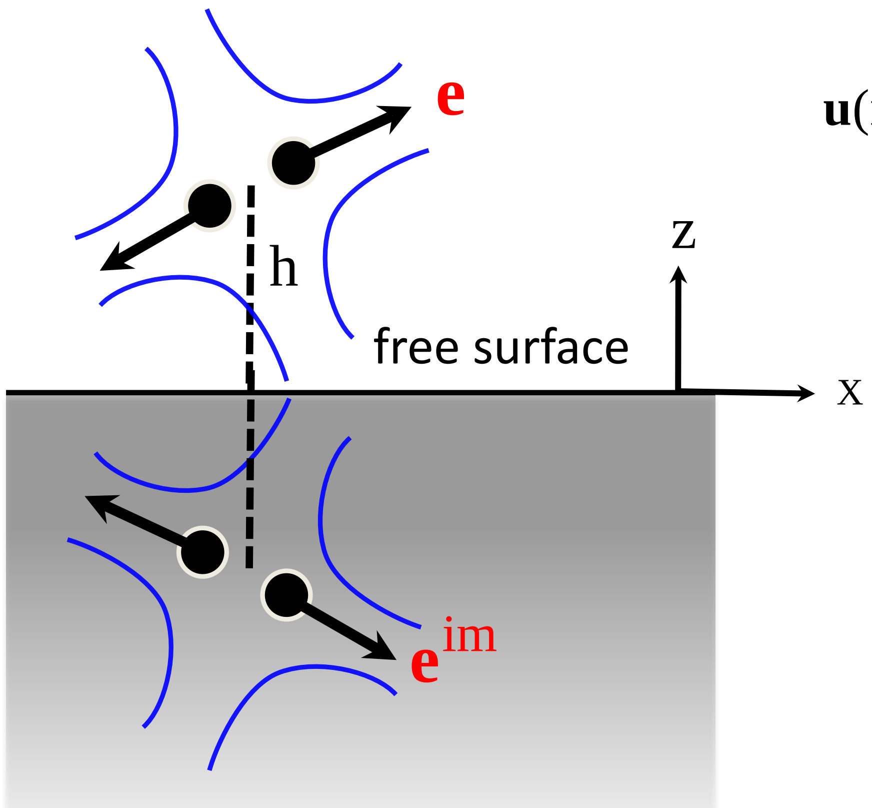
$$\boldsymbol{\Omega}_1^{adv}(\mathbf{R}_1) \approx \frac{1}{2}(\nabla\times\mathbf{u}_{inc})(\mathbf{R}_1) + \mathbf{e}_1\times\left(\mathbf{E}_{inc}(\mathbf{R}_1)\cdot\mathbf{e}_1\right)$$

Lauga & Powers

Rep. Prog. Phys. **72** (2009)

- Two pushers on not too diverging course tend to attract and reorient each other into parallel side-by-side motion
- Two pullers reorient antiparallel, swimming away from each other

Dipole swimmer near a surface



$$\mathbf{u}(\mathbf{r}) = \mathbf{u}_D(\mathbf{r} - \mathbf{R}_0; \mathbf{e}) + \mathbf{u}_D(\mathbf{r} - \mathbf{R}_0^{im}; \mathbf{e}^{im})$$

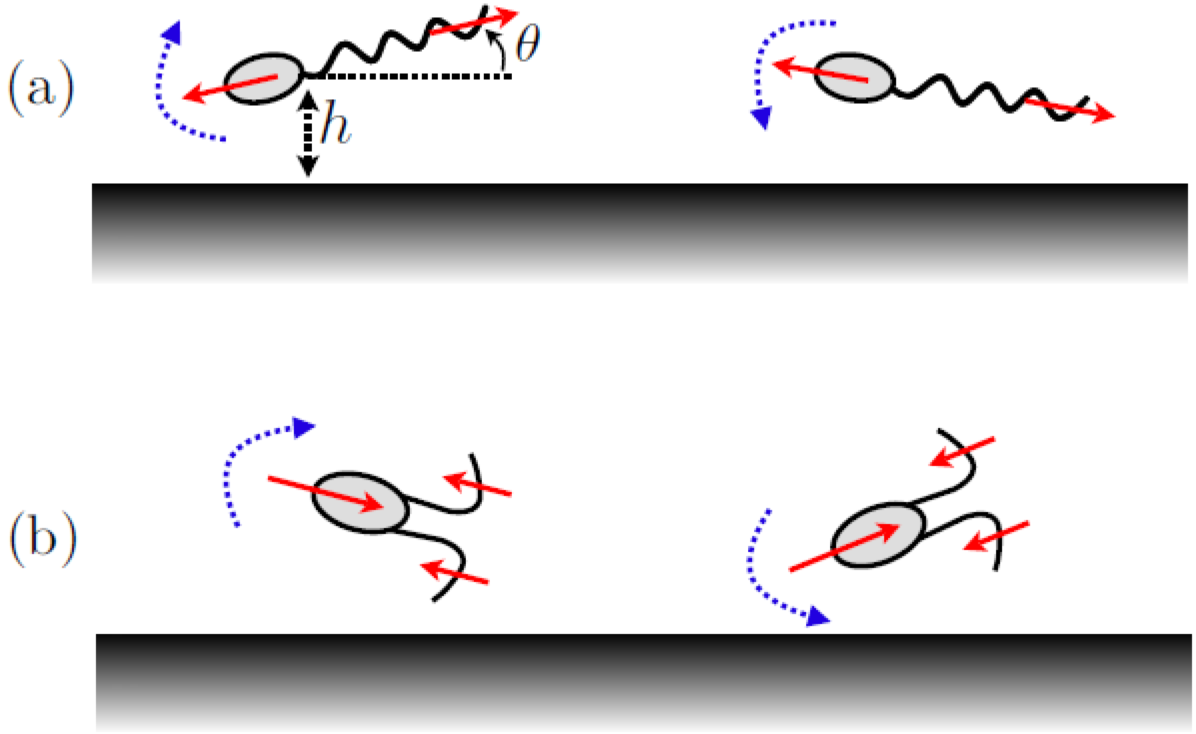
$$u_z^{adv}(\mathbf{R}_0) = u_{D,z}(\mathbf{R}_0 - \mathbf{R}_0^{im}; \mathbf{e}^{im})$$

$$= -\frac{p}{32\pi\eta h^2}\left[1 - 3\cos^2\theta\right]$$

$$\Omega_y^{adv}(\mathbf{R}_0) = -\frac{3p\sin(2\theta)}{128\pi\eta h^3}\left[2 - \cos^2\theta\right]$$

$$\cos\theta = \mathbf{e} \cdot \mathbf{e}_x$$

- Interface always attracts pusher and orients it parallel to surface
- Interface always repels puller, orienting it perpendicular to interface



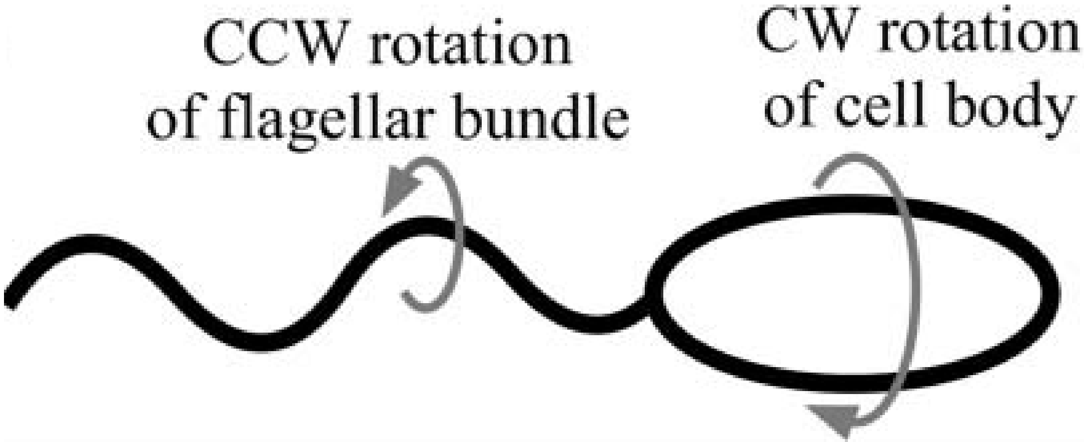
- These qualitative features are valid also for a partial-slip wall and liquid interface

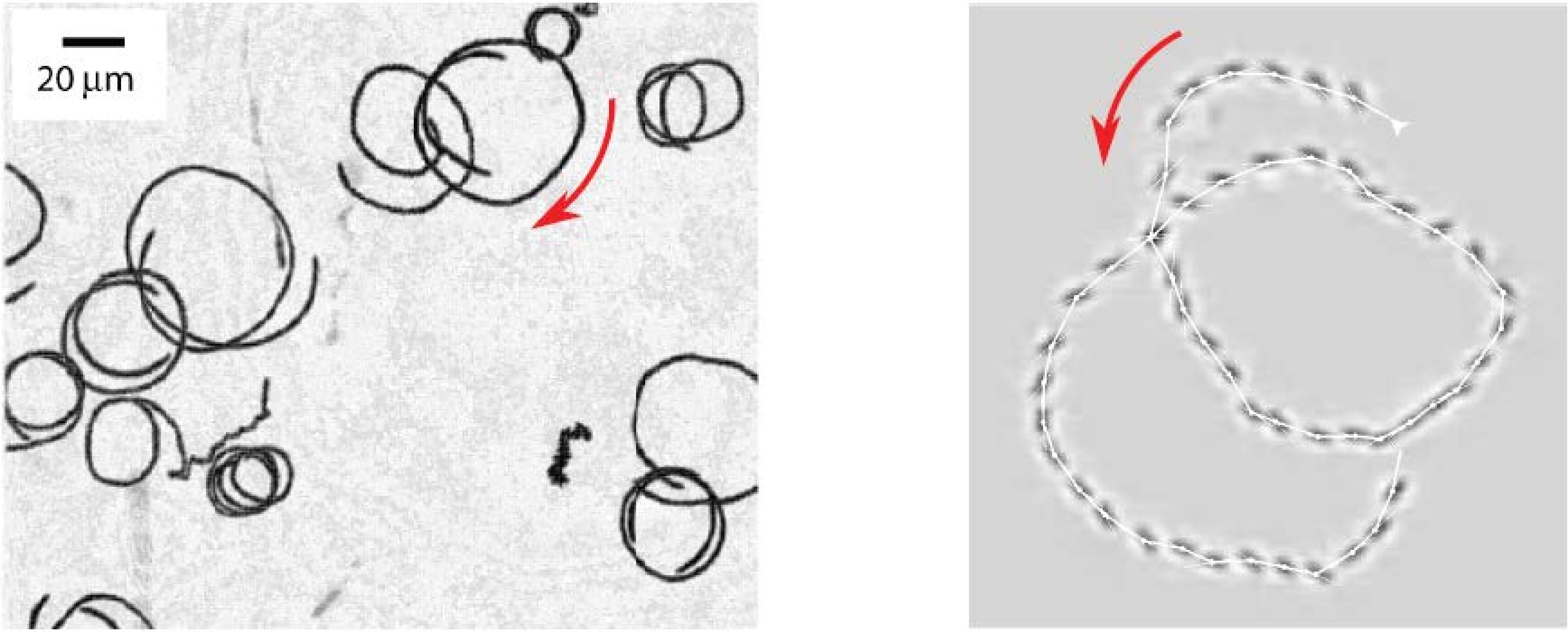
Berke et al., Phys. Rev. Lett. **101** (2008)

Lauga & Powers, Rep. Prog. Phys. **72** (2009)

Lopez & Lauga, Phys. Fluids **26** (2014)

No qualitative surface sensitivity of swimmer ?



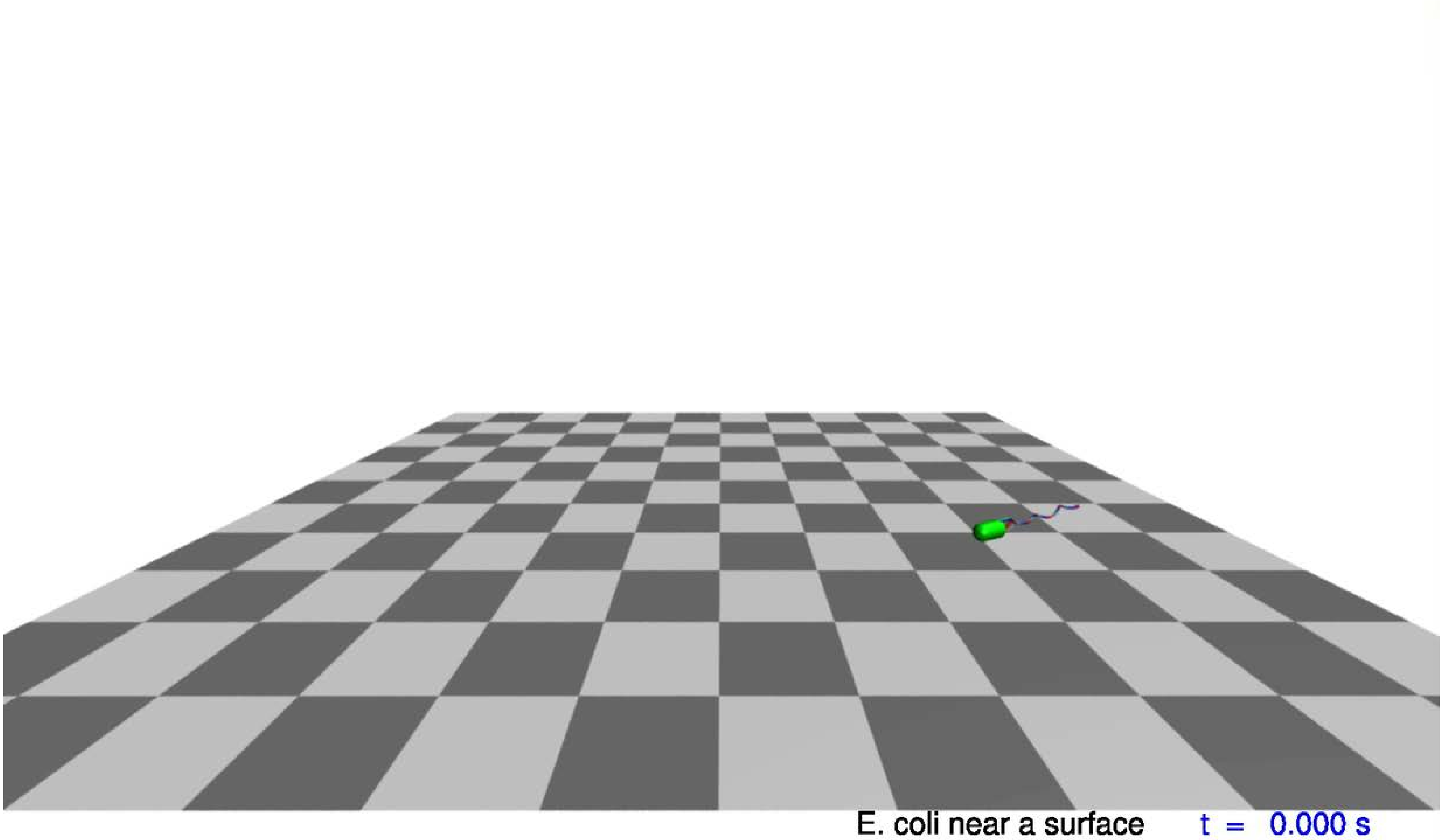


Solid wall (Clockwise)   Free surface (Counter- clockwise)

FIG. 1. Experimental evidence of clockwise motion for bacteria near a solid wall (left panel) [Reprinted with permission from Lauga *et al.*,[18] Biophys. J. **90**, 400–412 (2006). Copyright (2006) Biophysical Society], and counter-clockwise motion at a free surface (right panel) [Reprinted figure with permission from Di Leonardo *et al.*,[20] Phys. Rev. Lett. **106**, 038101 (2011). Copyright (2011) American Physical Society].

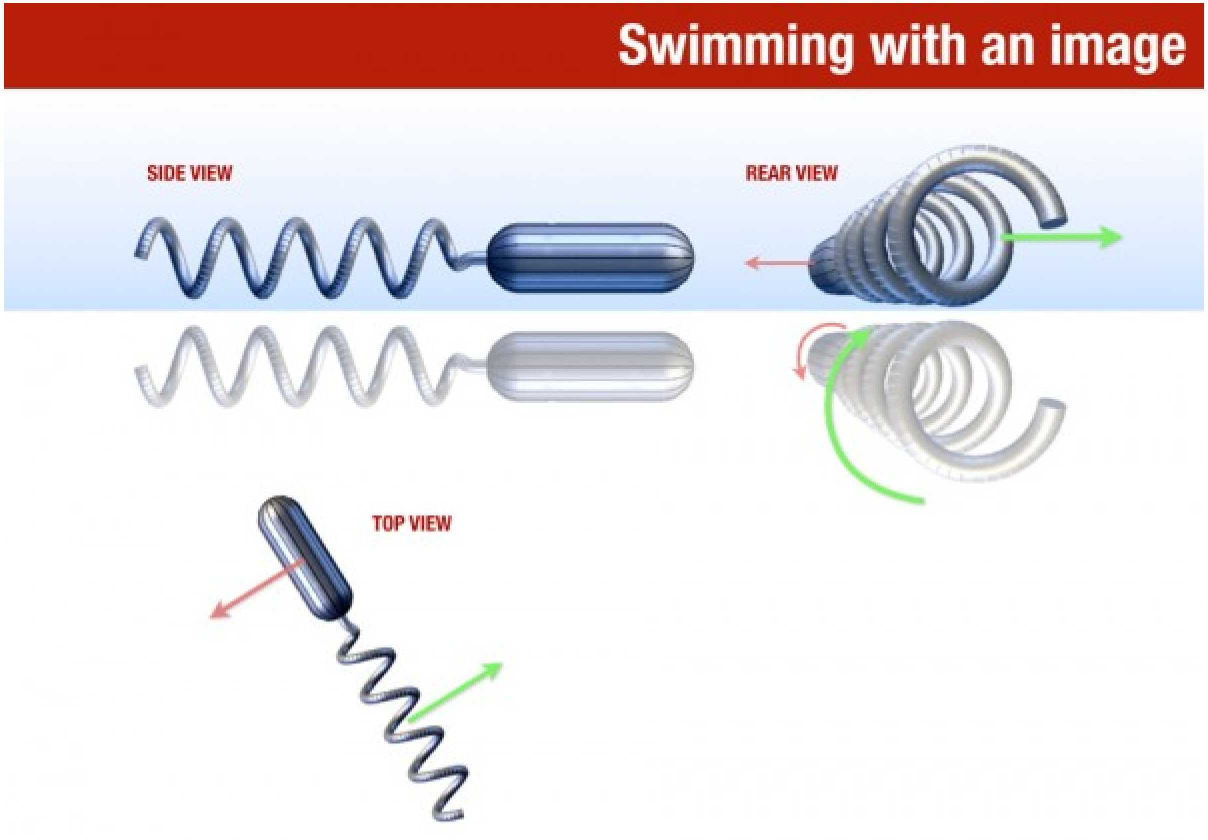
- Bacterium moves in circles, with orientation dependent on surface BC

MPC simulations of swimming E. coli (HI and Brownian motion included)



Courtesy: R. G. Winkler, J. Hu and G. Gompper, FZ Juelich, work submitted (2014)

Qualitative explanation

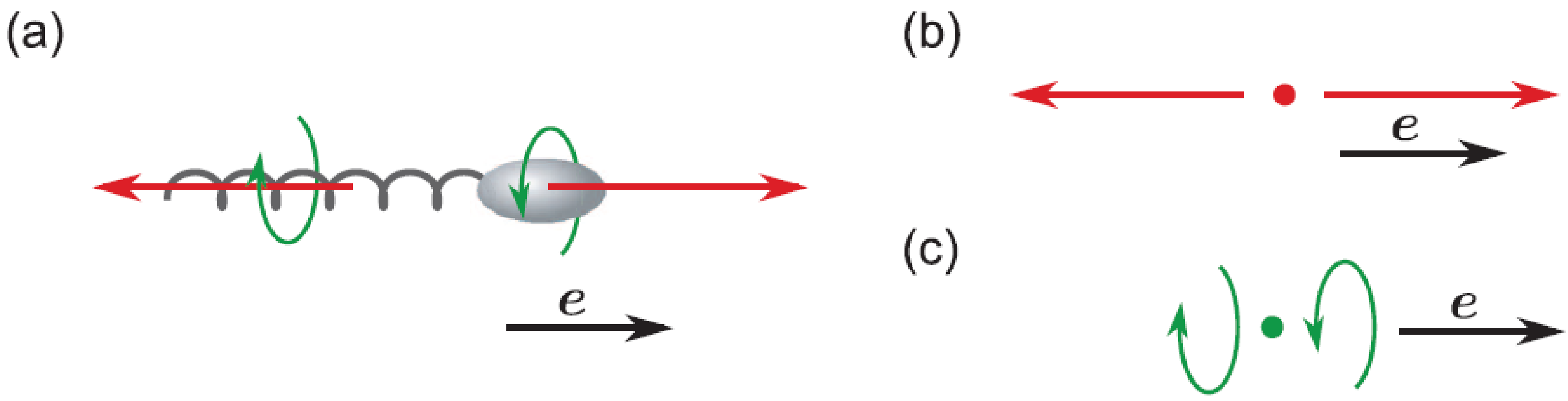


Di Leonardo et al., Phys. Rev. Lett. **106** (2011)

http://news.sciencemag.org/2011/01/nascar-drivers-microbes-forced-circle-forever-left

Approximative singularity solutions superposition calculation

- Rotlet dipole added to force dipole:

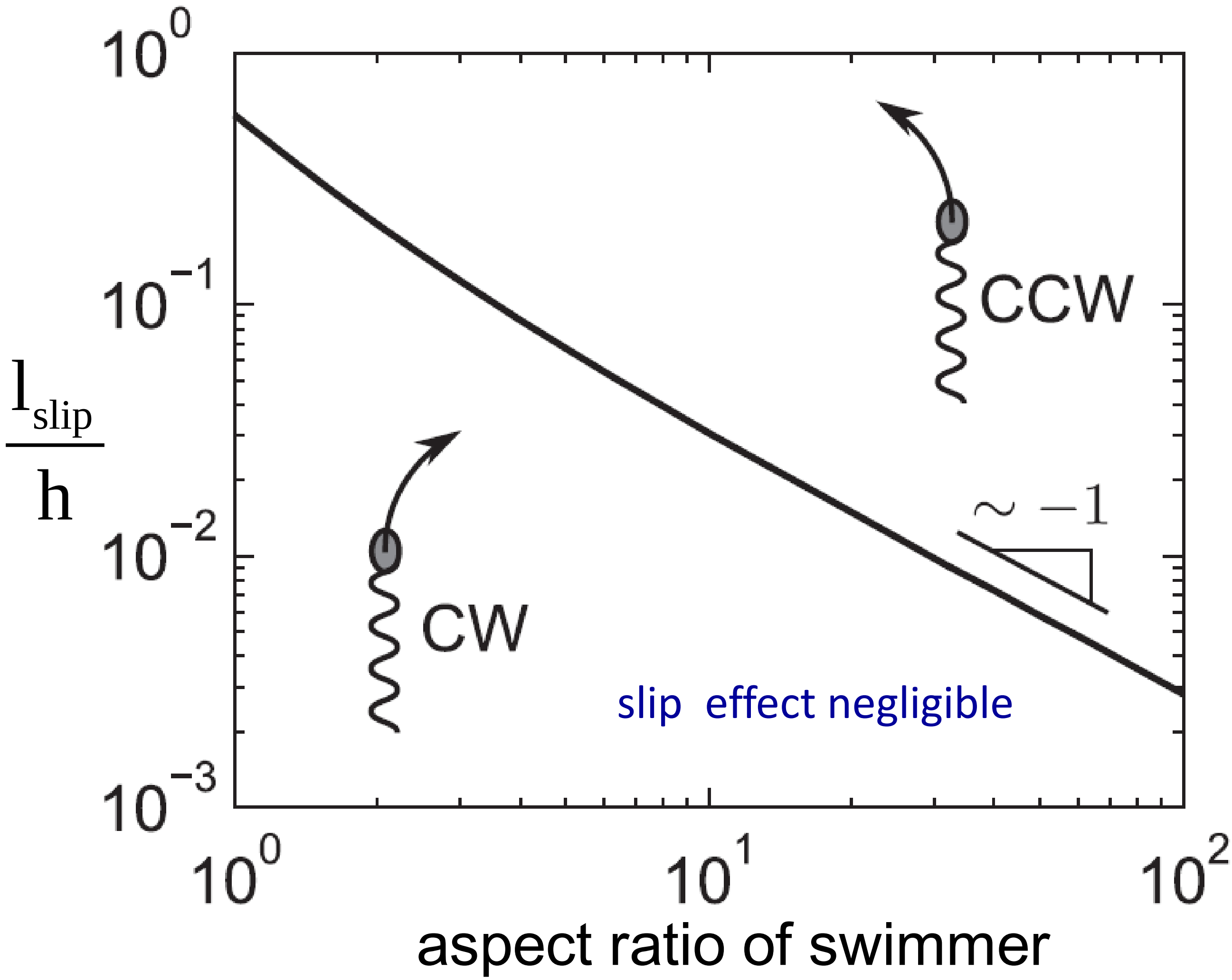


$$\mathbf{u}_{RD}(\mathbf{r}) \sim O(r^{-3})$$

- Applies for wall distances h > L

Lopez & Lauga, Phys. Fluids **26** (2014)

Critical values for partial – slip wall



Lopez & Lauga

Phys. Fluids **26** (2014)

- **Applications:** Biosensor for surface slip; guiding bacteria by surface stripes; Sorting bacteria according to circle radius

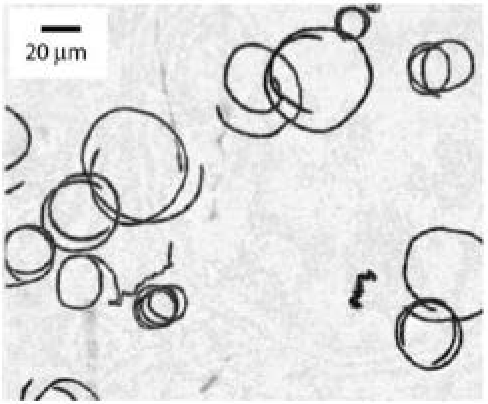
Just one concluding remark

Studying Low-Reynolds-number dynamic phenomena is fun,

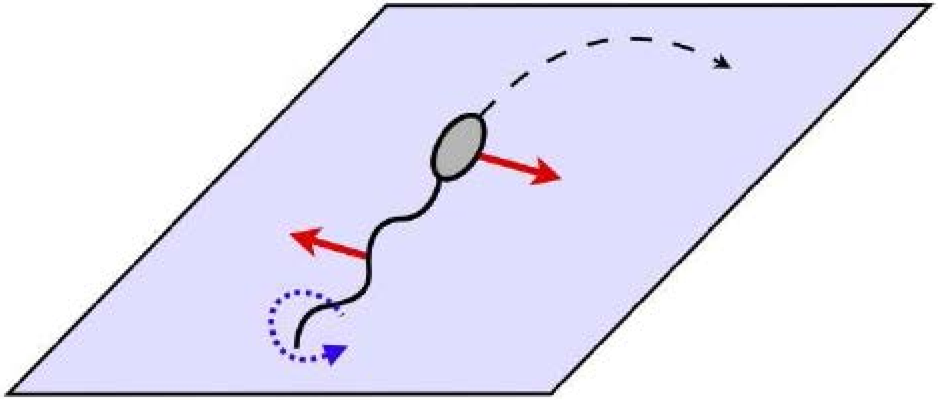
and often leads to surprising findings.

Thank you for your attention,

and to Peter Lang (ICS-3, FZ Jülich) for his kind invitation



(a)



(b)

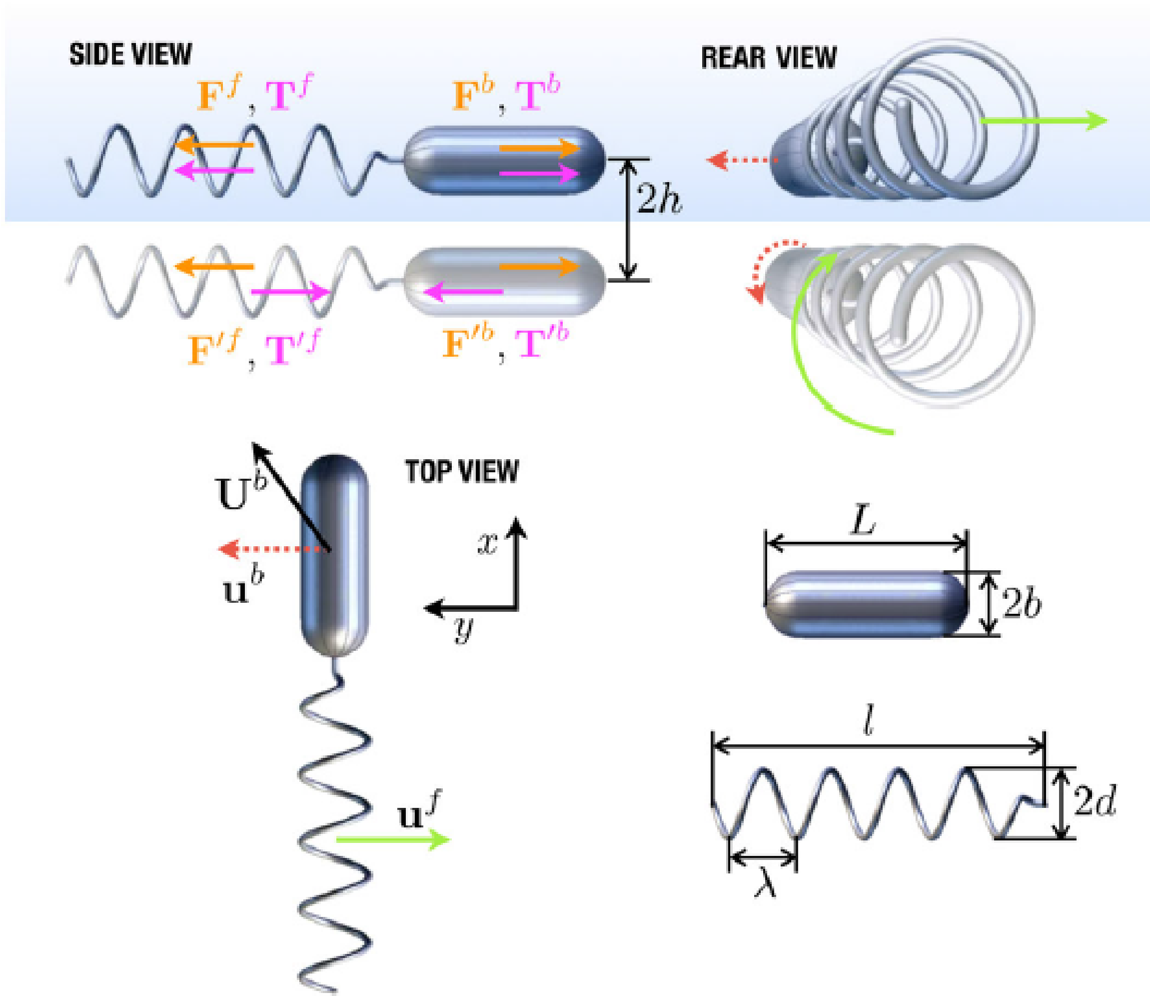


FIG. 3 (color online). Side, rear, and top views of the swimming bacterium and corresponding image bacterium on the opposite side of the interface. Solid green and dotted red arrows in rear and top views represent the flows produced by the rotating images of, respectively, the flagellar bundle and the body.

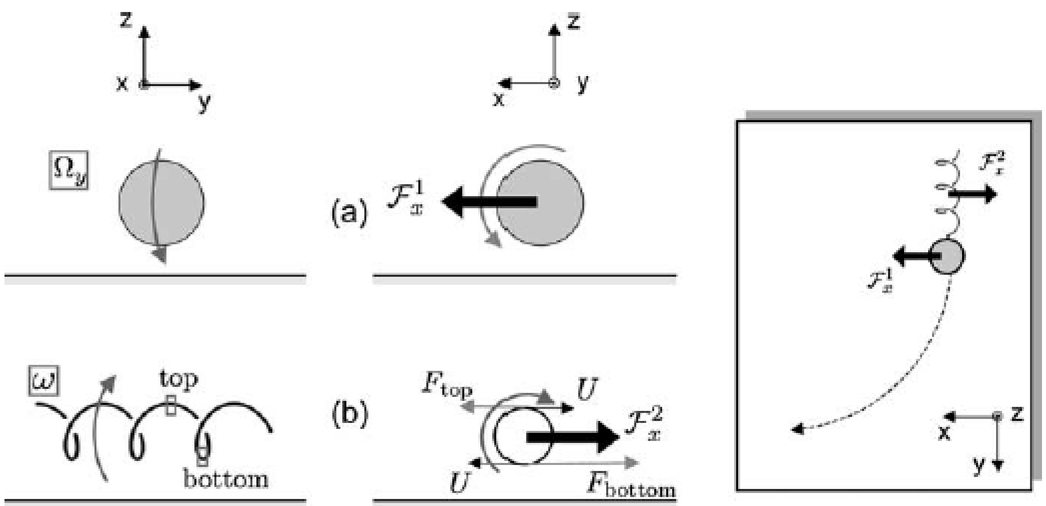
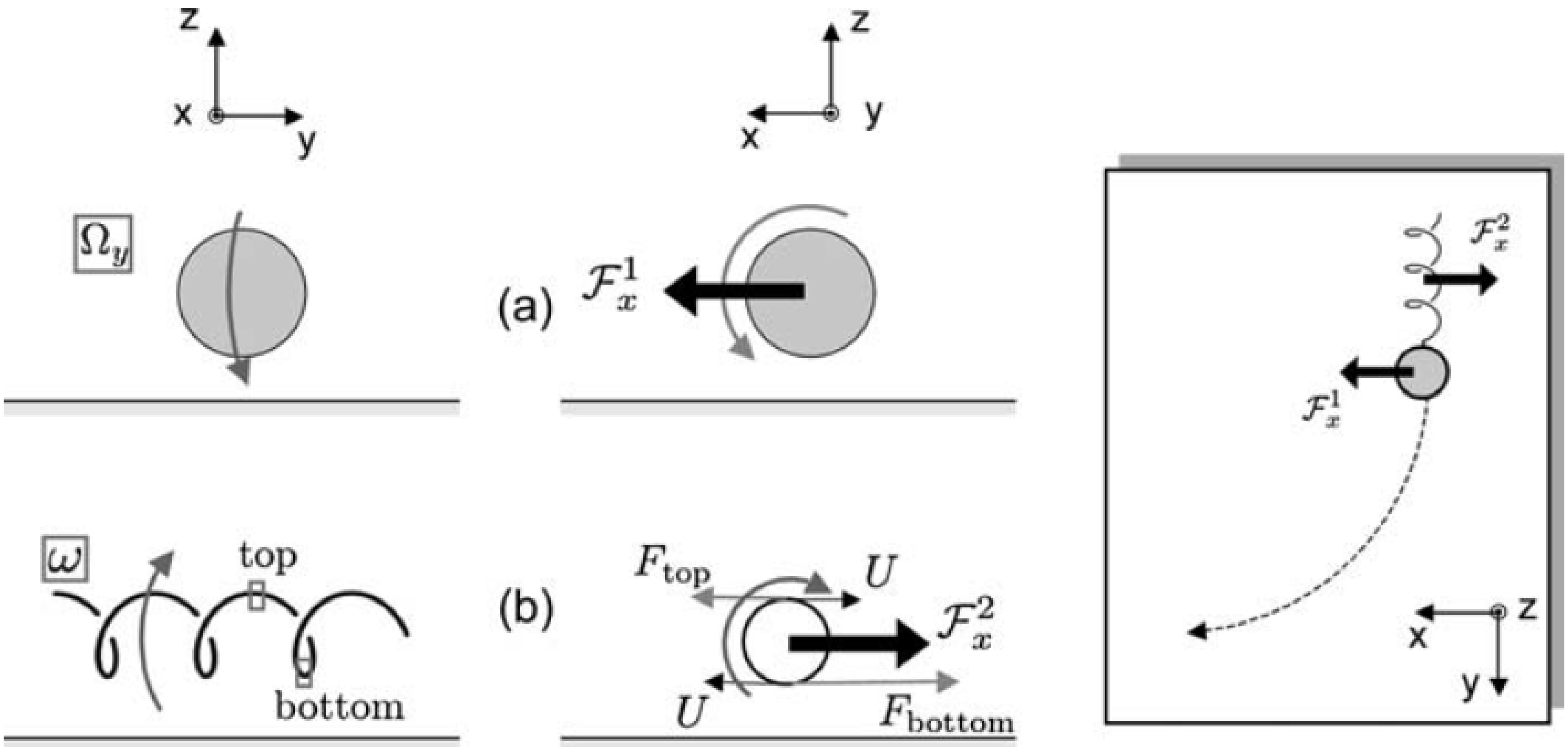


FIGURE 4 Physical picture (*side* and *front views*) for the out-of-plane rotation of the bacterium: (*a*) The positive *y*-rotation of the cell body leads to a net viscous *x*-force on the cell body, $\mathcal{F}_x^1 > 0$. (*b*) The negative *y*-rotation of the helical bundle leads to a net negative viscous *x*-force on the flagella, $\mathcal{F}_x^2 < 0$. The spatial distribution of these forces leads to a negative *z*-torque on the bacterium, which makes it rotate clockwise around the *z*-axis. Therefore, when viewed from above, the bacterium swims to its right.


z
x
y
Ω_y
(a)
$\mathcal{F}_x^1$
ω
top
bottom
(b)
F_{top}
U
$\mathcal{F}_x^2$
U
F_{bottom}
$\mathcal{F}_x^2$
$\mathcal{F}_x^1$
z
x
y

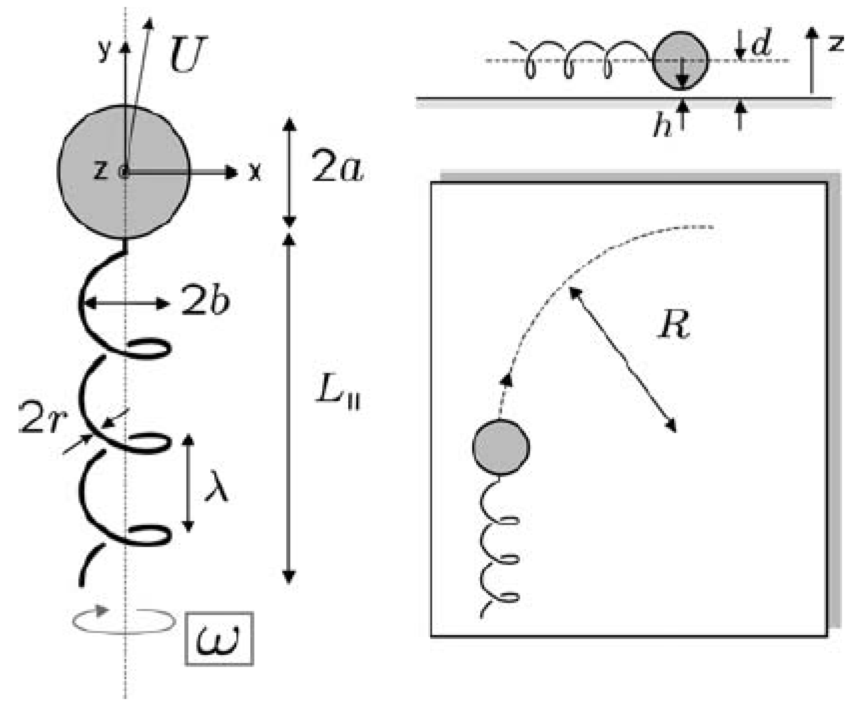
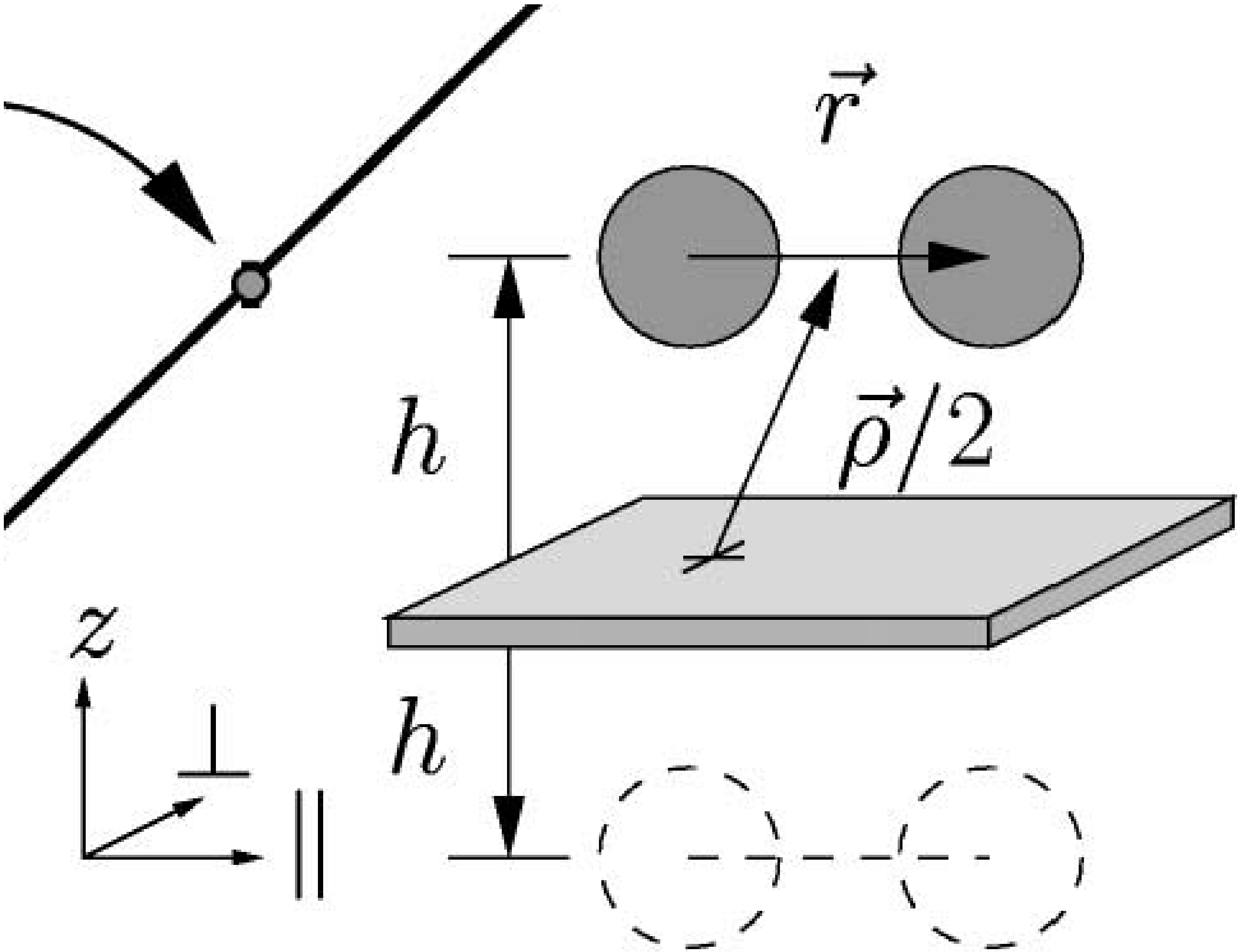


FIGURE 3 Setup and notations for the mechanical model of *E. coli* swimming near a solid surface.

5. Phoretic particle motion in external field



$\vec{r}$
h
$\vec{\rho}/2$
z
$\perp$
h
$\parallel$

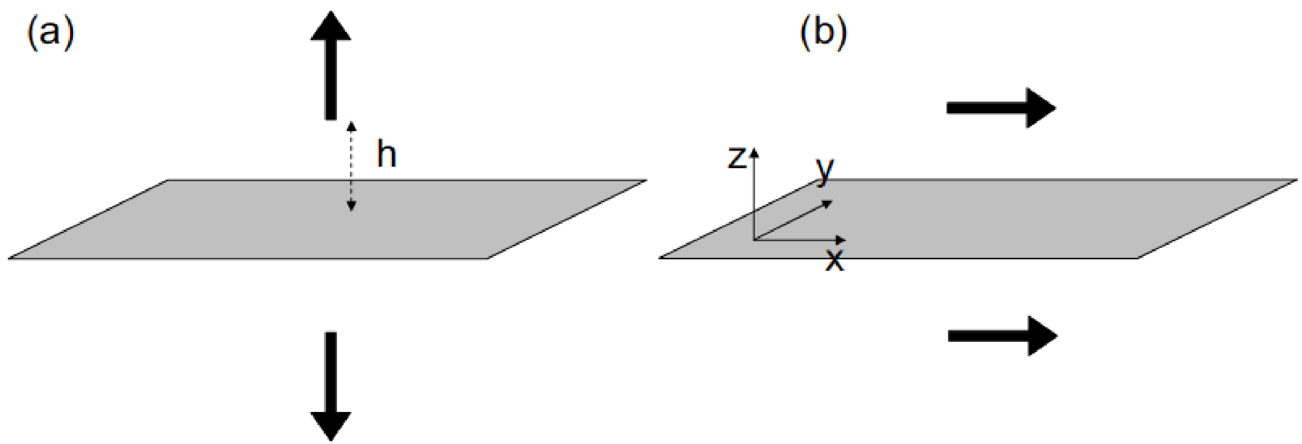


FIG. 1: First image of the Stokeslet; (a): Stokeslet (F, h) perpendicular to the surface; the first image is the Stokeslet $(-F, -h)$; (b): Stokeslet (F, h) parallel to the surface; the first image is the Stokeslet $(F, -h)$. Note that these would be the complete image systems if the surface was perfectly slipping $(\mathrm{Kn} = \infty)$.

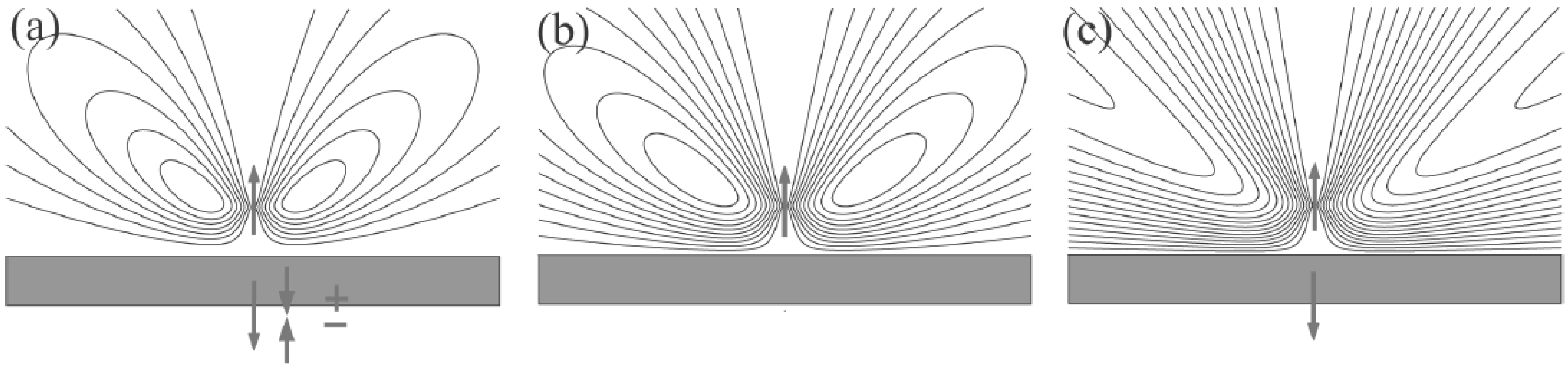


FIG. 2: Streamlines for a Stokeslet oriented perpendicular to a partial-slip wall, with (a) Kn = 0 (Blake's solution, no-slip), (b) Kn = 1, (c) Kn = ∞ (perfect slip). The streamlines are displayed in the plane which includes the Stokeslet and is perpendicular to the nearby surface.

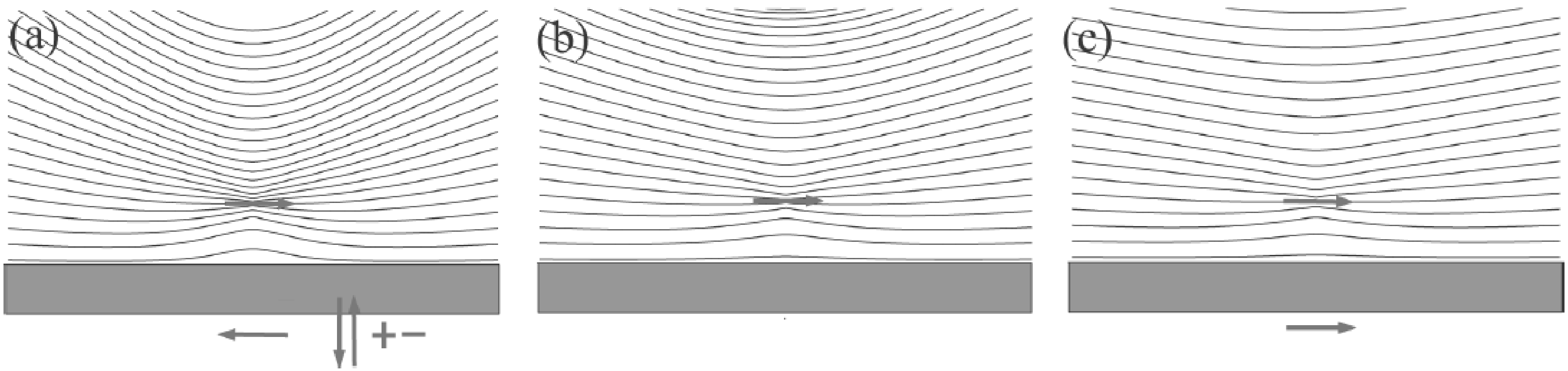


FIG. 3: Streamlines for a Stokeslet oriented parallel to a partial-slip wall, with (a) Kn = 0 (Blake's solution, no-slip), (b) Kn = 1, (c) Kn = ∞ (perfect slip). The streamlines are displayed in the plane which includes the Stokeslet and is perpendicular to the nearby surface.

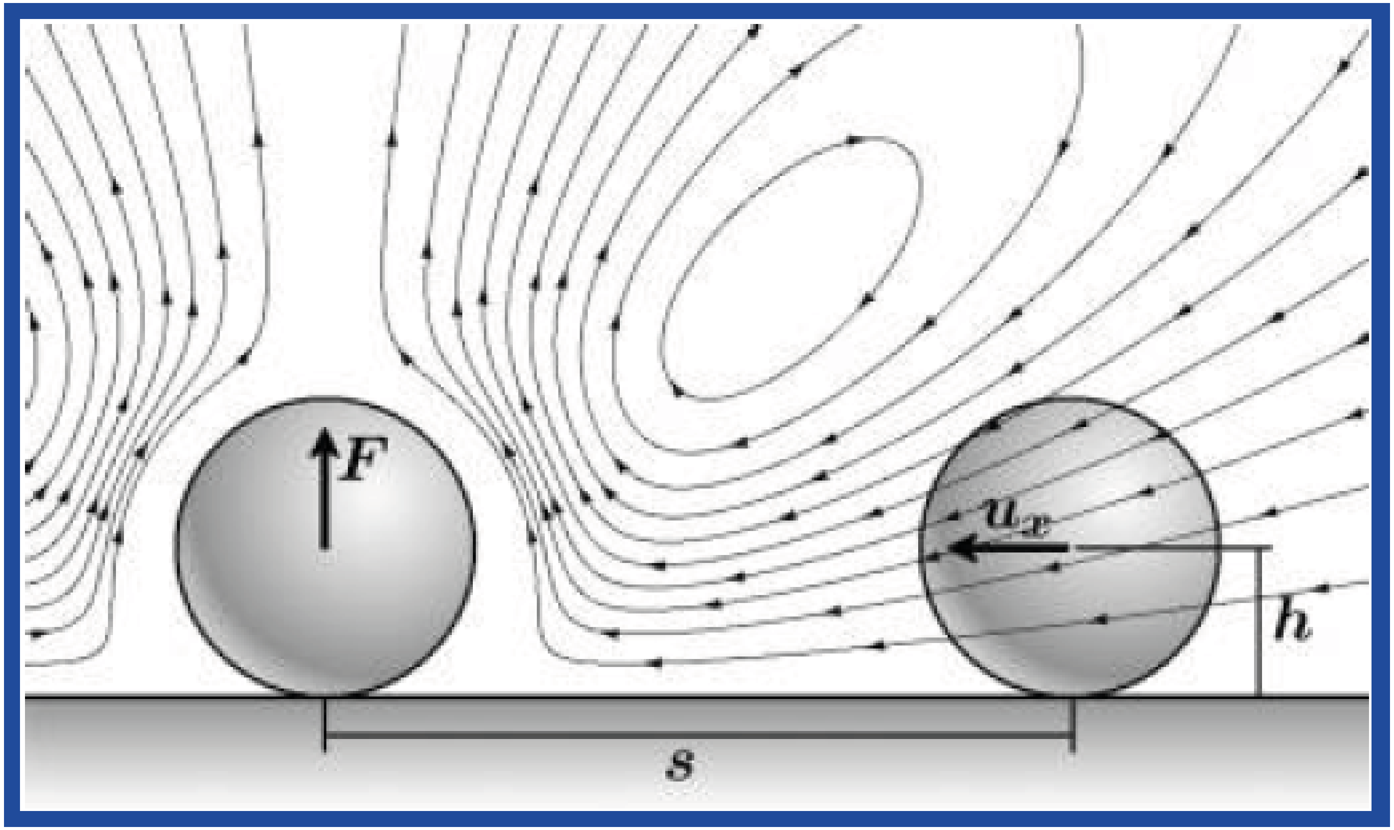


Figure 4. Streamlines of the flow generated by a point force pointing away from a plane wall with sticky boundary conditions. The resulting flow can be represented as the superposition of the Stokeslet flow produced by the force *F* plus an image singularity sister consisting of a Stokeslet, a Stokes dipole and a source dipole.[18]

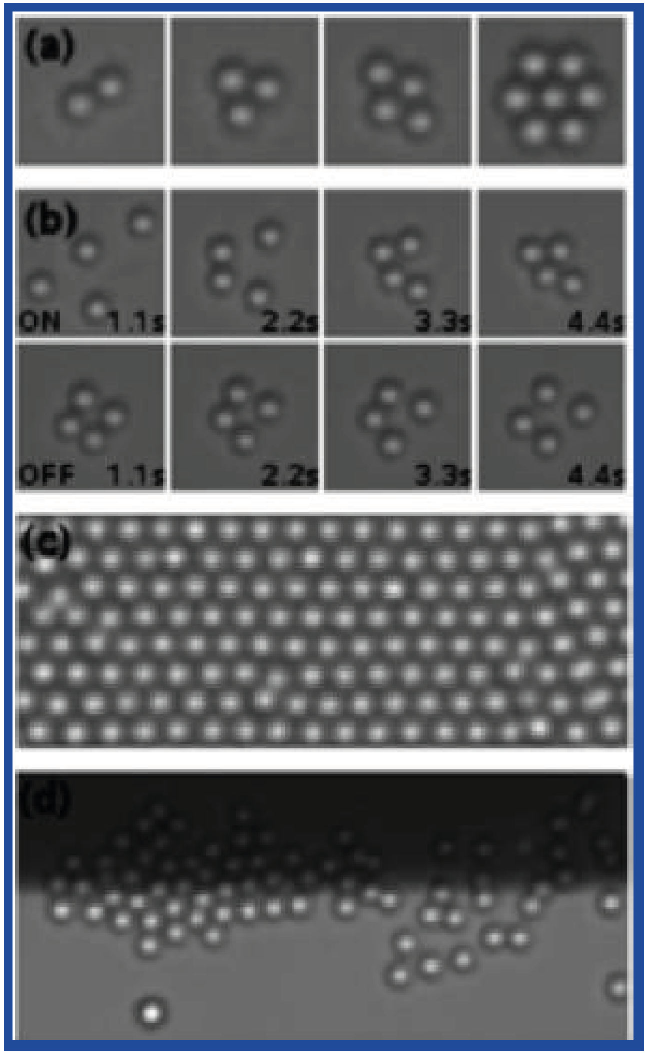
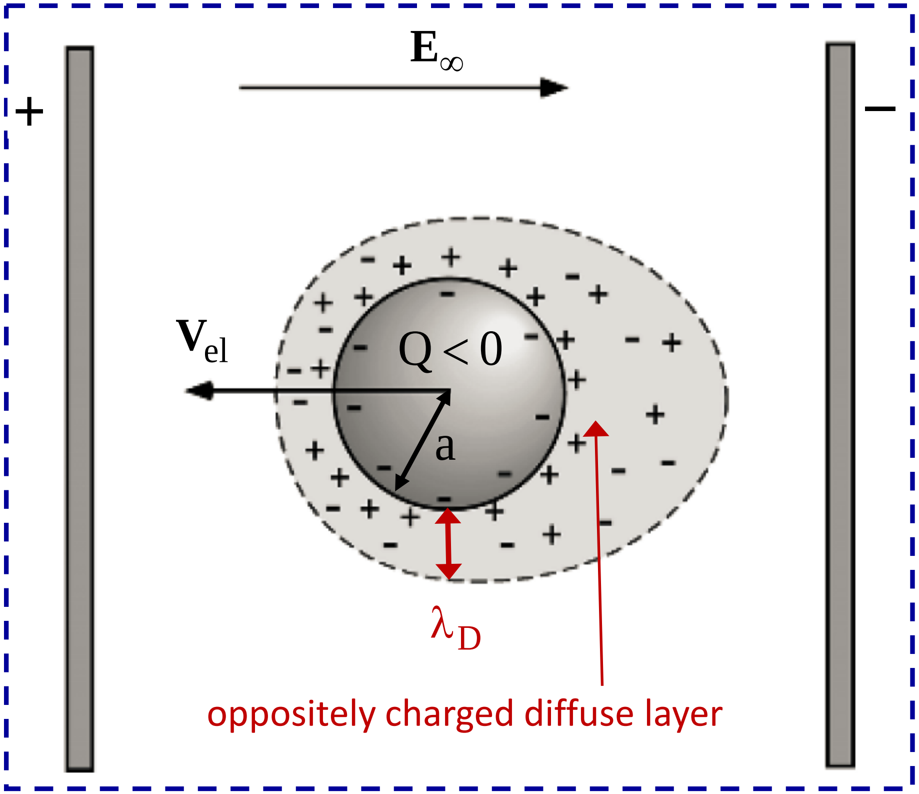


Figure 3. Reversible self-assembly of colloidal particles in a thermal gradient. (a) Particles confined to the bottom wall by thermophoresis assemble into close-packed structures. (b) Reversible aggregation (first row, temperature gradient on) and disintegration (second row, temperature gradient off) of close-packed clusters. (c) Highly concentrated samples assemble in a stable 2D crystal. (d) Close-packed structures form both on a dielectric charged surface (i.e., the cover glass (visible as the clearer band) and on a metallic surface (darker band) obtained through a gold coating that is 10 nm thick over the glass surface.

Non - conducting charged sphere in electrolyte exposed to uniform electric field



$$\mathbf{V}_{el} = \mu_{el}\,\mathbf{E}_\infty + O\left(|\mathbf{E}_\infty|^2\right)$$

- Hückel limit $\lambda_D \gg a$

$$F_{el}^0 + F_h^0 = Q\,E_\infty - 6\pi\eta_0 a V_{el}^0 = 0$$

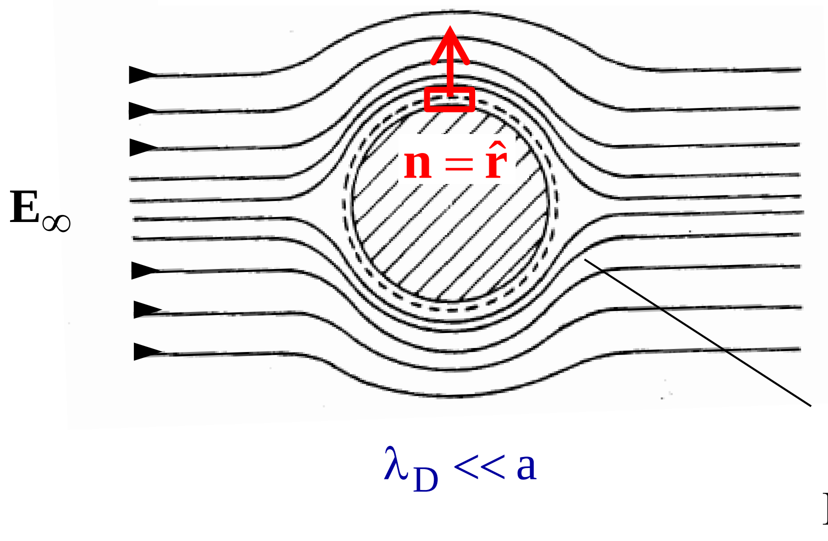
$$\mu_{el}^0 = \frac{Q}{6\pi\eta_0 a}$$

- electrokinetic effects for non-dilute diffuse microion layer only

- Retarding **electro-osmotic drag** by cross-streaming of oppositely charged fluid near sphere surface
- Retarding **polarization effect** force due to distortion of EDL away from spherical symmetry. Restructuring by microion diffusion and conduction, and by solvent convection is non-instantaneous

Smoluchowski limit and flat plane electro-osmosis

Helmholtz – Smoluchowski limit (1879 & 1903)

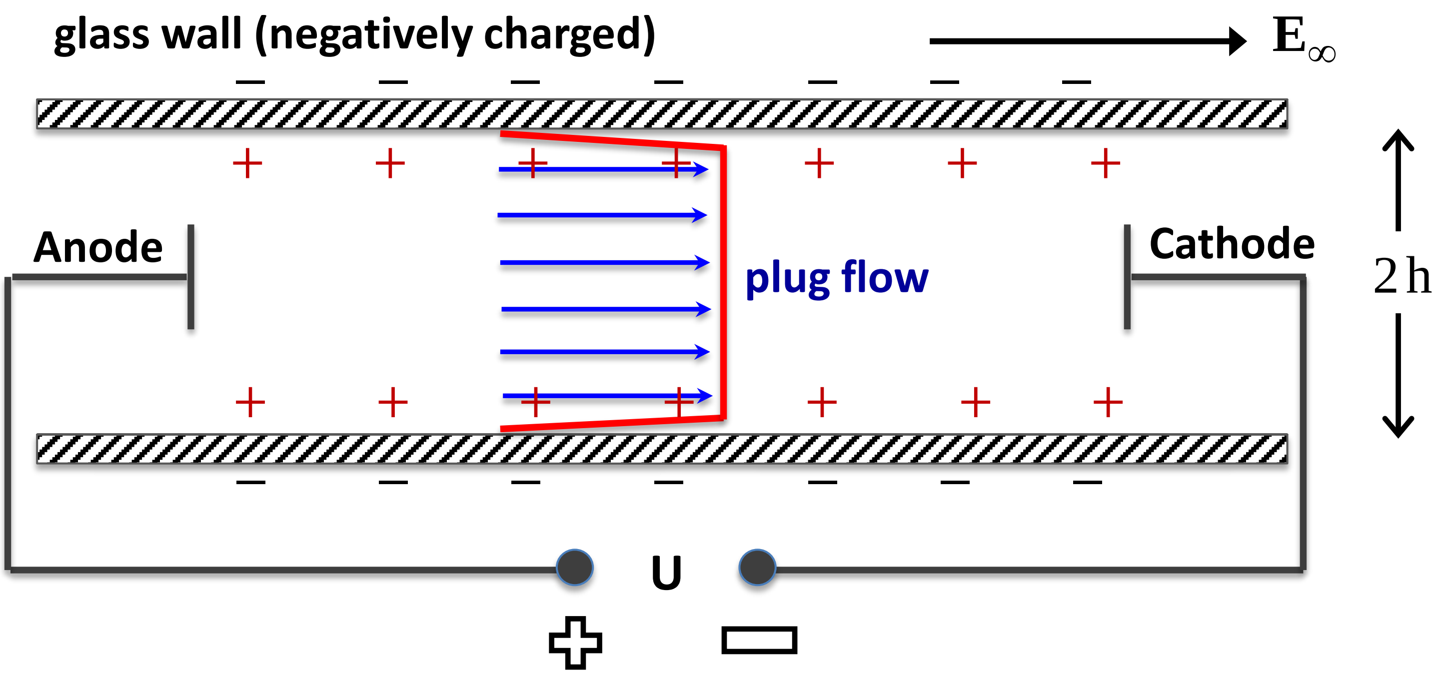


- field within ultrathin EDL **tangentially curved around** non – conducting sphere

$$\mathbf{E}_{out}(\mathbf{r} \in \mathrm{EDL}) \propto (\mathbf{1} - \mathbf{n}\,\mathbf{n}) \cdot \mathbf{E}_\infty$$

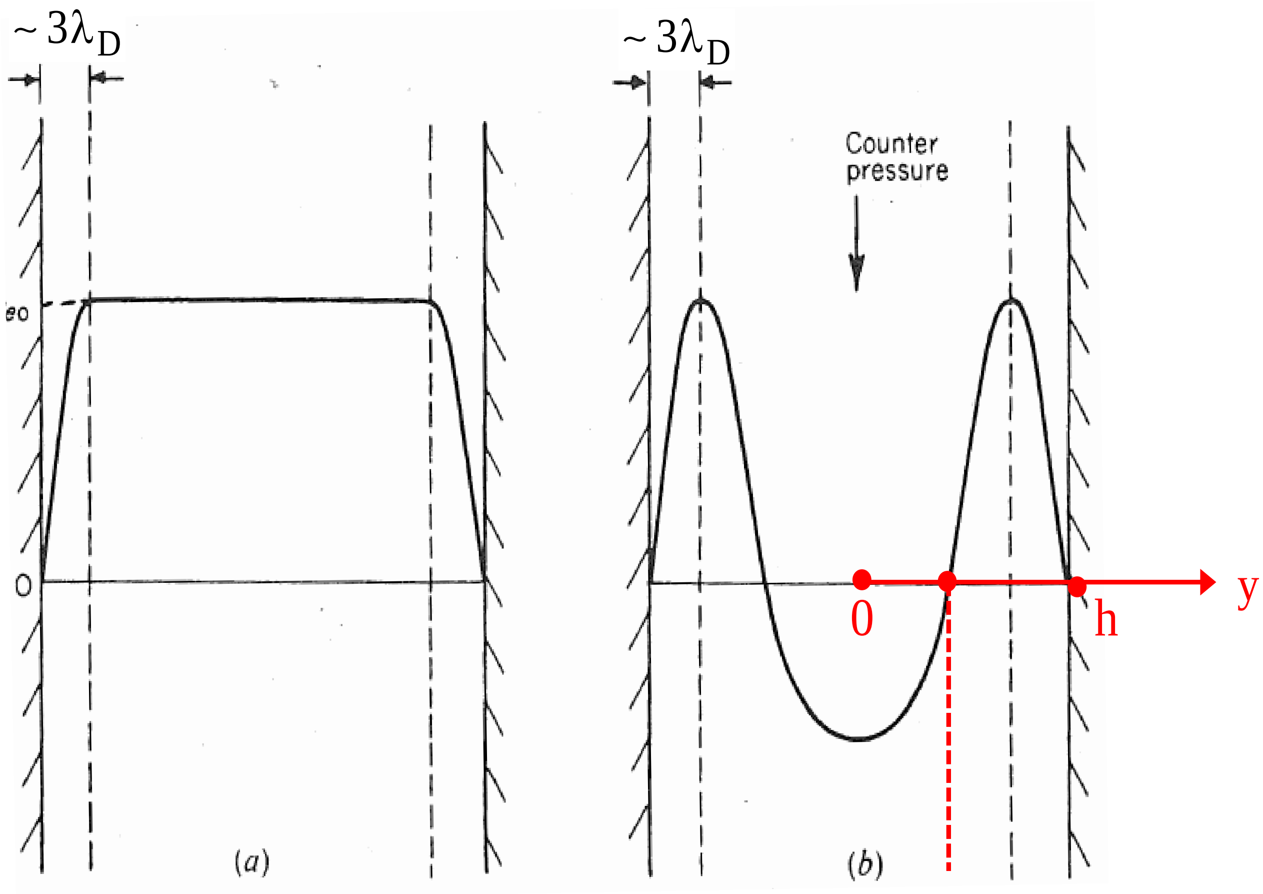
- Electro-osmosis: flow of charged electrolyte fluid past stationary (particle) surface

Electro - osmotic plug flow in an open capillary tube



- Electroosmosis used in microfluid devices to drive aqueous media through narrow micro - channels where Low-Reynolds number fluid dynamics applies to

Open (a) versus closed (b) electro-osmotic cells



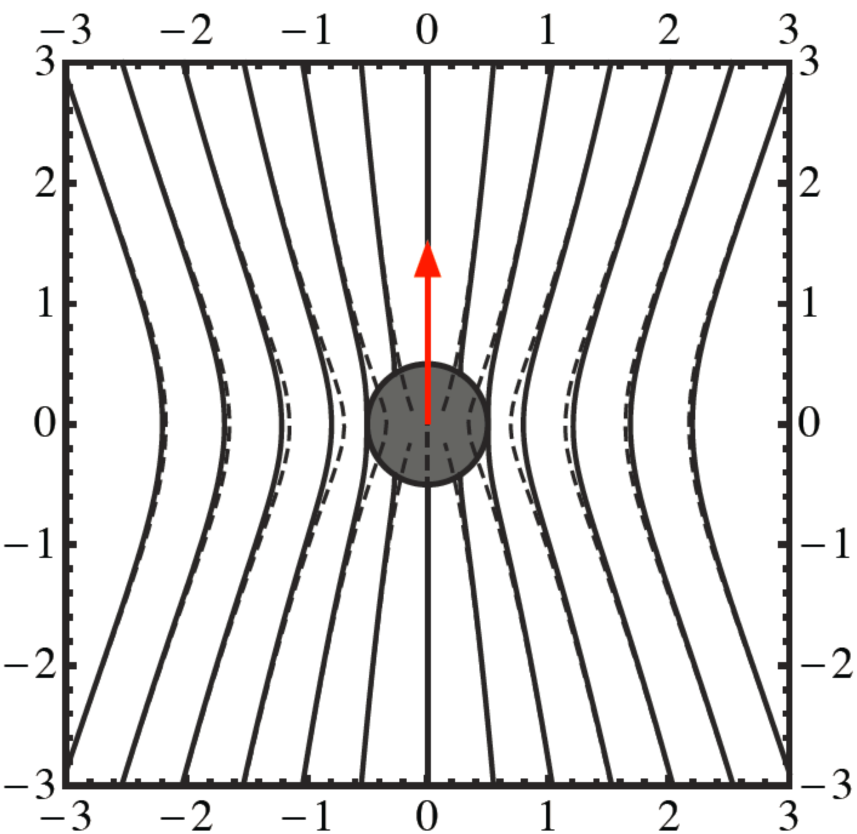
$$\int_A dS\,\mathbf{u} = \mathbf{0}$$

$h/\sqrt{3}$ if $h \gg \lambda_D$

- Absolute electrophoretic velocity measured in zero flow plane (Malvern Zeta-sizer)

- Sphere in quiescent, infinite fluid, no-slip BC
- Sphere stationary (rest frame)

$\mathbf{u}_\infty = \mathbf{0}$



$\mathbf{u}_\infty = -\mathbf{V}$