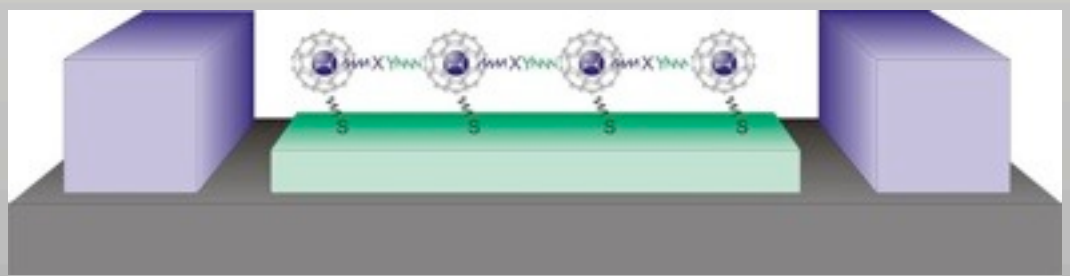
lecture WS ’15

Quantum Computers
- how to make them work -
Shor’s algorithm

C. Meyer, C.M. Schneider



prime factorization

algorithm found by Peter Shor 1994

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generalized to an algorithm for order finding

prime factorization

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generalized to an algorithm for order finding

key step is the quantum Fourier transform (QFT)

discrete Fourier transform (DFT)

Lieven Vandersypen, PhD thesis: http://arxiv.org/abs/quant-ph/0205193

DFT

unitary transform that converts a string of N complex numbers x_j to a string of N complex numbers y_k

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j \, \mathbf{e}^{i2\pi jk/N}$$

inverts periodicity (input string period r, output string N/r)

1 0 0 0 1 0 0 0 ⟶ 1 0 1 0 1 0 1 0

converts off-sets into phase factors

0 1 0 0 0 1 0 0 ⟶ 1 0 -i 0 -1 0 i 0

quantum Fourier transform

Lieven Vandersypen, PhD thesis: http://arxiv.org/abs/quant-ph/0205193

number of qubits $n = \log_2 N$ (for N=8, three qubits)

amplitude of $|000\rangle$ represents first complex number, $|001\rangle$ the 2nd

states $|000\rangle, |001\rangle, \ldots, |111\rangle$ are labelled $|0\rangle, |1\rangle, \ldots, |7\rangle$

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{i2\pi jk/N} |k\rangle$$

0 1 0 0 0 1 0 0 ⟶ 1 0 -i 0 -1 0 i 0

$(|1\rangle + |5\rangle)\sqrt{2} \longrightarrow (|0\rangle - i|2\rangle - |4\rangle + i|6\rangle)/2$

quantum Fourier transform

Nielsen/Chuang S. 218

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$j = j_1 2^2 + j_2 2^1 + j_3 2^0$ (short: $j = j_1 j_2 j_3$) $0.j_1 j_2 j_3 = j_1/2 + j_2/4 + j_3/8$

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$$|j_1 j_2 j_3\rangle \rightarrow \frac{1}{2^{n/2}} \left[\left(|0\rangle + e^{2\pi i\, 0.j_3} |1\rangle \right) \left(|0\rangle + e^{2\pi i\, 0.j_2 j_3} |1\rangle \right) \left(|0\rangle + e^{2\pi i\, 0.j_1 j_2 j_3} |1\rangle \right) \right]$$

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reverse order of output qubits: $j_1^{in} = j_3^{out}$, $j_3^{in} = j_1^{out}$

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H

quantum Fourier transform

Nielsen/Chuang S. 218

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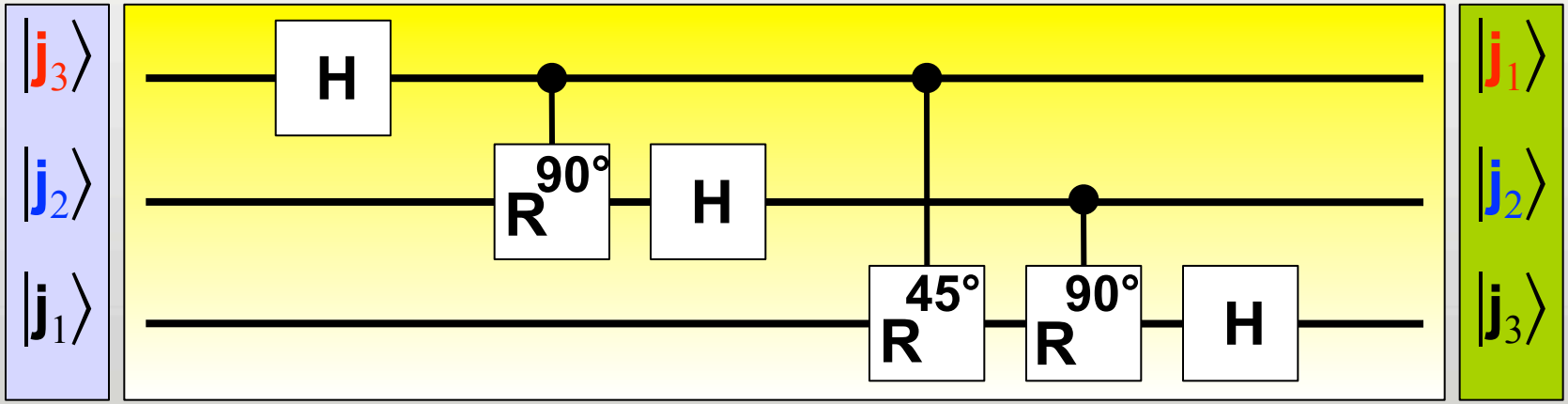
H | $R_{23}^{90°}$ | H

QFT quantum circuit

Weinstein et al.: quant-ph/99060059v1 (1999)



$$H = e^{\frac{i}{\hbar}\pi I_x} e^{\frac{i}{\hbar}\frac{\pi}{2} I_y}$$

$$R_1 = e^{-\frac{i}{\hbar}\phi I_z} = \begin{pmatrix} e^{-i\frac{\phi}{2}} & \\ & e^{i\frac{\phi}{2}} \end{pmatrix}$$

$$R_{jk} = e^{-\frac{i}{\hbar}\frac{\pi}{2} I_y^{j,k}} e^{\frac{i}{\hbar}\phi I_x^{j,k}} e^{\frac{i}{\hbar}\frac{\pi}{2} I_y^{j,k}}$$

$$e^{\frac{i}{\hbar}\pi I_\theta{}^j} e^{\frac{i}{\hbar} J t I_z^j I_z^k} e^{\frac{i}{\hbar}\pi I_\theta{}^j}$$

$$= e^{-\frac{i}{\hbar}\frac{\pi}{2} I_y} e^{\frac{i}{\hbar}\phi I_x} e^{\frac{i}{\hbar}\frac{\pi}{2} I_y}$$

QFT implementation

Weinstein et al.: Phys. Rev. Lett. 86,1889 (2001)

- ^{13}C labeled spins of Alanine as qubits (T_1>1.5s, T_2>400ms)
- Preparation of pseudo-pure state $|000\rangle$

$|111\rangle\langle 000|$

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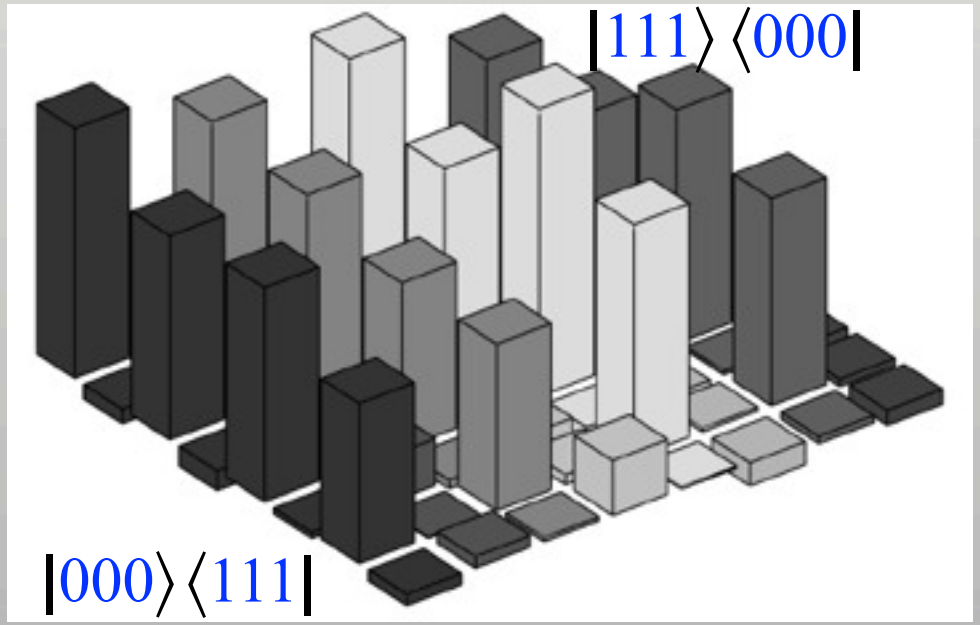

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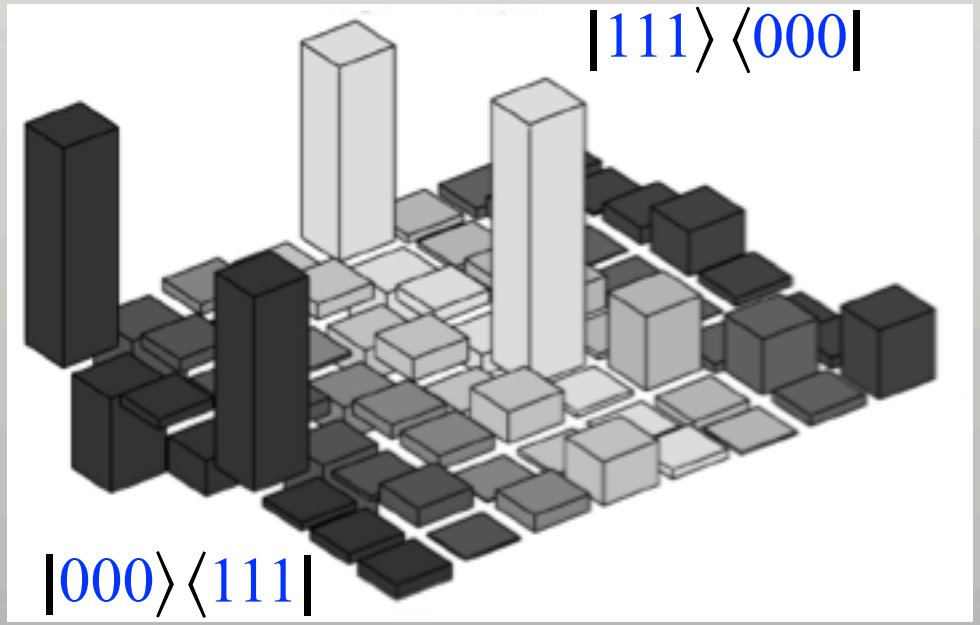
- Preparation of pseudo-pure state $|000\rangle$
- Preparation of input state $|000\rangle+|010\rangle+|100\rangle+|110\rangle$
- Apply QFT => $|000\rangle+|001\rangle$ and SWAP => $|000\rangle+|100\rangle$



Order finding

Lieven Vandersypen, PhD thesis: http://arxiv.org/abs/quant-ph/0205193

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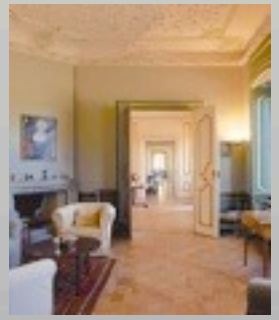
- M rooms with one entrance and one exit







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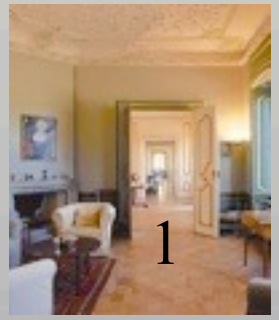

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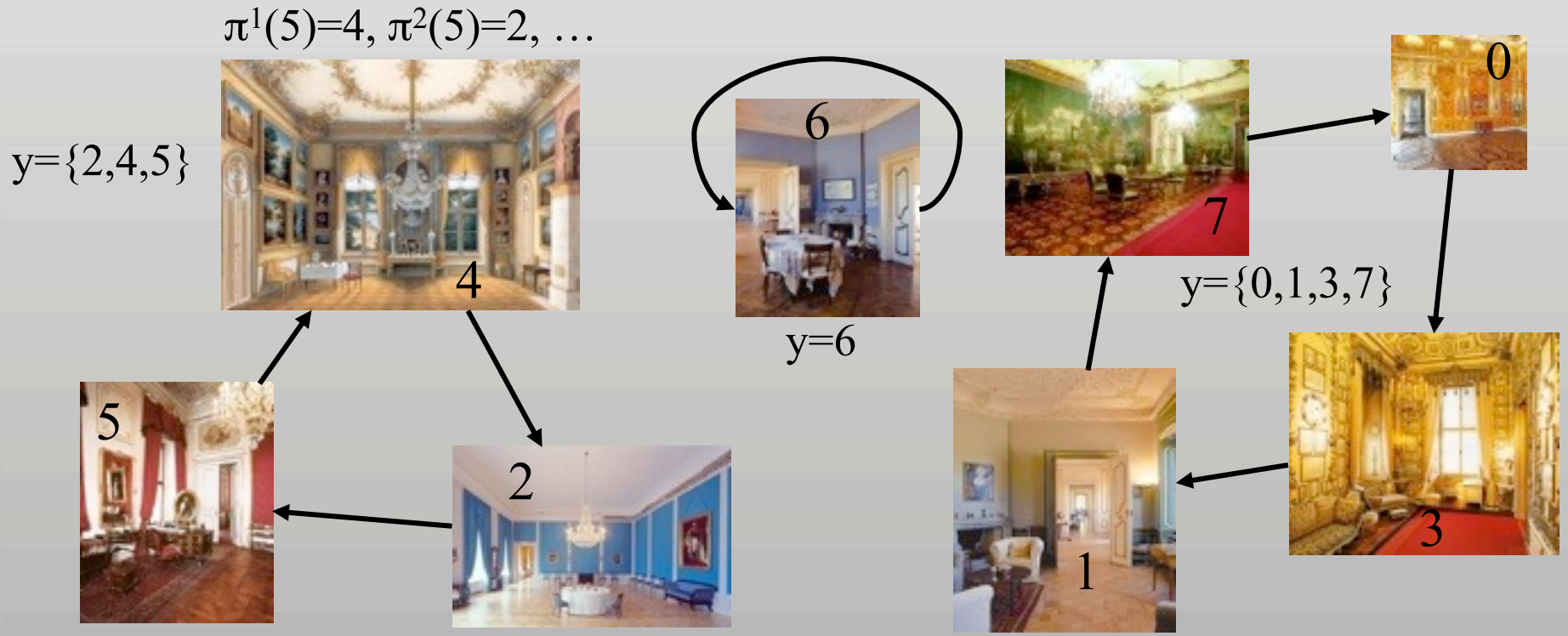

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- M rooms with one entrance and one exit
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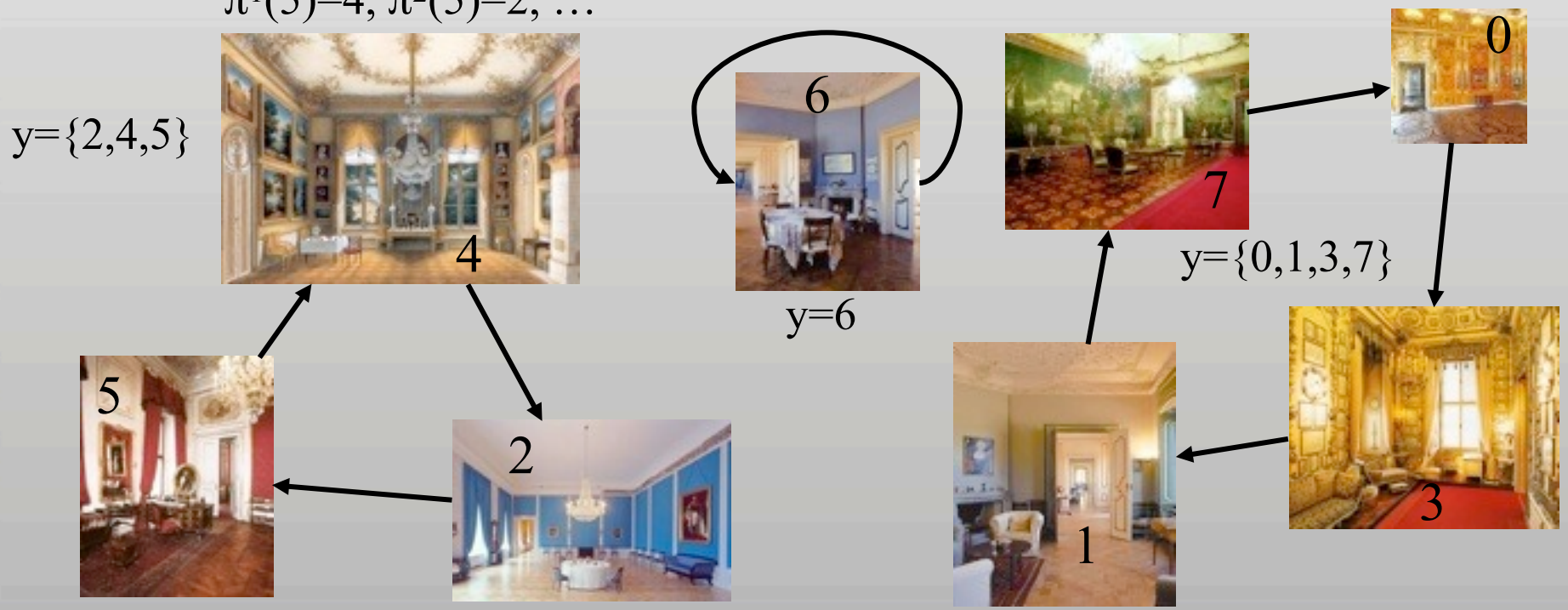
- How often to change room until back at start?

$\pi^1(5)=4$, $\pi^2(5)=2$, …



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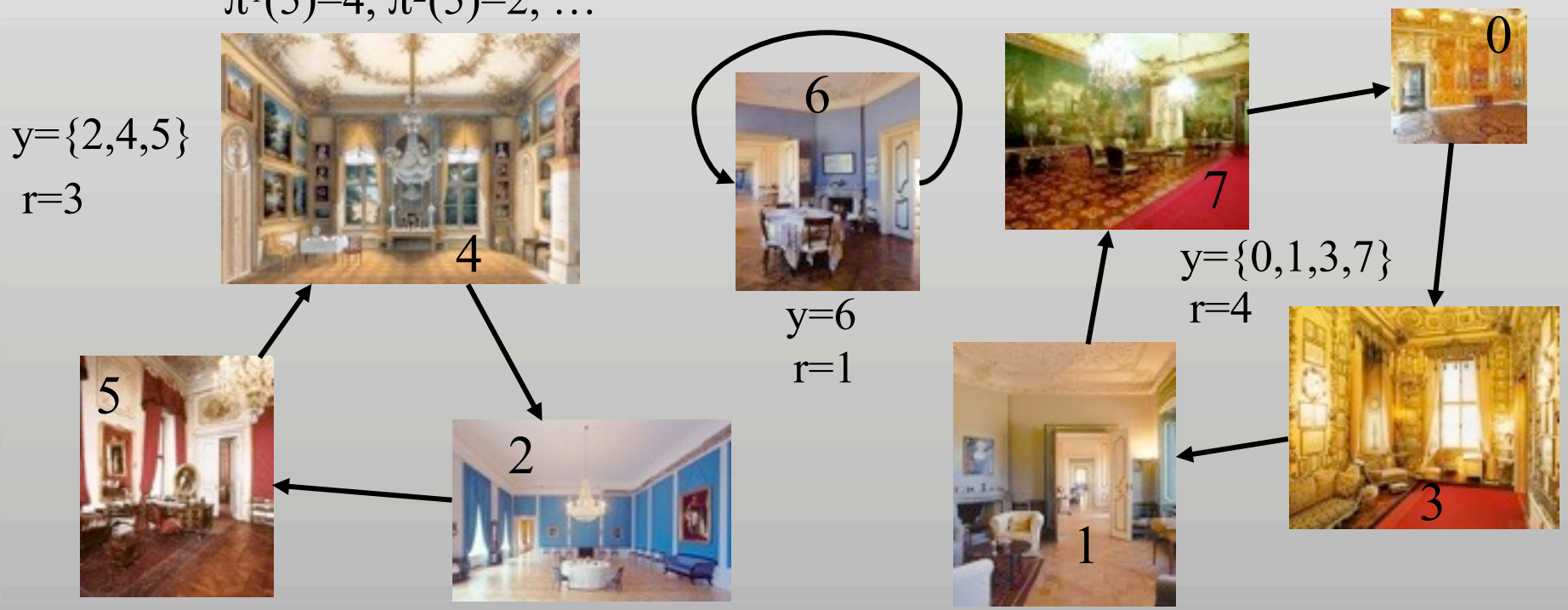

Order finding: quantum circuit

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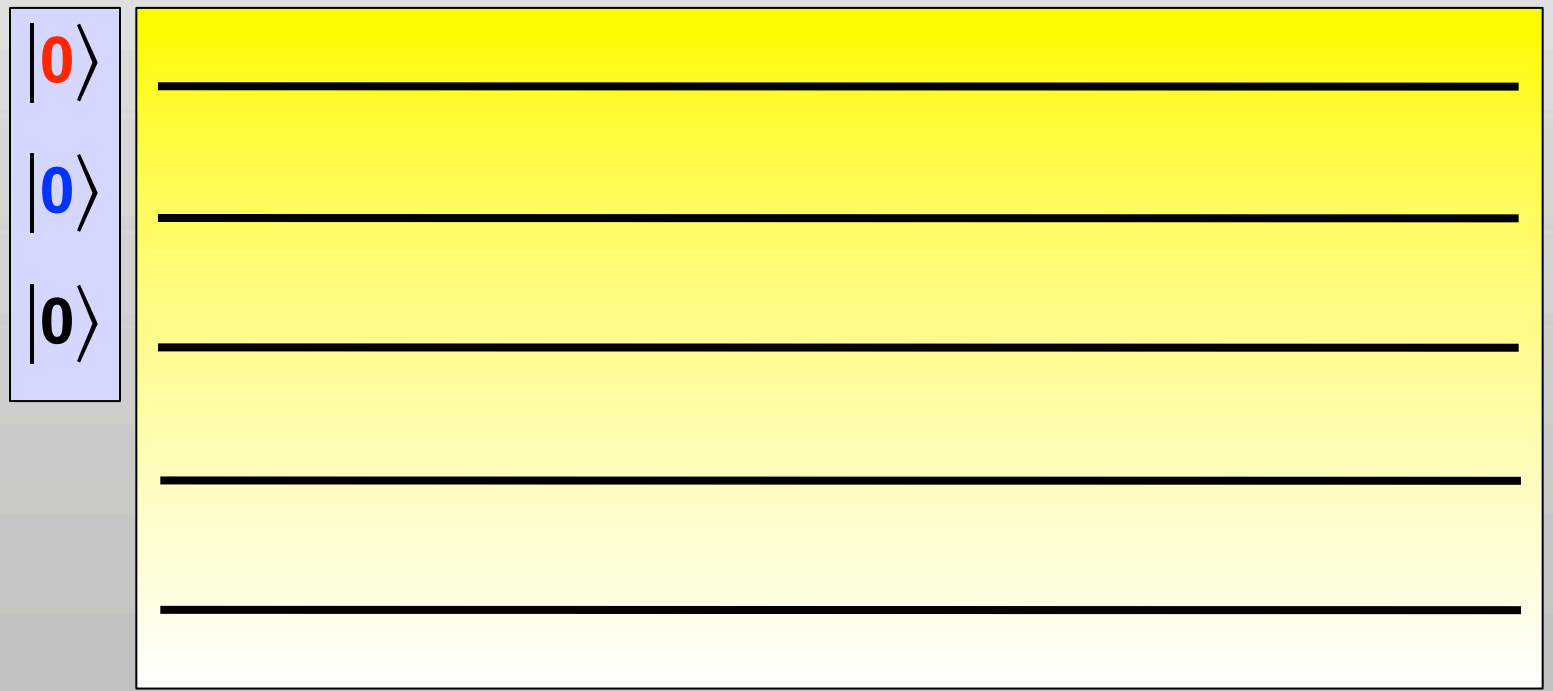

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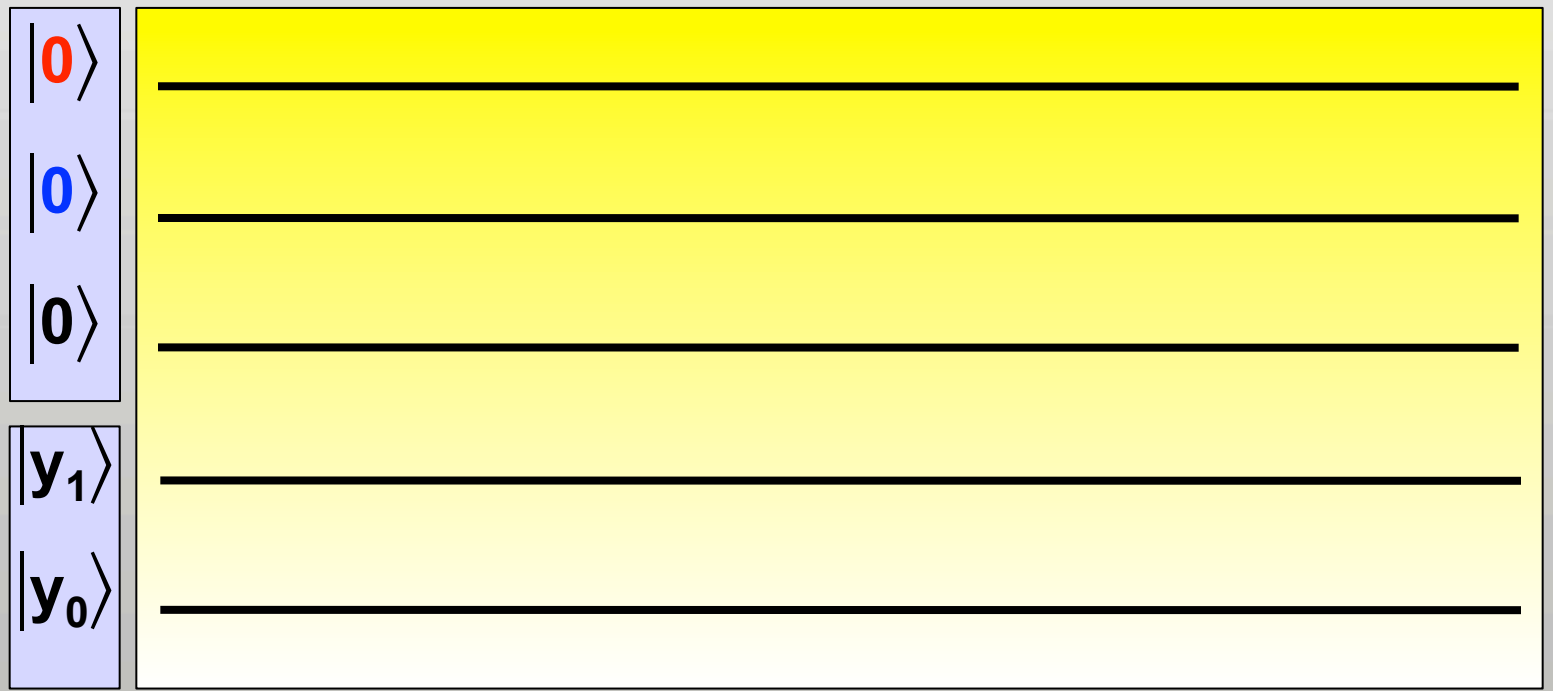

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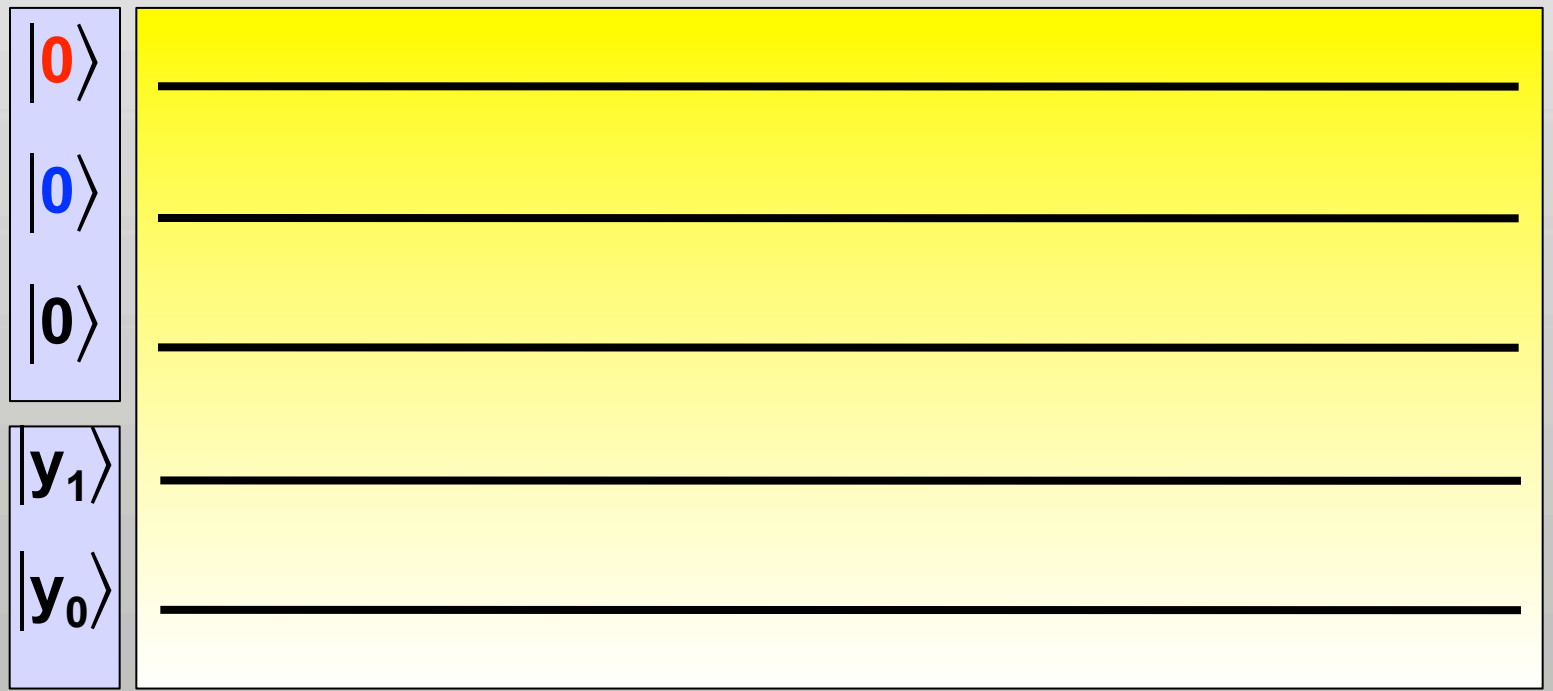

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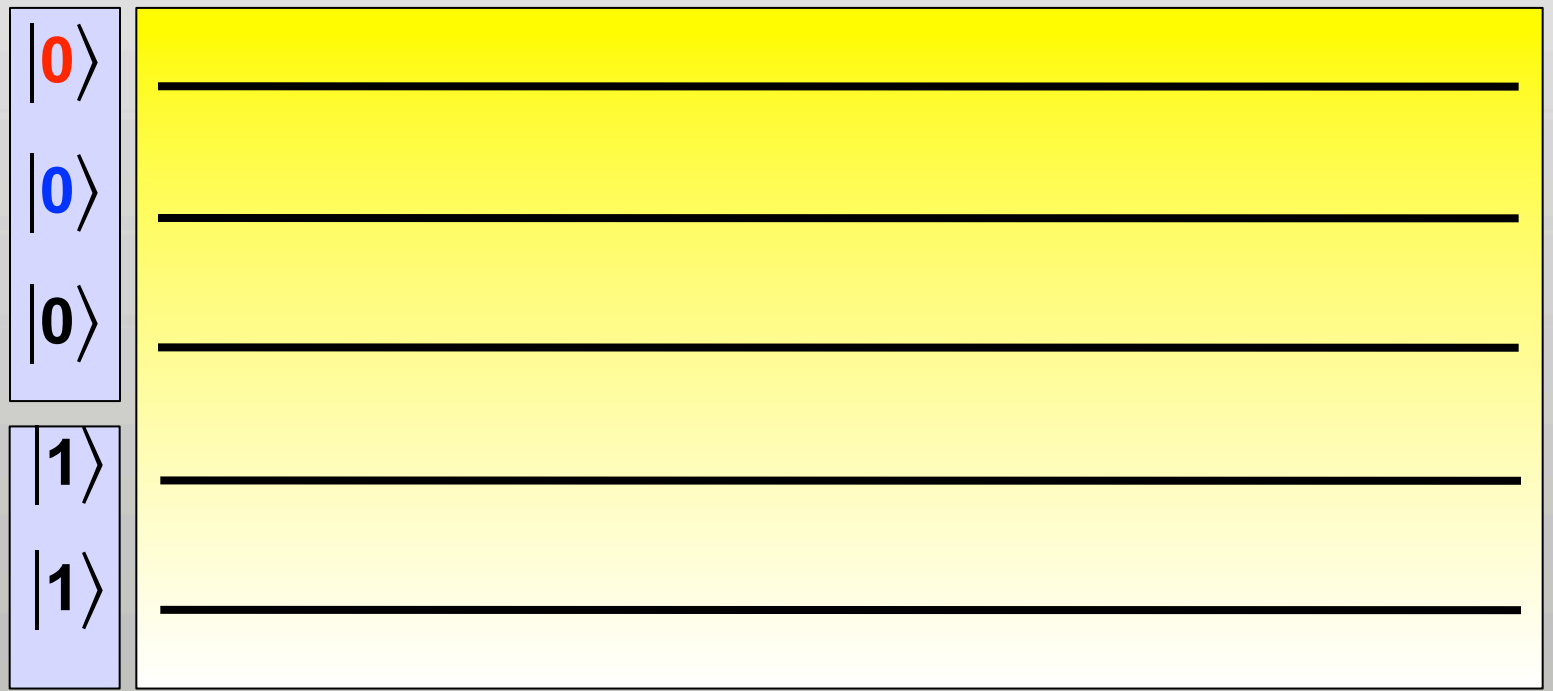

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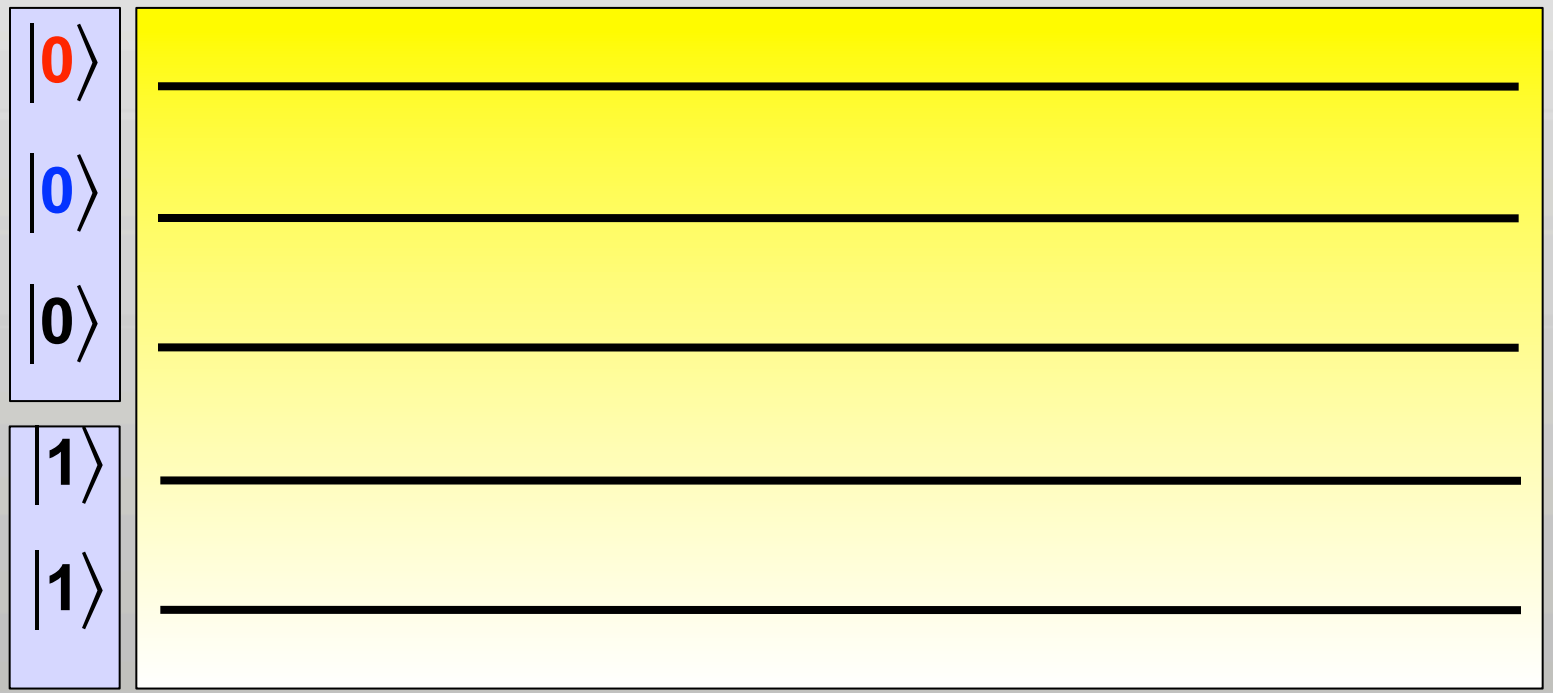

$|\psi_0\rangle = |000\rangle|11\rangle$

Order finding: quantum circuit

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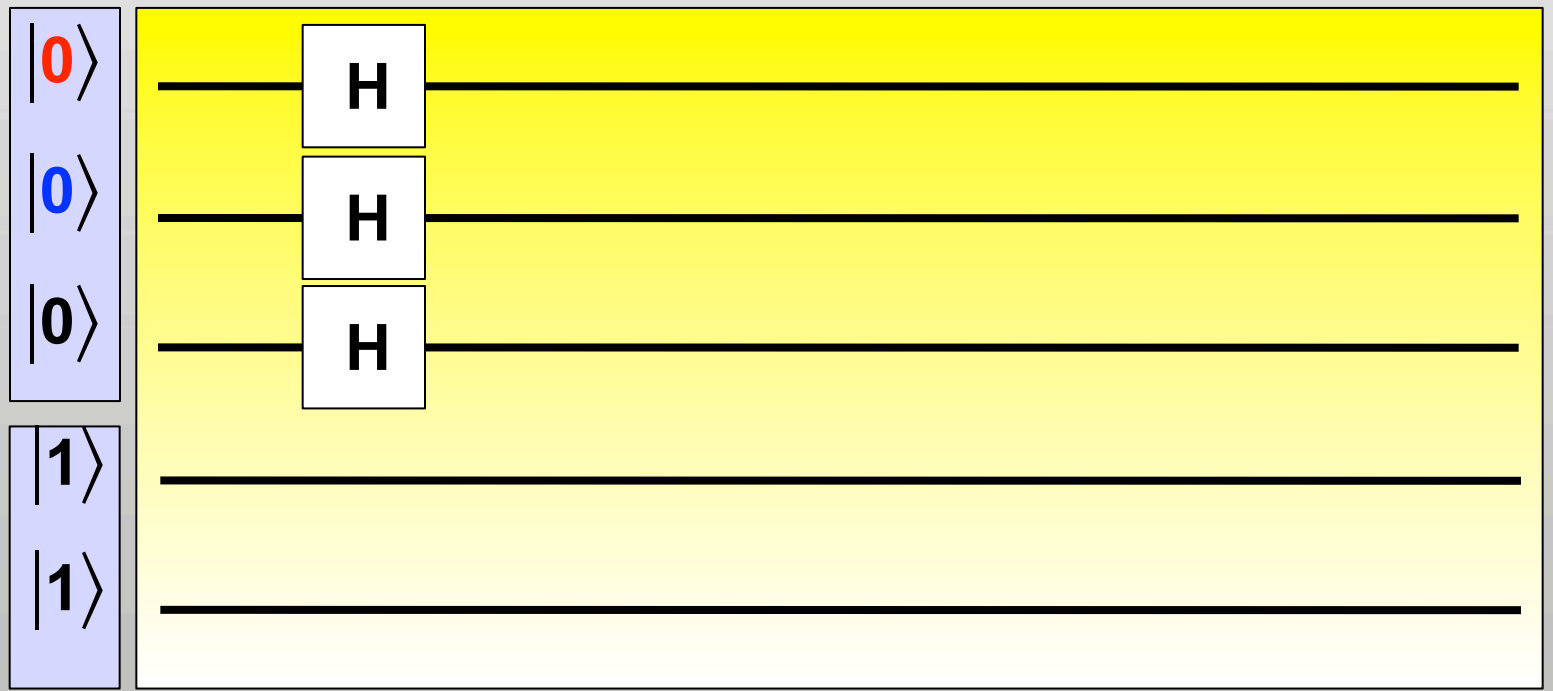

$$|\psi_1\rangle = (|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle + |7\rangle)|11\rangle$$

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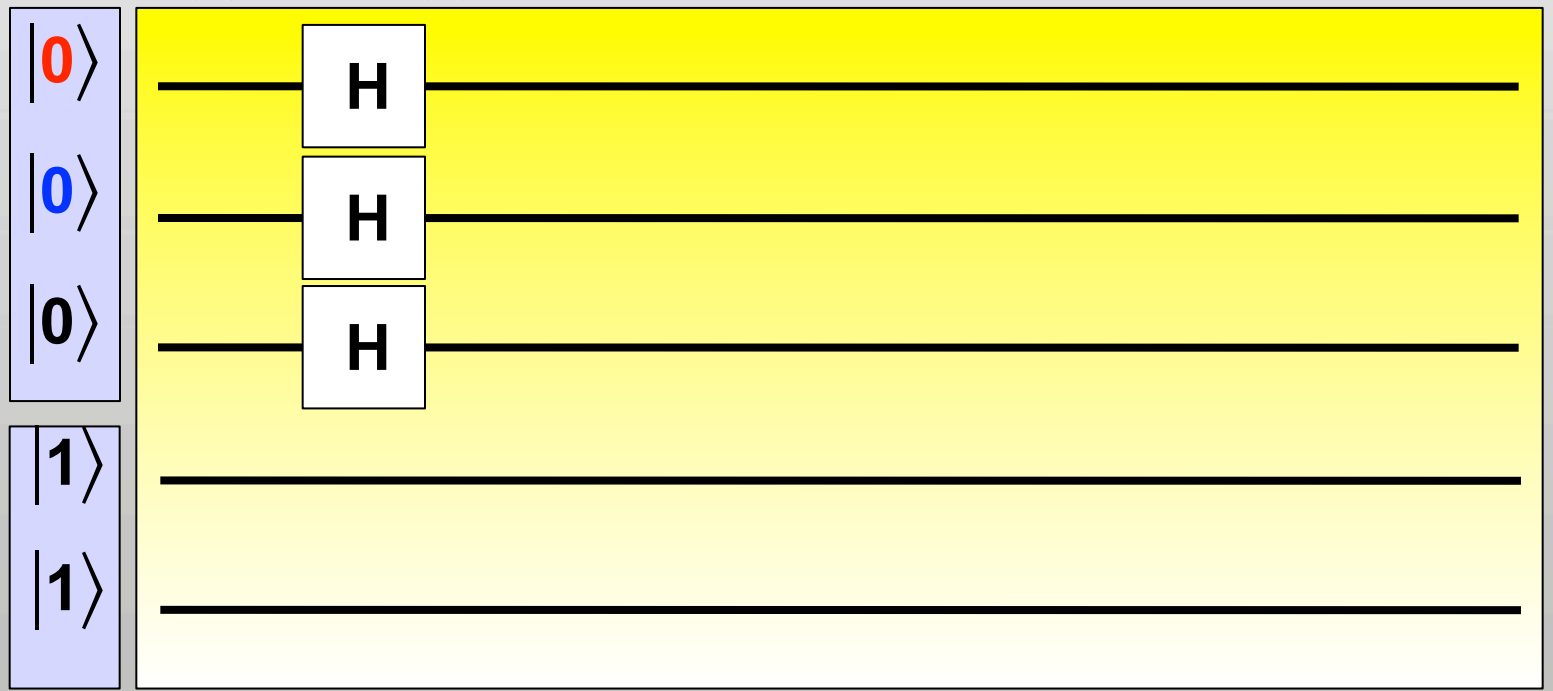

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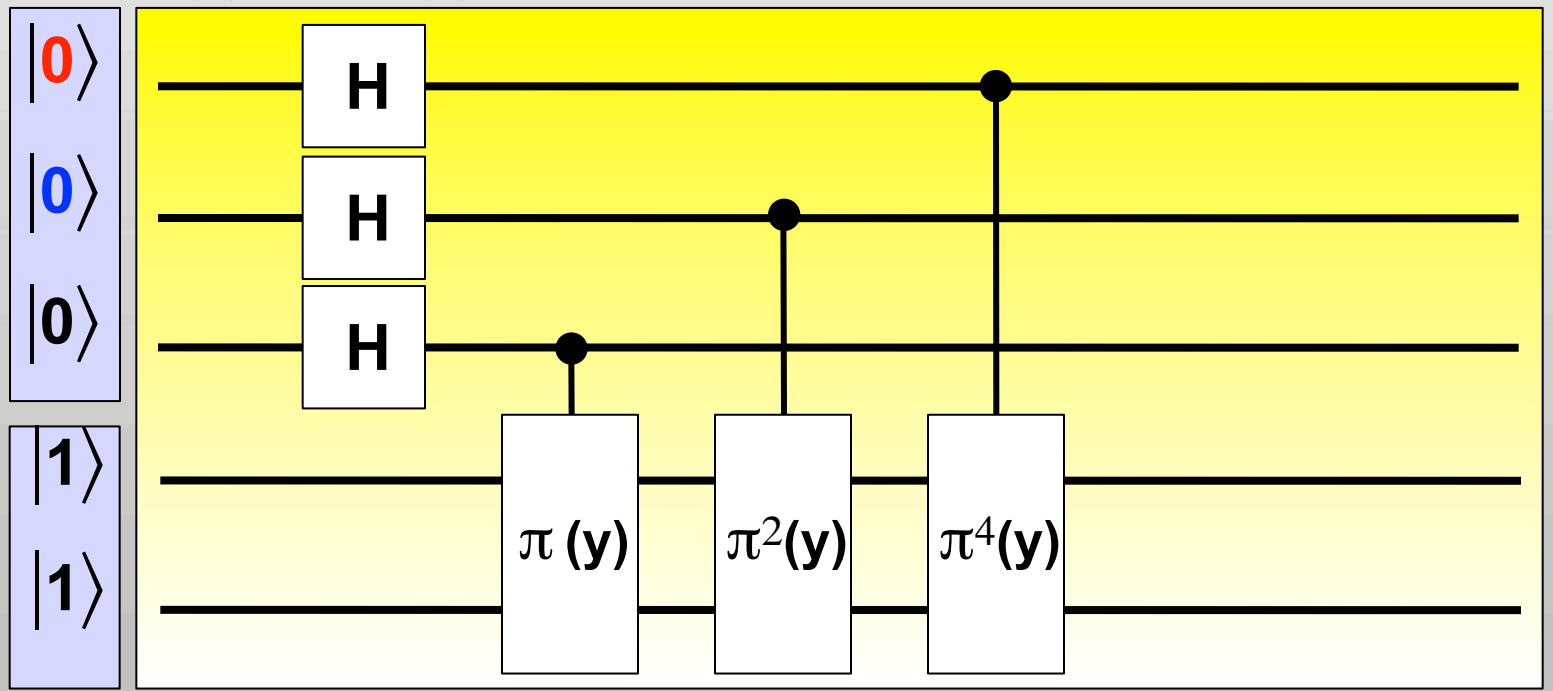

$$|\psi_2\rangle = (|0\rangle + |2\rangle + |4\rangle + |6\rangle)|1\rangle + (|1\rangle + |3\rangle + |5\rangle + |7\rangle)|3\rangle$$

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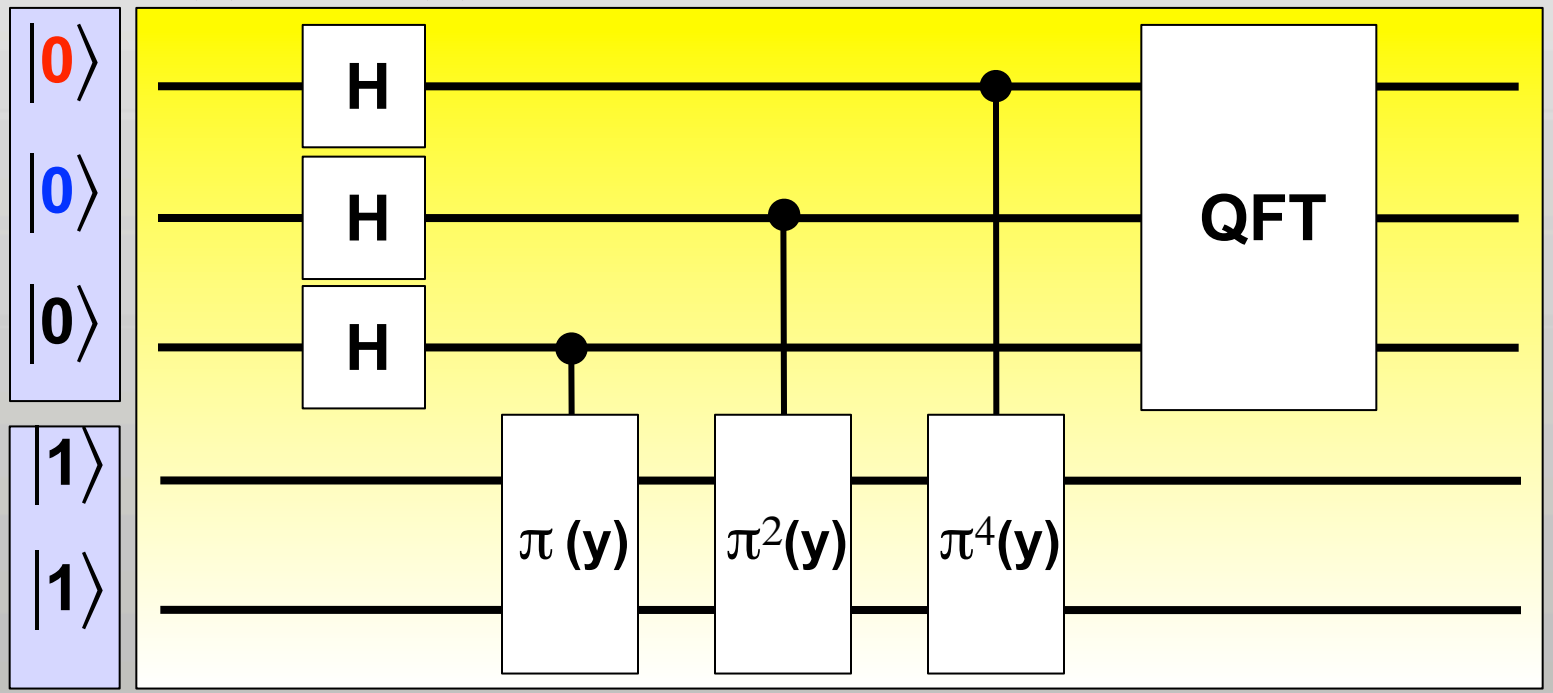

$$|\psi_3\rangle = (|0\rangle + |4\rangle)|1\rangle + (|0\rangle - |4\rangle)|3\rangle$$

Order finding: quantum circuit

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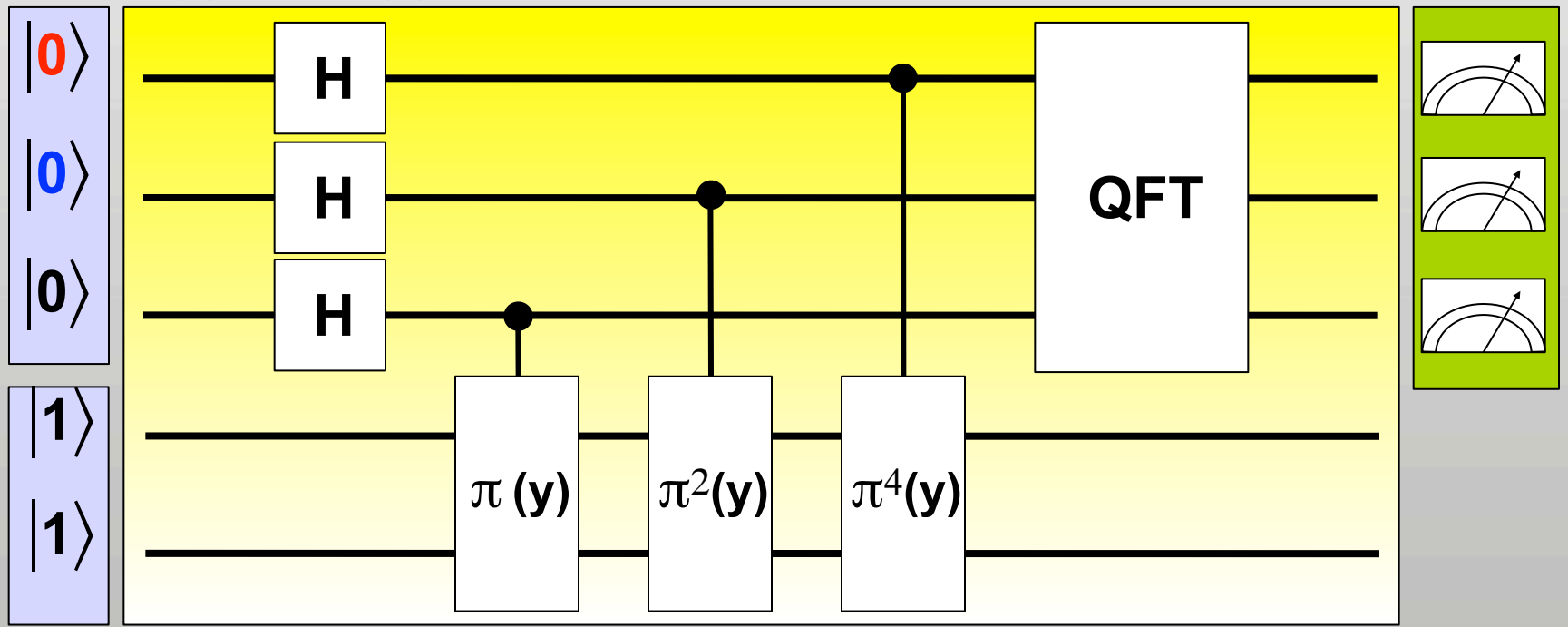

$|\psi_3\rangle = (|0\rangle + |4\rangle)|1\rangle + (|0\rangle - |4\rangle)|3\rangle$

measurement yields multiple of $2^n/r \Rightarrow r=2$

Order finding for n qubits

Lieven Vandersypen, PhD thesis: http://arxiv.org/abs/quant-ph/0205193

register 1 number of transitions between "rooms"

register 2 number of starting room

1) $|0\rangle|0\rangle$

2) $H^{\otimes n} \rightarrow \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} |j\rangle|0\rangle$

3) $f(j)=\pi^j(0) \rightarrow \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} |j\rangle|f(j)\rangle$

4) QFT r1 $\rightarrow \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} \sum_{k=0}^{2^n-1} e^{i2\pi jk/N} |k\rangle|f(j)\rangle \rightarrow \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} \sum_{j=0}^{2^n-1} e^{i2\pi jk/N} |k\rangle|f(j)\rangle$

5) Measure the amplitude of $|k\rangle \rightarrow \left(\sum_{j=0}^{2^n-1} e^{i2\pi jk/N} \right)^2$ (large for k=c*N/r)

Shor's algorithm

L. Vandersypen et al.: Nature 414, 883 (2001)

number to factorize: N
(e.g. N=15)

integer a does NOT divide N
(a=2,4,7,8,11,13,14)

$f(x,y)=\pi^x(y)=a^x y \bmod N$

one factor is $\gcd(a^{r/2}\pm 1,N)$
(probability >0.5)

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$a^4 \bmod 15 = 1,\ a \in \{2,7,8,13\} \Rightarrow a^{2k} \bmod 15 = 1,\ k \geq 2$

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f(x) needs x_0 and x_1

Shor's algorithm

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f(x) needs x_0 and x_1

register 2 needs 4 qubits $m=(\log_2 N)$

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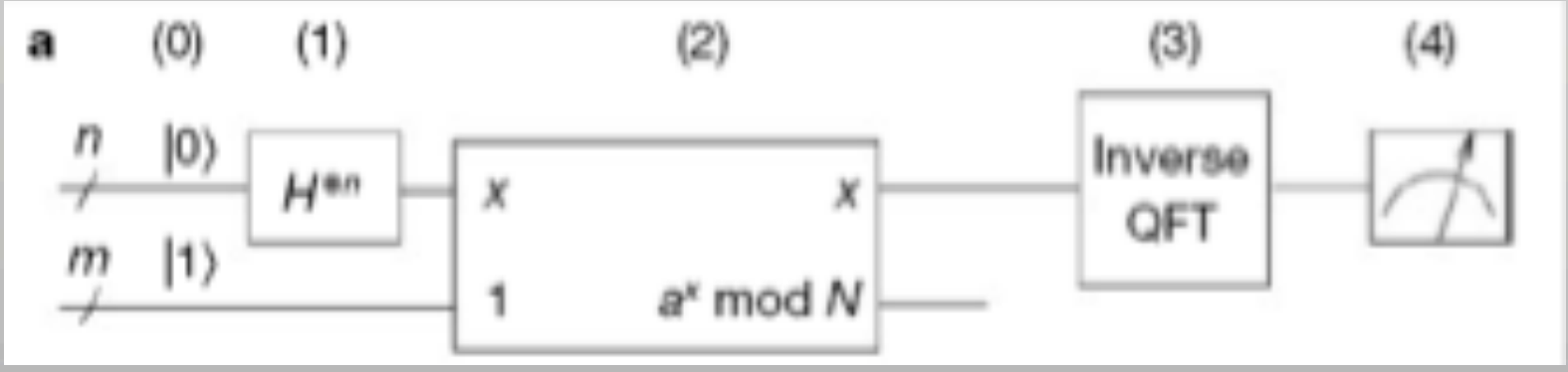
f(x) needs x_0 and x_1

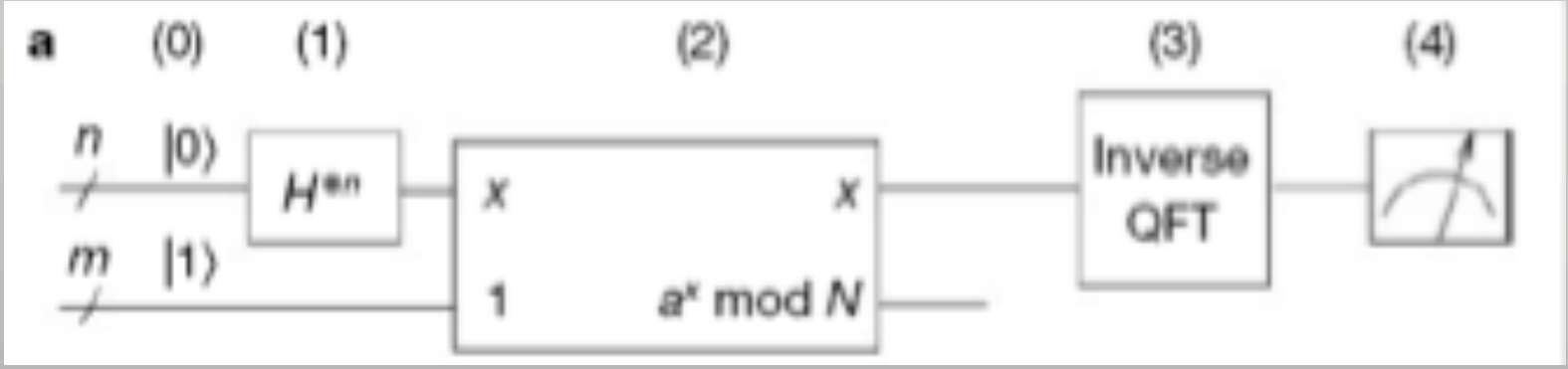
register 1 needs 2 qubits (take 3) $n=2m$

register 2 needs 4 qubits $m=(\log_2 N)$

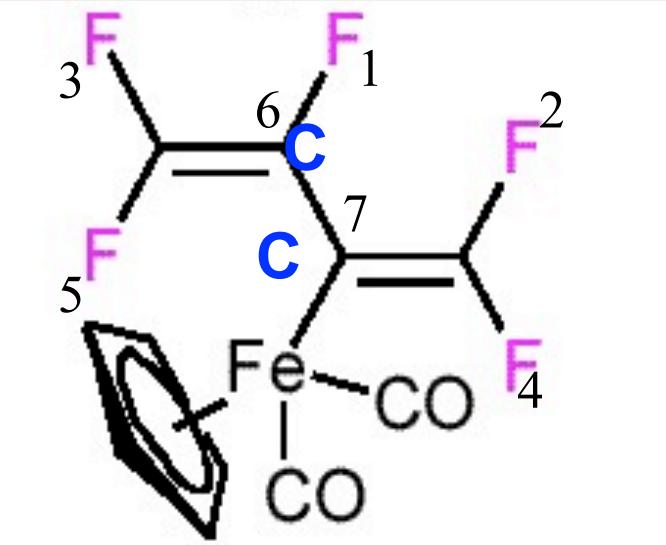
Implementation of Shor's

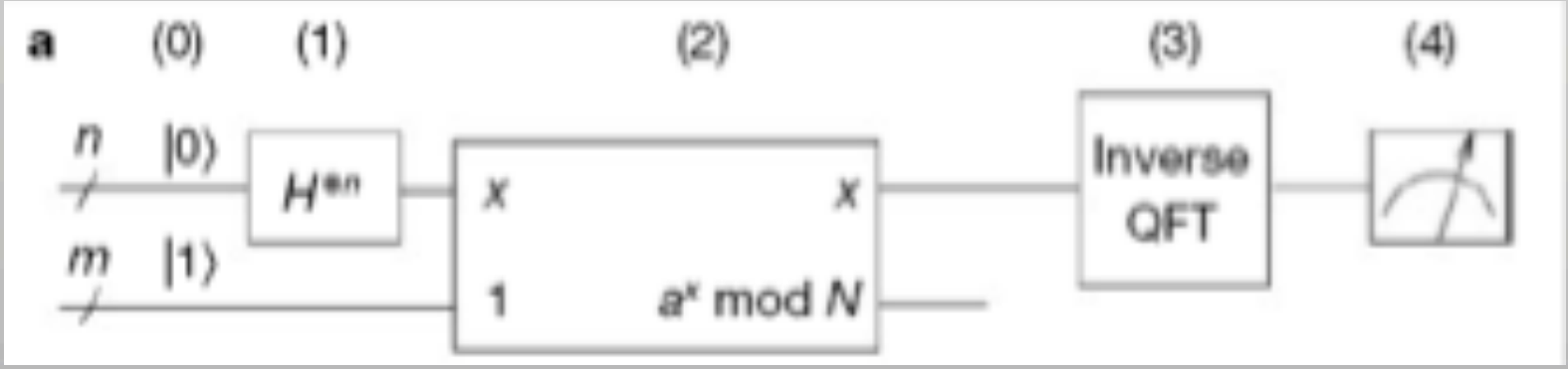


Implementation of Shor's

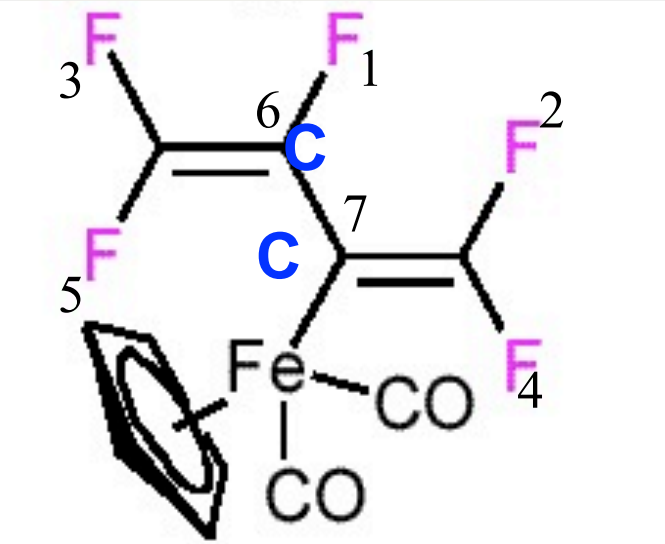


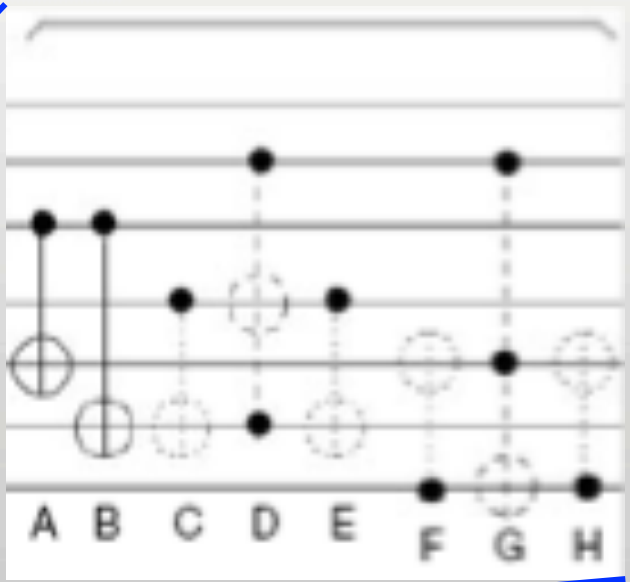
Implementation of Shor's

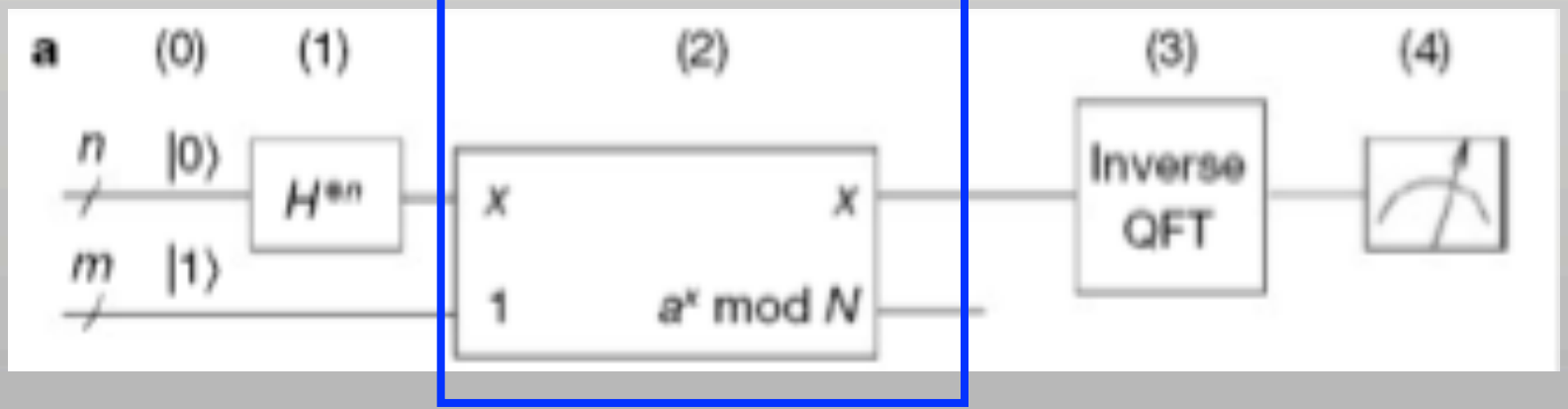




Implementation of Shor's

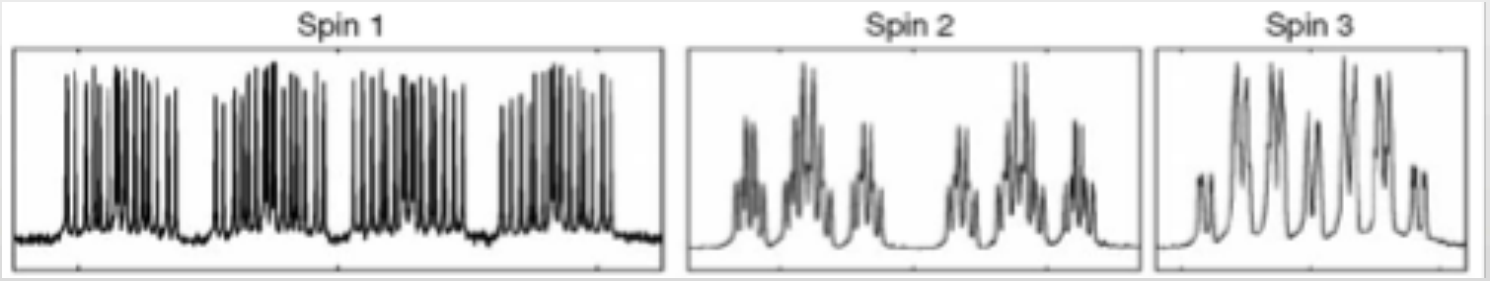




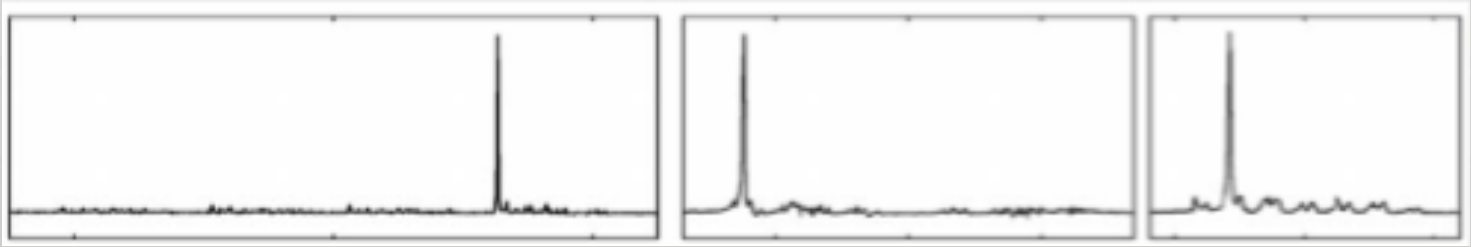


Measurement

thermal spectra

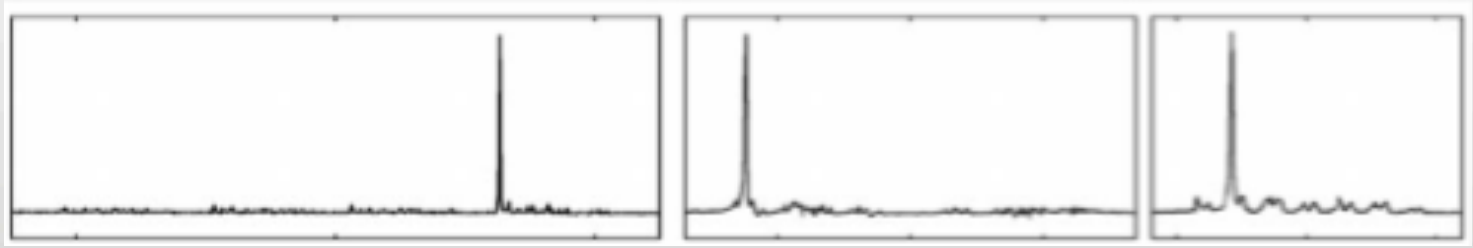


effective pure state



Measurement

effective pure state



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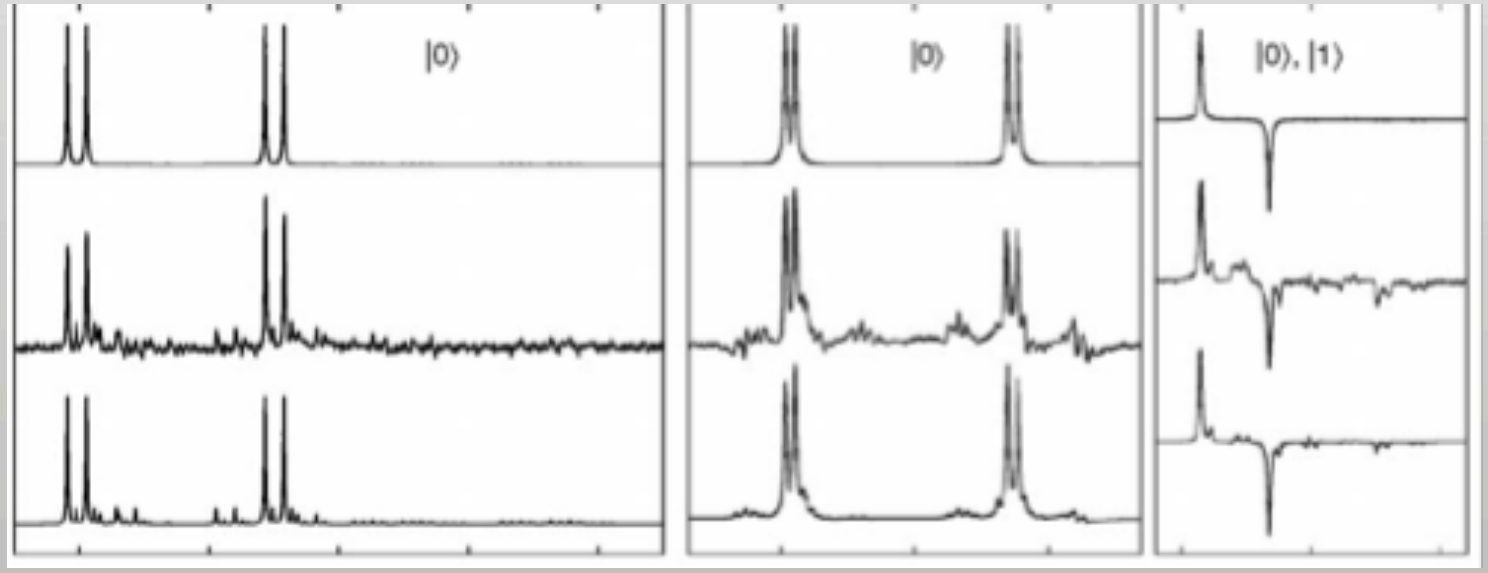
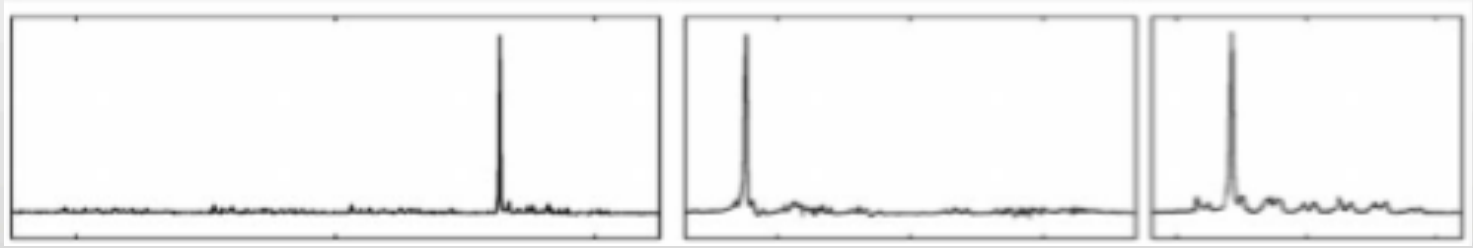


Measurement

effective pure state



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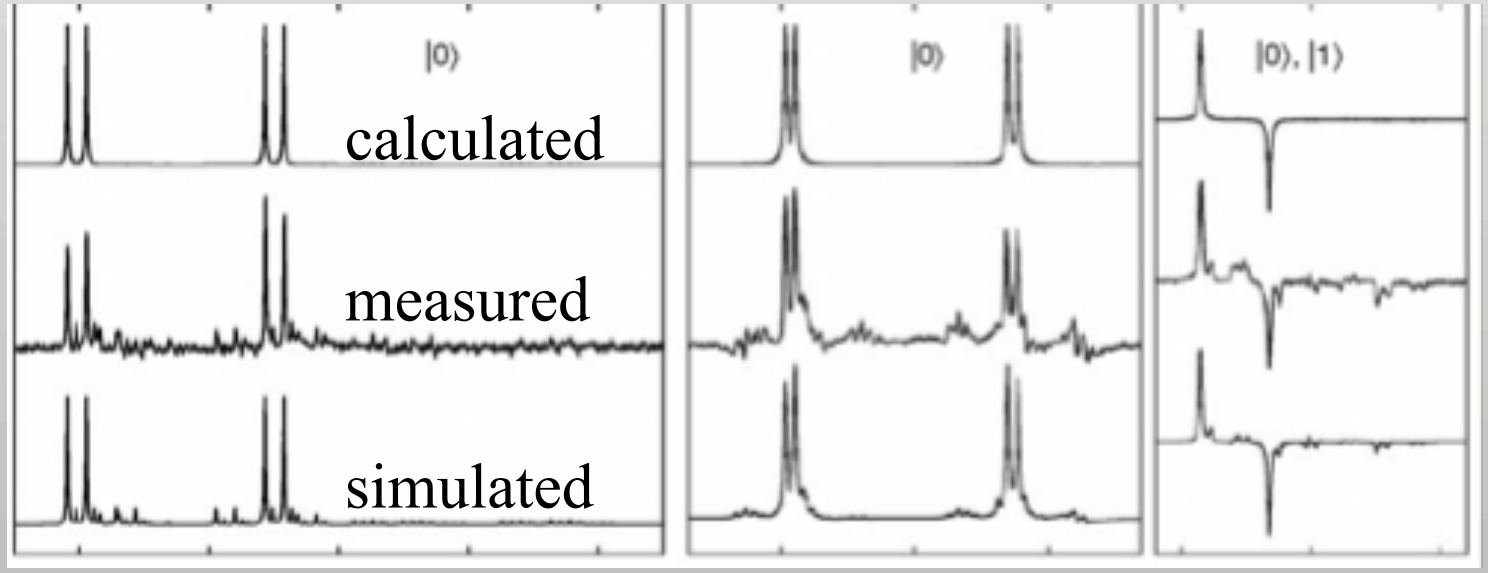
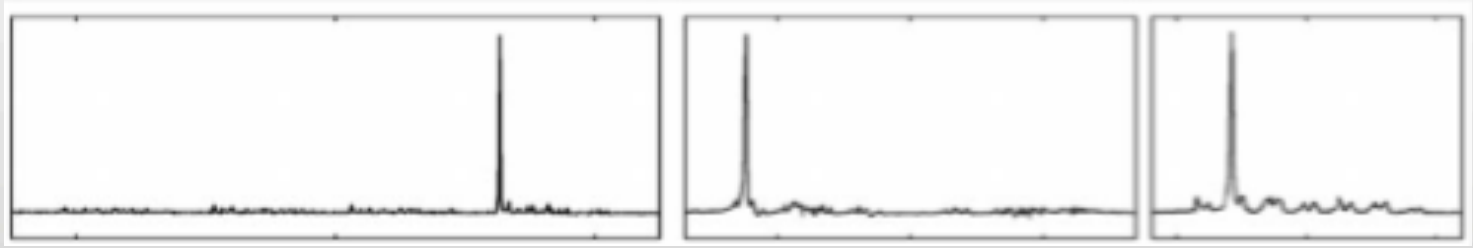


Measurement

effective pure state



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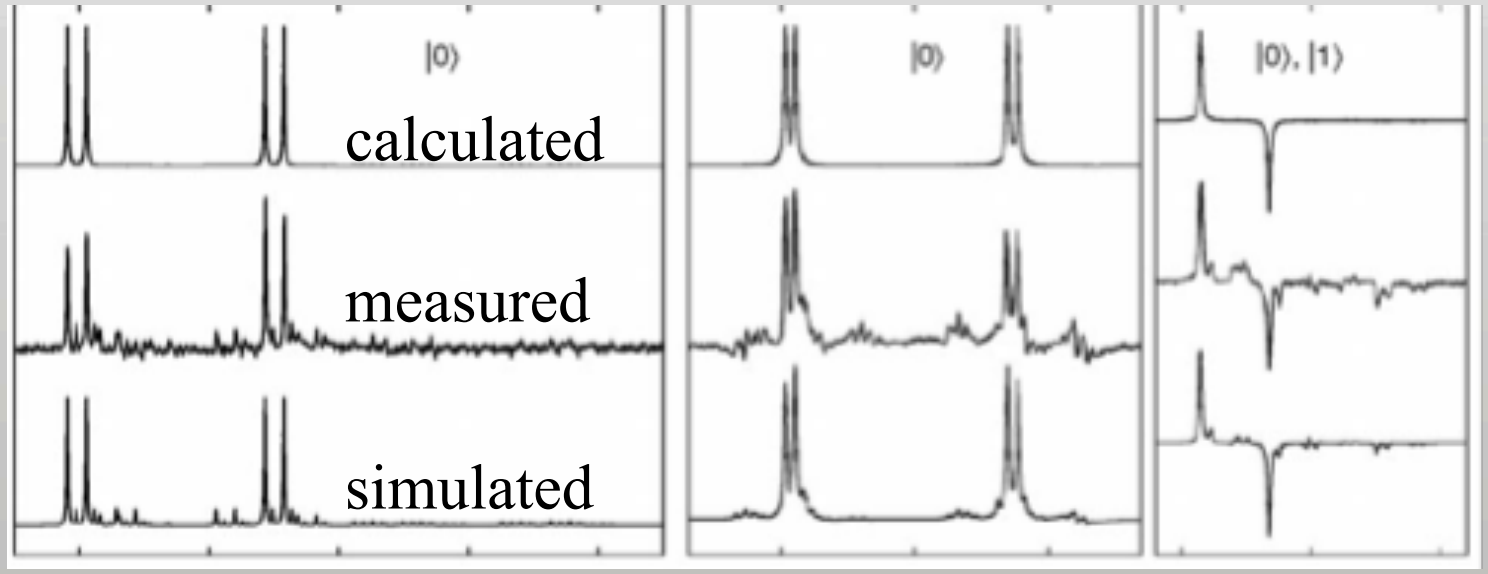
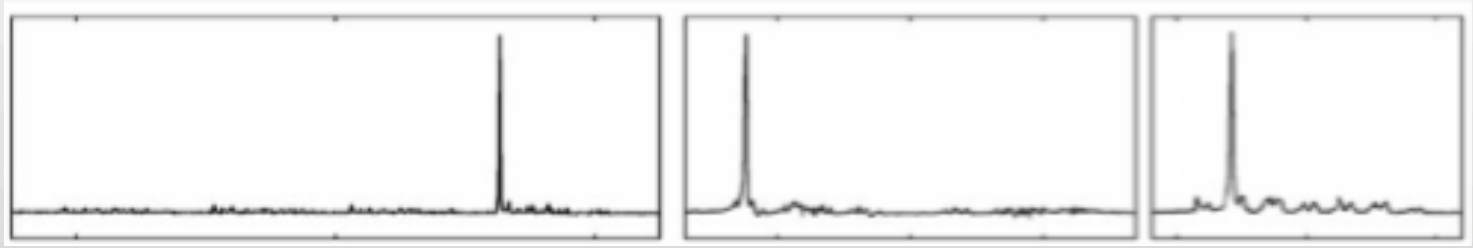


$x_0=0$ $x_1=0$ $x_2=|0\rangle+|1\rangle$

$x=|000\rangle+|100\rangle$

Measurement

effective pure state



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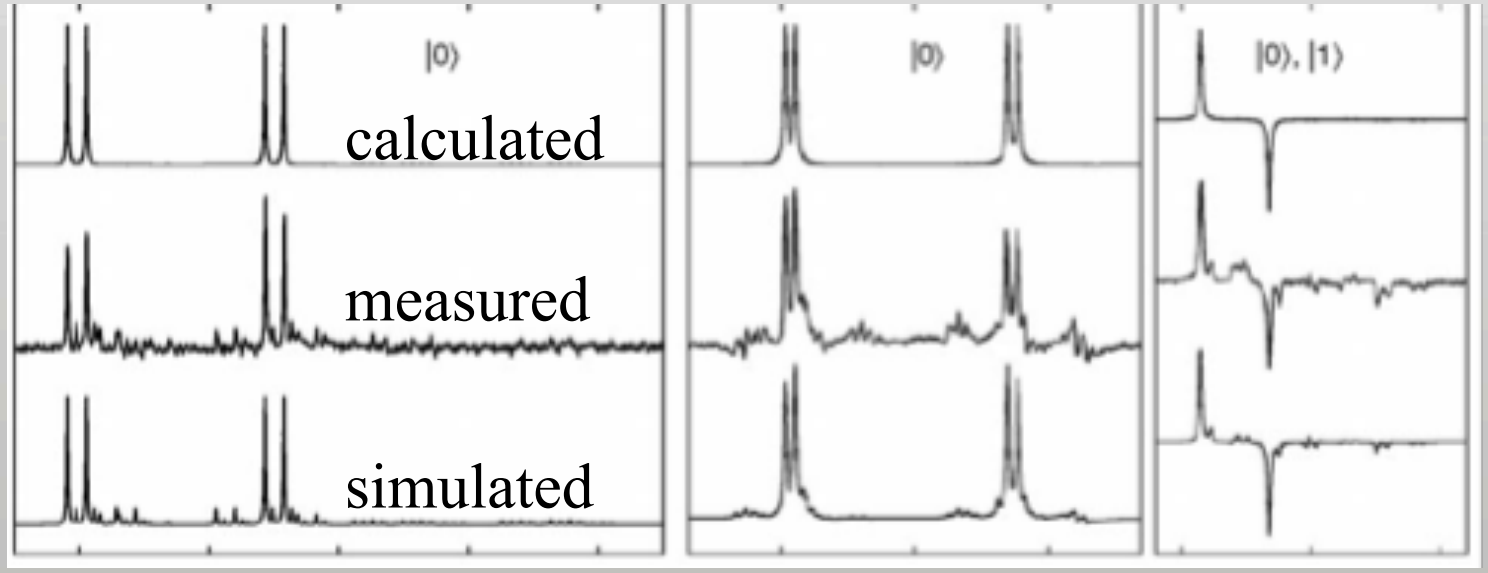
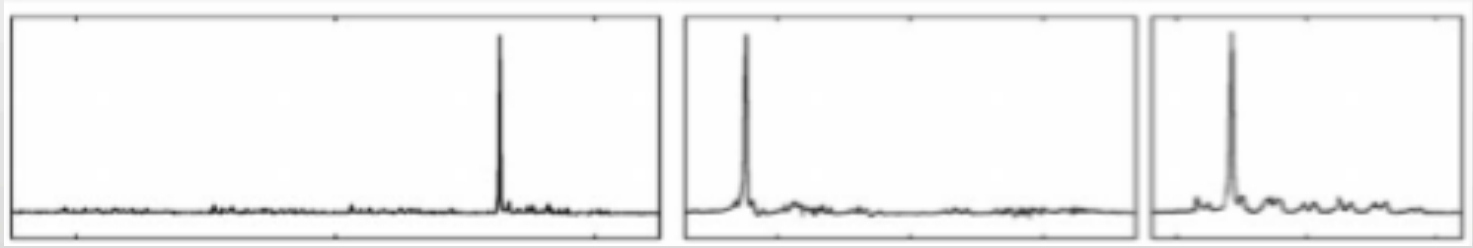


$x_0=0$ $x_1=0$ $x_2=|0\rangle+|1\rangle$

$x=|000\rangle+|100\rangle = |0\rangle+|4\rangle$

Measurement

effective pure state



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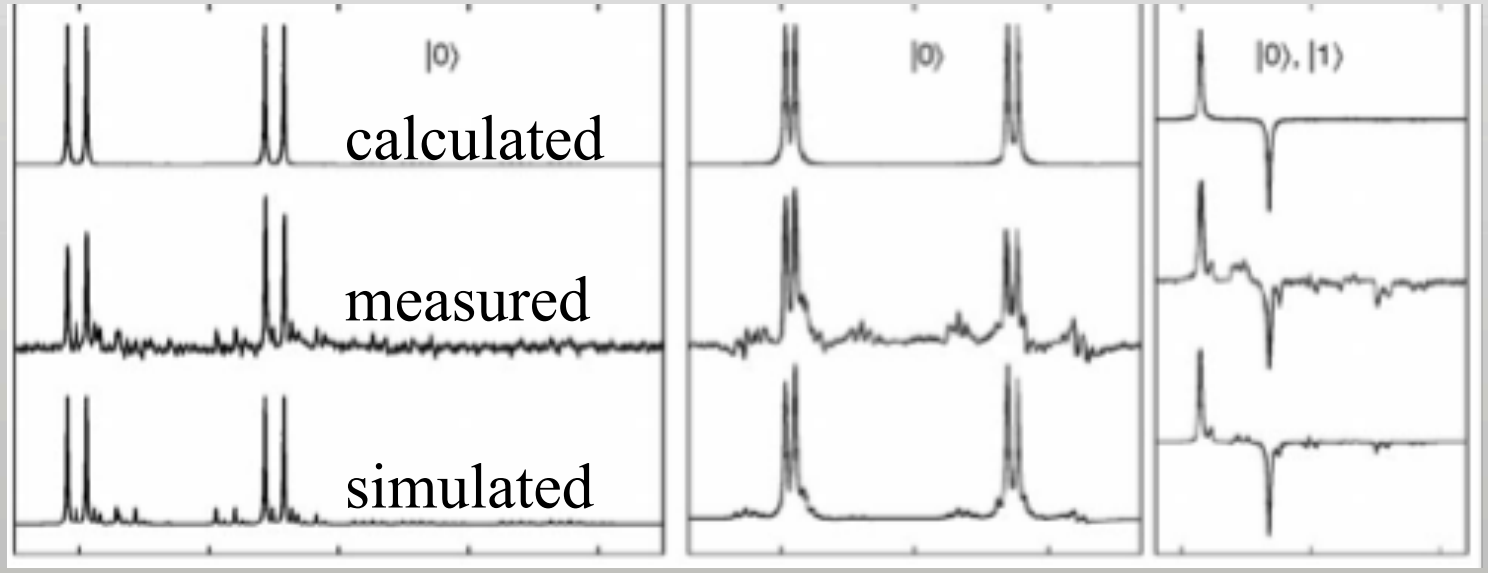

$x_0=0$ $x_1=0$ $x_2=|0\rangle+|1\rangle$

$x=|000\rangle+|100\rangle = |0\rangle+|4\rangle \Rightarrow r=2^n/4=2$

Measurement

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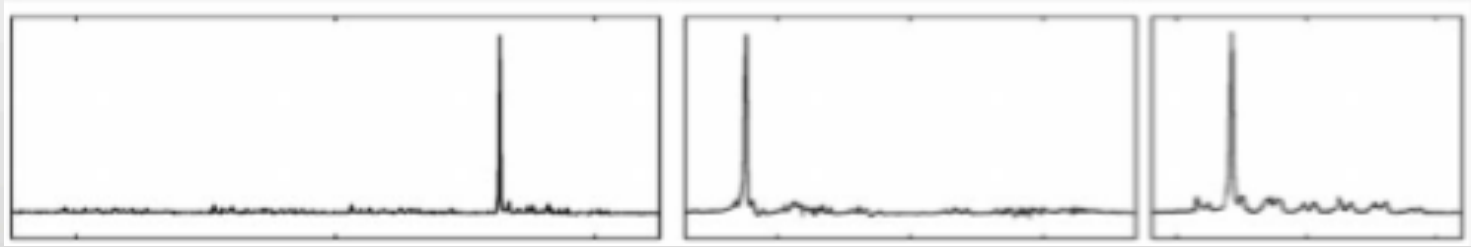

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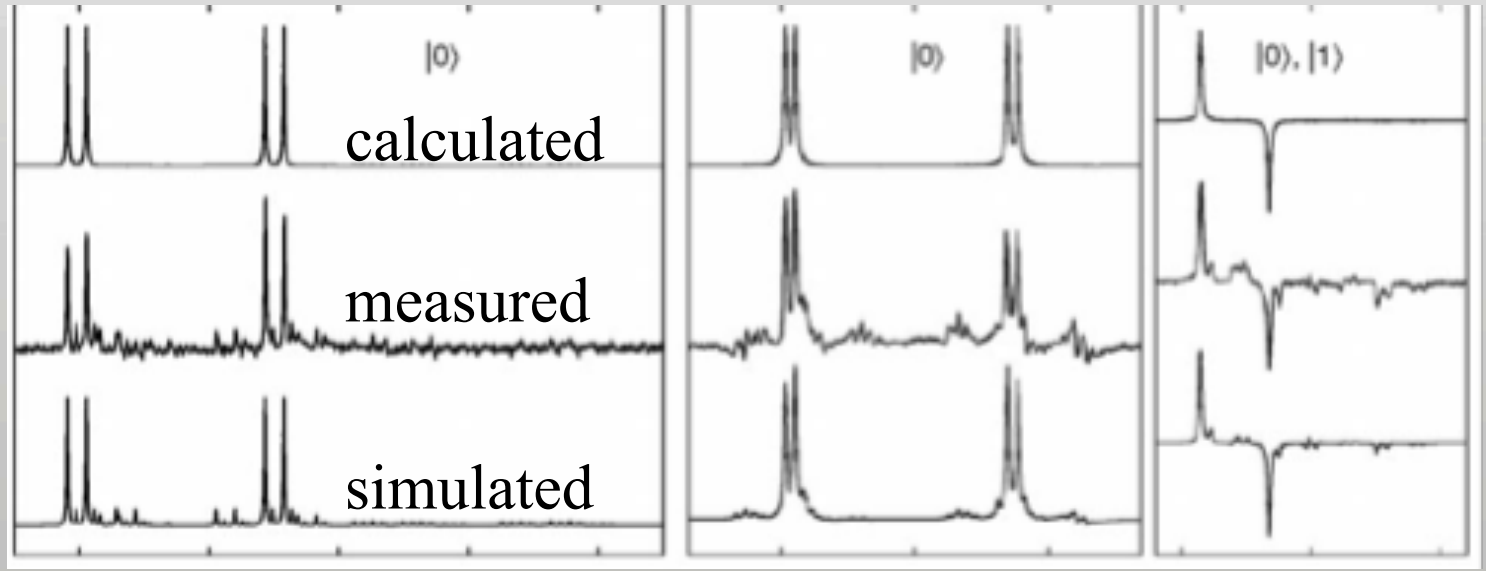

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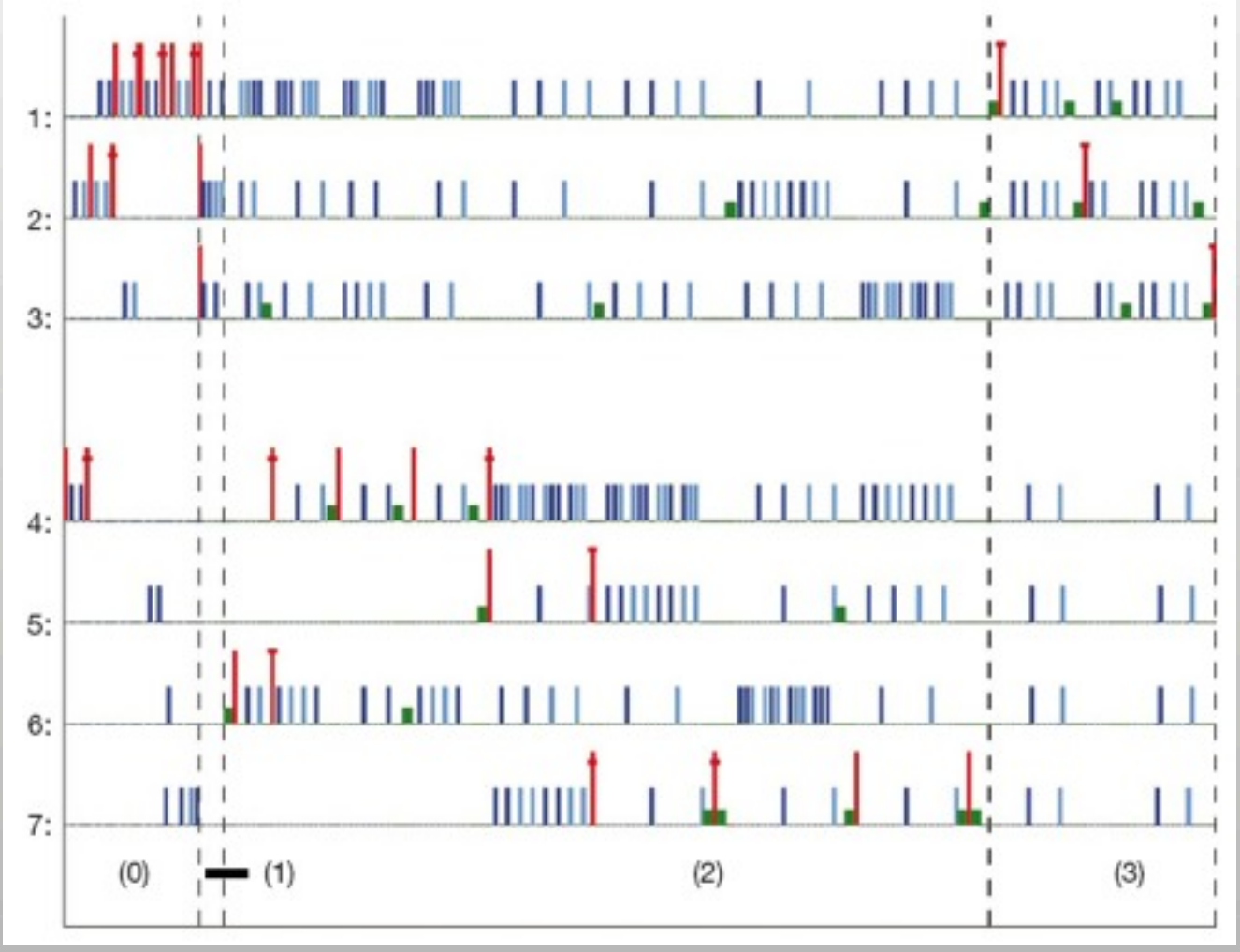
$x=|000\rangle+|100\rangle = |0\rangle+|4\rangle \Rightarrow r=2^n/4=2$ $\gcd(11^{2/2}\pm1,15)=3,5$

Pulse sequence

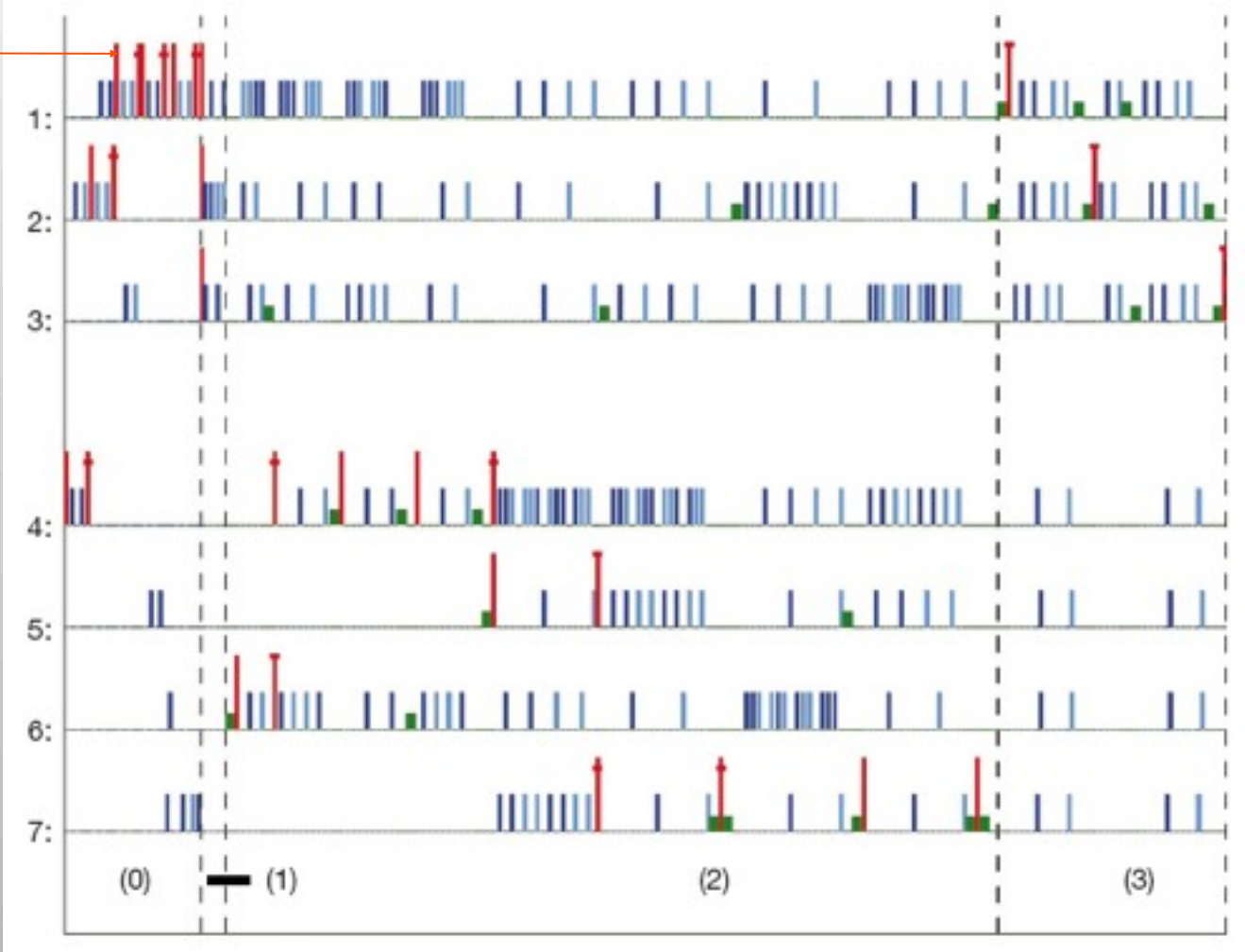
total ~300 pulses



Pulse sequence

total ~300 pulses

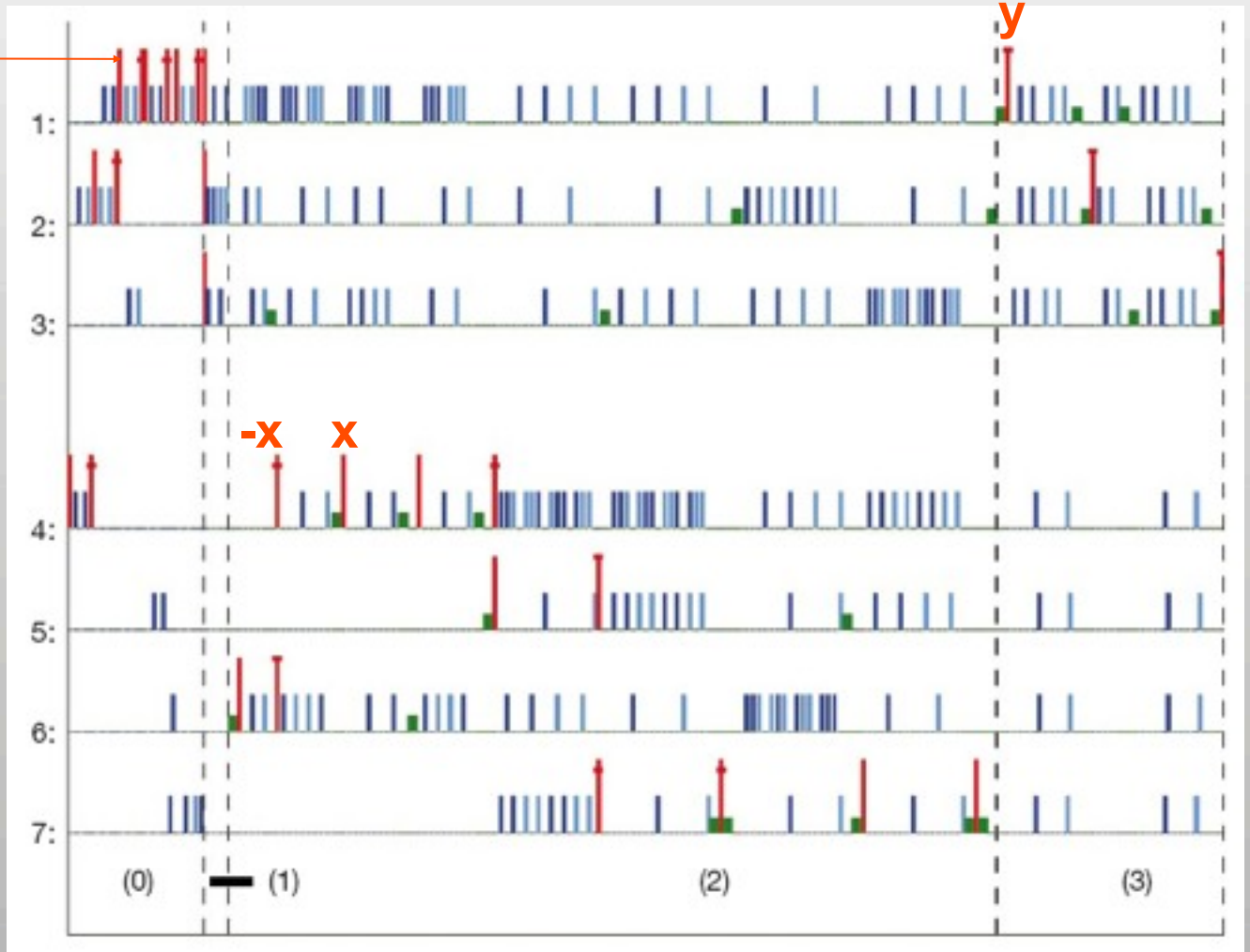
90°pulse



Pulse sequence

total ~300 pulses

90°pulse



Pulse sequence

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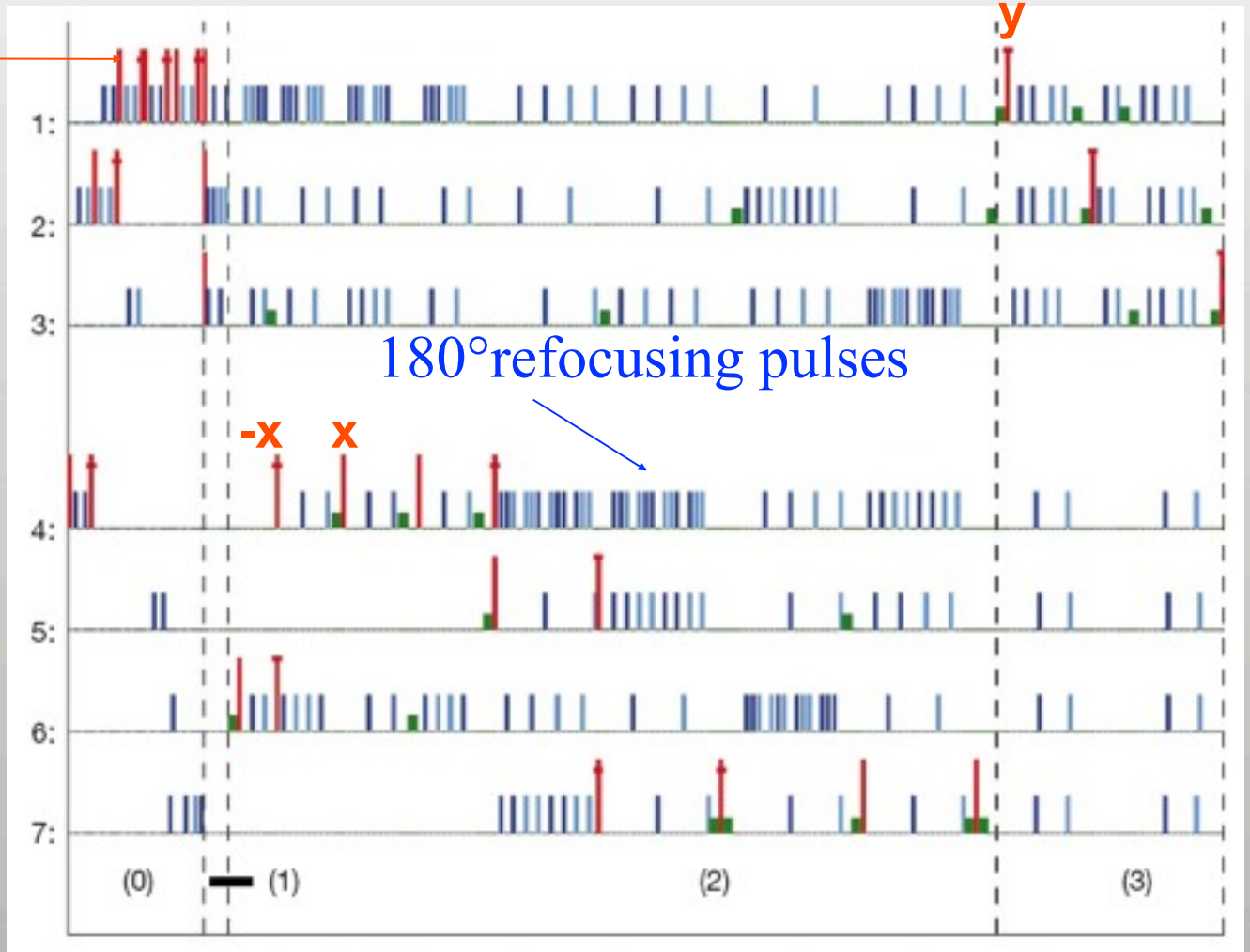
90°pulse



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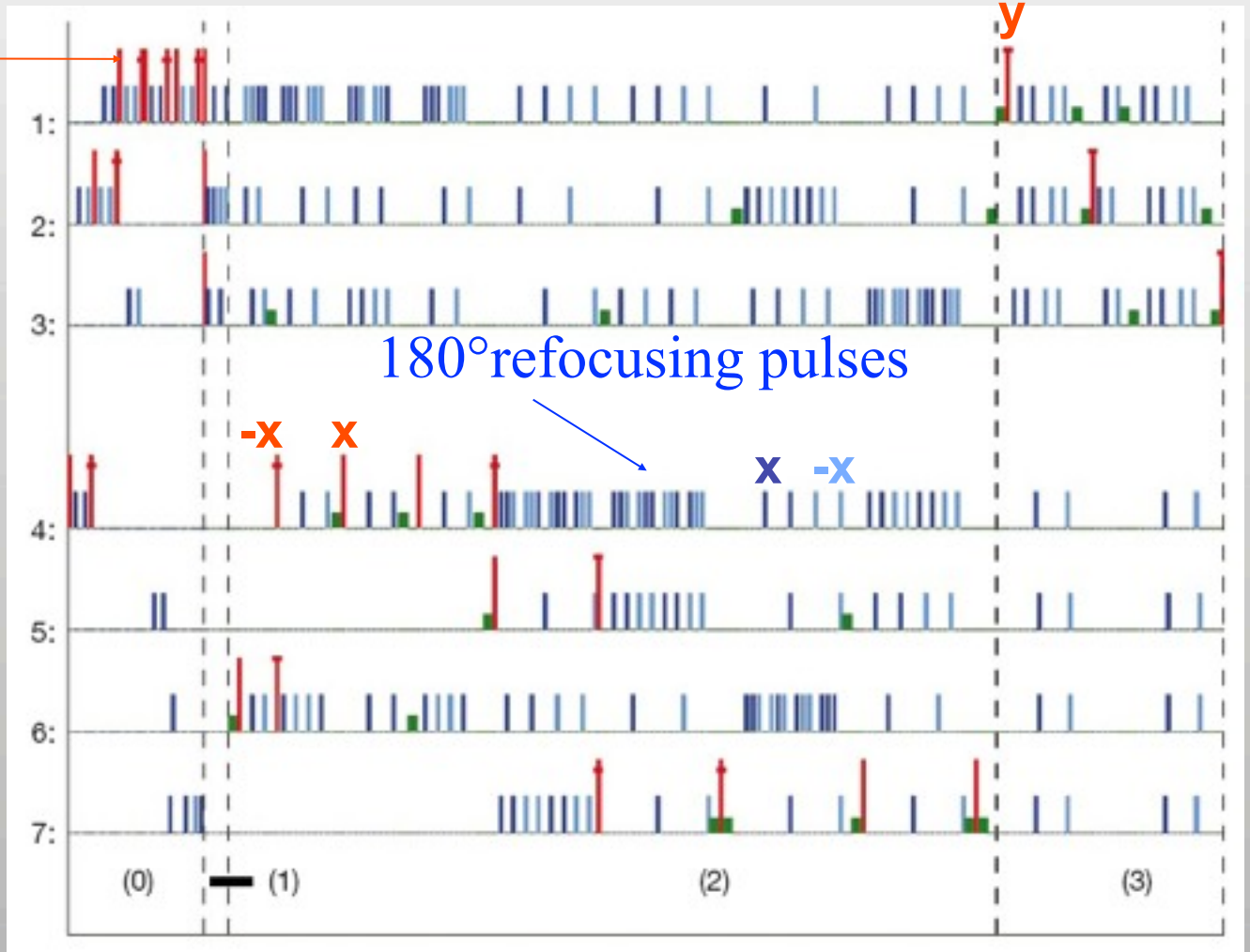

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