# The hadronic contribution to the muon anomalous magnetic moment

*Zoltan Fodor*[‡ ⋆], *Eric B. Gregory*[‡], *Christian Hoelbling*[‡], *Stefan Krieg*[‡ ⋆], *Laurent Lellouch*[♯ $], *Rehan Malak*[♯ $], *Craig McNeile*[‡ ††], *Alfonso Sastre*[‡], *Kalman Szabo*[‡]

We present preliminary lattice results for the leading-order hadronic contribution to the muon anomalous magnetic moment, calculated with HEX-smeared clover fermions. In our calculation we include 2+1-flavor ensembles with pions at the physical mass.

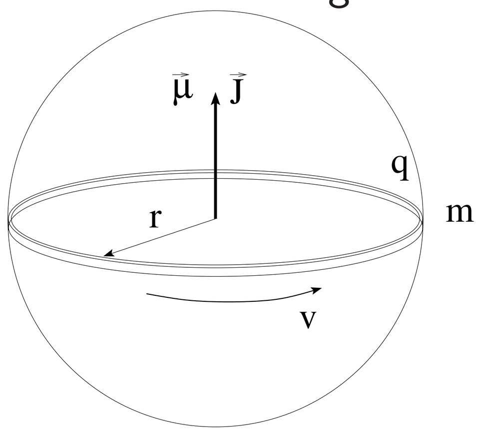
[‡]University of Wuppertal, [⋆]Forschungszentrum Jülich, [♯]Aix-Marseille Universitè, [$]Universitè de Toulon, [††]University of Plymouth

## Introduction: What is $g_\mu - 2$?

The $g$-factor is a dimensionless proportionality factor relating a particle's magnetic moment $\vec{\mu}$ to its angular momentum $\vec{J}$:

$$\vec{\mu} = g\frac{q}{2m}\vec{J}$$

For a classical ring with charge $q$ and mass $m$:




$$|\vec{\mu}| = IA = \left(\frac{qv}{2\pi r}\right)\left(\pi r^2\right) = \frac{qvr}{2}$$

$$\left|\vec{J}\right| = mvr$$

$$\vec{\mu} = \tfrac{q}{2m}\vec{J} \quad \to \; g = 1$$

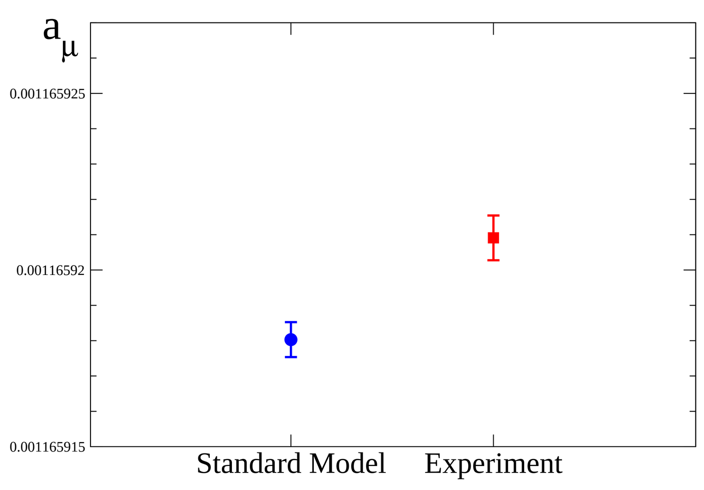
No $r$- or $v$-dependence, so a classical rotating charged sphere also has $g = 1$

- Dirac's equation for relativistic quantum mechanics of fermions predicts spin-$\frac{1}{2}$ particles such as electrons and muons have *intrinsic spin*, a type of angular momentum which is *twice* as effective at producing magnetic moment, *i.e*, $g = 2$!
- Quantum Field Theory gives a more complete picture, describing a soup of virtual particle-antiparticle pairs fluctuating in and out of existence and interacting with the $\mu$ (or $e$) and contributing to its magnetic moment. Thus $g_\mu = 2.002....$!

$a_\mu \equiv \frac{g_\mu - 2}{2}$ is known as the *muon anomalous magnetic moment*.

## Does the Standard Model explain measured $g_\mu - 2$?

- The Standard Model (SM) describes the known particles which compose the matter in the universe: quarks, leptons, neutrinos, the Higgs boson, and gauge bosons which mediate the electromagnetic, weak and strong interactions.
- The SM has been exceptionally successful at describing the results of experiments, but it is thought that it may someday be replaced by a more fundamental theory, incorporating a quantum description of gravity and perhaps currently unknown particles and interactions.
- The LHC has so far failed to find direct evidence of "Beyond Standard Model" (BSM) particles or interactions.
- A viable BSM theory must explain experimental results for which the SM fails.



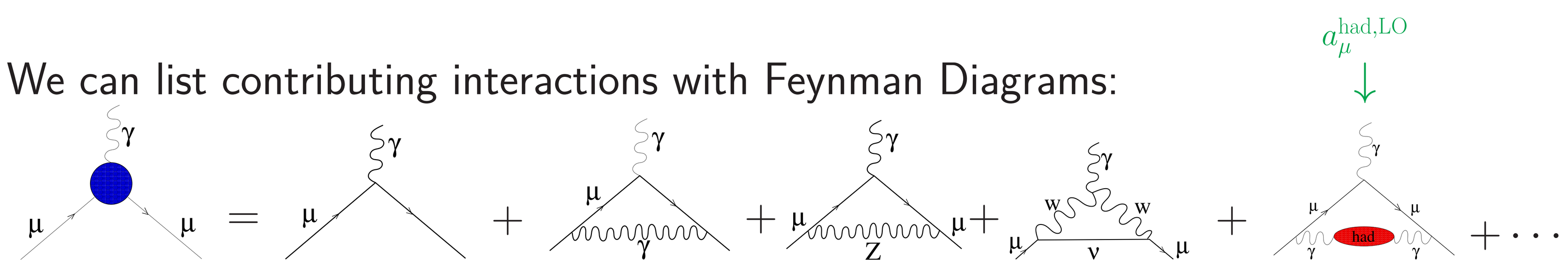

- $a_\mu$ is determined both experimentally and theoretically to $\sim 500$ ppb.
- Is $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(63)(49) \times 10^{-11}$ a hint of BSM physics? Or noise?
- Experimental precision will improve to 140 ppb with "Muon $g_\mu - 2$" experiment at Fermilab.

## Calculating $a_\mu$ from the Standard Model

$a_\mu^{\rm SM}$ is calculated as the sum of contributions from electromagnetic, weak and strong forces to the interaction between a photon $\gamma$ and muon $\mu$.

$$a_\mu^{\rm SM} \equiv \frac{g_\mu - 2}{2} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$$

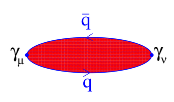
We can list contributing interactions with Feynman Diagrams:




At $\sim 1\%$ precision, calculation of the hadronic contribution $a_\mu^{\rm had}$ seems precise, but hadronic interactions contribute:

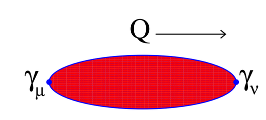
- $0.006\%$ of $a_\mu^{\rm SM}$
- 99.98% of the uncertainty of $a_\mu^{\rm SM}$

Current calculations of $a_\mu^{\rm had}$ are based on phenomenology. Can Lattice QCD do better?

In the right-most diagram above,  represents all possible quark and gluon interactions connecting two photons, the hadronic vacuum polarization $\hat{\Pi}(Q^2)$. can be calculated on the lattice to give the *leading-order hadronic contribution* $a_\mu^{\rm had,LO}$.

## $a_\mu^{\rm had,LO}$ from Lattice QCD

Recipe for leading order contribution, called hadronic vacuum polarization (HVP):

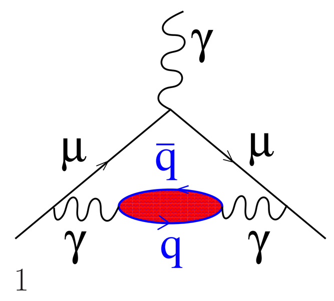



$$\equiv \Pi_{\mu\nu}(Q) = \sum_{x-y} e^{iQ(x-y)}\langle J_\mu(x) J_\nu(y)\rangle$$

- ⋆ Generate statistical ensemble of lattice gauge field configurations
- ⋆ On each configuration, measure correlators of electromagnetic current operators $J_\nu(x) \sum_{i=1}^{N_f} Q_i \overline{\psi}^i(x)\gamma_\mu\psi^i(x)$ (which couple to photons).
- Fourier transform to get a function $\Pi_{\mu\nu}(Q)$ of discrete lattice momentum $Q$,
- Calculate the vacuum-subtracted HVP scalar through:

$$\Pi_{\mu\nu}(Q) = \left(Q^2\delta_{\mu\nu} - Q_\mu Q_\nu\right)\Pi(Q^2)$$

- Fit the discrete data points $\Pi(Q^2)$ to a smooth function $\Pi_{\rm sm}(Q^2)$, subtract the $Q^2 = 0$ value to get $\hat{\Pi}_{\rm sm}(Q^2) \equiv 4\pi\alpha\left[\Pi_{\rm sm}(Q^2) - \Pi_{\rm sm}(0)\right]$ and integrate:




$$a_\mu^{\rm had,LO}(Q^2) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 f(Q^2)\hat{\Pi}_{\rm sm}(Q^2),$$

with $f(Q^2) = \frac{m_\mu^2 Q^2 Z(Q^2)^3\left(1 - Q^2 Z(Q^2)\right)}{1 + m_\mu^2 Q^2 Z(Q^2)^2}$ and $Z = -\frac{Q^2 - \sqrt{Q^4 + 4m_\mu^2 Q^2}}{2m_\mu^2 Q^2}$

⋆ Performed at JSC.

## $a_\mu^{\rm had,LO}$ from Lattice QCD

### Configuration Parameters

**2-HEX** $(N_f = 2+1)$

| | $am_{ud}^{\rm bare}$ | $am_s^{\rm bare}$ | volume | # cfgs | $M_\pi$ (GeV) |
|---|---|---|---|---|---|
| | | | $\beta = 3.31,\ a^{-1} = 1.697$ GeV | | |
| ⋆ | -0.09933 | -0.0400 | $48^3 \times 48$ | 928 | 0.136(2) |
| | -0.09300 | -0.0400 | $24^3 \times 48$ | 210 | 0.255(2) |
| | | | $\beta = 3.5,\ a^{-1} = 2.131$ GeV | | |
| ⋆ | -0.05294 | -0.0060 | $64^3 \times 64$ | 83 | 0.130(2) |
| † | -0.04900 | -0.0120 | $32^3 \times 64$ | 216 | 0.250(2) |
| | -0.04900 | -0.0060 | $32^3 \times 64$ | 110 | 0.258(2) |
| | -0.04630 | -0.0120 | $32^3 \times 64$ | 212 | 0.308(2) |
| | | | $\beta = 3.61,\ a^{-1} = 2.561$ GeV | | |
| | -0.03000 | -0.0042 | $32^3 \times 48$ | 188 | 0.332(4) |
| | | | $\beta = 3.7,\ a^{-1} = 3.026$ GeV | | |
| | -0.02700 | 0.0000 | $64^3 \times 64$ | 208 | 0.182(2) |

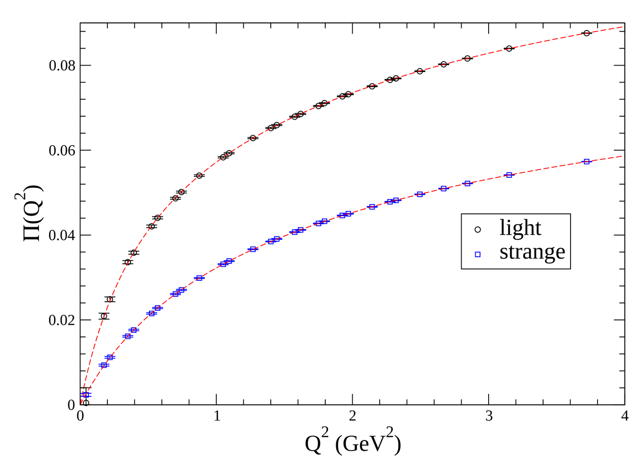
**3-HEX** $(N_f = 4)$

| $am_u^{\rm bare}$ | $am_d^{\rm bare}$ | $am_s^{\rm bare}$ | $am_c^{\rm bare}$ | volume | # cfgs | $M_\pi$ (GeV) |
|---|---|---|---|---|---|---|
| | | | $\beta = 3.2,\ a^{-1} = 1.897$ GeV | | | |
| -0.0806 | -0.0794 | -0.033 | 0.71 | $32^3 \times 64$ | 240 | 0.250 |

We do 5-parameter fits using

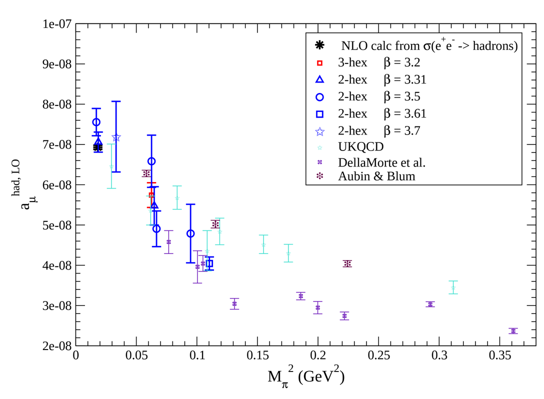
$$\Pi(Q^2) = a + \frac{b_1}{Q^2 + c_1} + \frac{b_2}{Q^2 + c_2}$$

⋆ physical mass pions.

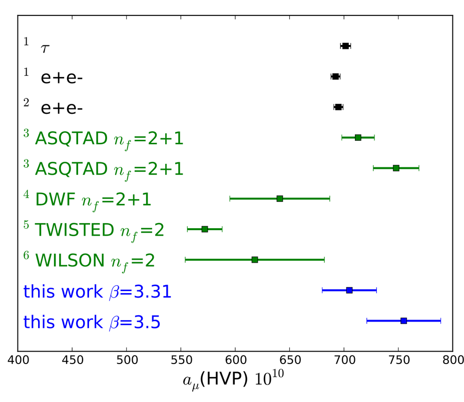



HVP $\hat{\Pi}(Q^2)$ for light $u$, $d$ and strange $s$ quarks for †.

## Results & future plans



Results as a function of pion mass $M_\pi$ compared to other lattice calculations.




Our results from physical pion mass simulations compared to other lattice and phenomenological determinations.

[1]M. Davier, *et al.*(2011)
[2]K. Hagiwara, *et al.*(2011)
[3]C. Aubin and T. Blum (2007)
[4]P. Boyle,*et al.*(2012)
[5]X. Feng, *et al.* (2011)
[6]M. Della Morte *et al.* (2012)

### Conclusions

- We have made a preliminary calculation of the connected part of $a_\mu^{\rm had,LO}$ on the lattice with pions at the physical mass.
- Results are consistent with phenomenological determinations from $e^+e^-$ and $\tau$ decay data.
- Preliminary results presented at Lattice 2013 conference and in [1].
- Results from further ensembles are forthcoming, as are estimates of the "disconnected diagram contribution" to $a_\mu^{\rm had,LO}$

[1] E. B. Gregory, *et al.*, "Leading-order hadronic contributions to $g_\mu - 2$,", PoS(LATTICE 2013)302, arXiv:1311.4446.