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Pedestrian dynamics

Implementation and analysis of ODE solvers

September 30, 2010 | Timo Hülsmann

Pedestrian dynamics: Implementation and analysis of ODE solvers

Parts of this talk

- Part 1: Introduction to pedestrian dynamics
- Part 2: Preserving data locality with Space-Filling Curves (Excursus)
- Part 3: Implementation and analysis of ODE solvers

Pedestrian dynamics

Part I: Introduction to pedestrian dynamics

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Pedestrian dynamics

Introduction

- Why pedestrian dynamics?
- Classification of Models
- Generalized Centrifugal Force Model (GCFM)

Why pedestrian dynamics?

- Enhancement of safety in complex buildings and mass events
- Simulation and optimization of evacuations
- Improvement of comfort in public buildings (airports, railway stations, shopping malls, etc.)
- Minimal travel times and maximum capacities

Classification of Models

Macroscopic Models

- System is described by mean values of characteristics: Conservation laws for density, flow, etc.

Microscopic Models

- Each pedestrian is treated seperately.
- Can be space-continuous (system of ODEs) or space-discrete (cellular automaton).
- Can be rule-based or force-based.

Generalized Centrifugal Force Model (GCFM)

Forces

- Driving force (carries pedestrians to desired direction with desired speed)
- Repulsive forces from other predestrians and walls (to avoid collisions)
- All Forces add up to the complete force acting on the pedestrian:

$$\vec{F}_i = \sum_{i \neq j}^{N} \vec{F}_{ij}^{rep} + \sum_{B} \vec{F}_{iB}^{rep} + \vec{F}_i^{driv}$$

Forces

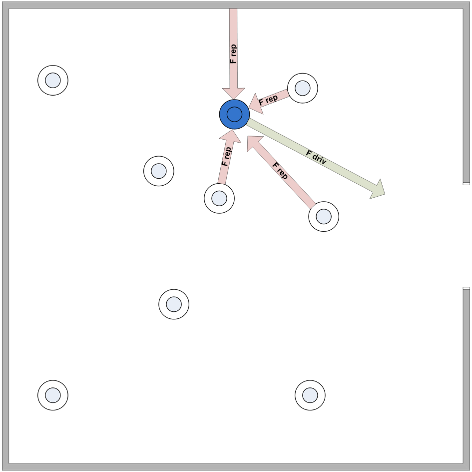

Figure: Example of forces acting on a pedestrian

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Forces

- The repulsive force:

$$\vec{F}_{ij}^{rep} = -m_i K_{ij} \frac{(\eta V_i^0 + V_{ij})^2}{\text{dist}_{ij}} \vec{e}_{ij}\,, \quad \vec{R}_{ij} = \vec{R}_j - \vec{R}_i\,, \quad \vec{e}_{ij} = \frac{\vec{R}_{ij}}{\|\vec{R}_{ij}\|}$$

$$V_{ij} = \frac{1}{2}((\vec{V}_i - \vec{V}_j)\cdot\vec{e}_{ij} + |(\vec{V}_i - \vec{V}_j)\cdot\vec{e}_{ij}|)\,, \quad K_{ij} = \frac{1}{2}\frac{\vec{V}_i \cdot \vec{e}_{ij} + |\vec{V}_i \cdot \vec{e}_{ij}|}{\|\vec{V}_i\|}$$

- The driving force:

$$\vec{F}_{iB}^{driv} = m_i \frac{\vec{V}_i^0 - \vec{V}_i}{\tau}$$

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Pedestrian dynamics

Part II: Preserving data locality with Space-Filling Curves (Excursus)

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Pedestrian dynamics

Data locality

- Idea: Reduce cache misses using data locality
- Space-Filling Curves
- Test cases
- Results
- Conclusion

Idea: Reduce cache misses using data locality

- We want to optimize the runtime. The aim is to simulate faster than realtime.
- In the implementation of the GCFM Linked-Cells are used to determine pedestrians' neighbourhood.
- Only the repulsive forces from pedestrians situated in the immediate neighbourhood are considered in the calculations.
- Data of nearby pedestrians should also be nearby in memory.
- Space-Filling Curves order 2-dimensional data s.t. locality is preserved.

Space-Filling Curves

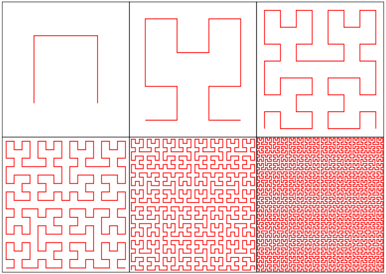
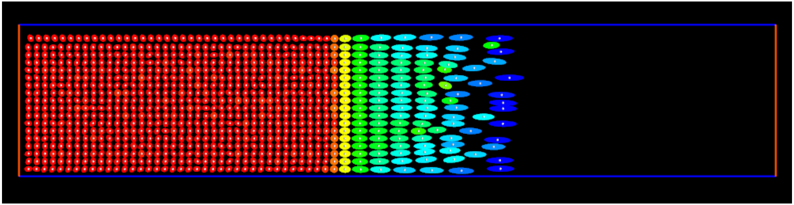


Figure:
http://upload.wikimedia.org/
wikipedia/commons/3/3a/Hilbert_curve.png

Hilbert curve

- Construction is a recursive process
- Continuous and surjective
- Good locality-preserving behavior

Test cases



- A corridor was used for testing and 1152 pedestrians were ordered
  - columnwise
  - rowwise
  - along a hilbert curve
  - random

Results

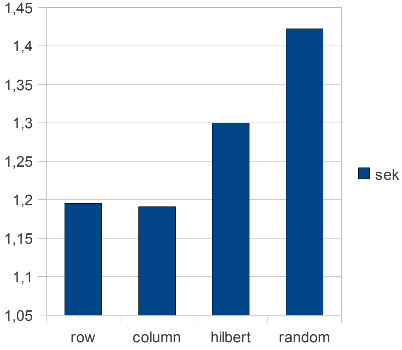


Figure: First time-step

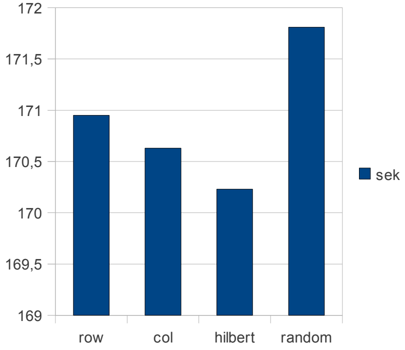


Figure: Complete simulation

Conclusion

- In the first time-step columnwise and rowwise ordering are faster than ordering along a hilbert curve.
- Random ordering has the longest simulation time.
- Ordering along a hilbert curve has the shortest simulation time, but differences are not significant.
- Further investigations showed that the main time in the simulation is spent on calculating ellipses for the computation of $dist_{ij}$ in the repulsive forces.
- In an optimized program the effects of ordering along a hilbert curve may become more visible.

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Pedestrian dynamics

Part III: Implementation and analysis of ODE solvers

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Pedestrian dynamics

ODE solvers

- Idea: Increasing step-sizes with more sophisticated solvers
- ODEs for GCFM
- Adaptive Runge-Kutta-Fehlberg Methods
- Velocity Verlet Method
- Test cases
- Results
- Conclusion
- Problems

Idea: Increasing step-sizes with more sophisticated solvers

- A timestep in the GCFM has high computational effort.
- Bigger time-steps could directly decrease the runtime of the simulation.
- Only the explicit Euler-Method was used in the implementation of the GCFM which is of order 1.
- Methods of order > 1 may allow bigger time-steps.

ODEs for GCFM

- System to solve:

$$\frac{d}{dt}\vec{R}_i = \vec{V}_i\,, \quad m_i \frac{d}{dt}\vec{V}_i = \vec{F}_i\,, \quad i = 1, \ldots, N$$

- In the simulation $m_i = 1$ is used, so this can be rewritten as:

$$\vec{x} = (R_{1,x}, R_{1,y}, V_{1,x}, V_{1,y}, \ldots, R_{N,x}, R_{N,y}, V_{N,x}, V_{N,y})^T$$

$$\vec{f}(\vec{x}) = (V_{1,x}, V_{1,y}, F_{1,x}, F_{1,y}, \ldots, V_{N,x}, V_{N,y}, F_{N,x}, F_{N,y})^T$$

$$\frac{d}{dt}\vec{x} = \vec{f}(\vec{x})$$

- We can apply standard numerical integration techniques.

Adaptive Runge-Kutta-Fehlberg 2(3) Method

- We want to integrate the system $\frac{d}{dt}\vec{x} = \vec{f}(\vec{x})$ with step-size h.
- The Runge-Kutta-Fehlberg 2(3) Method reads as follows:

$$\vec{x}_1 = \vec{x}_0 + h\sum_{i=1}^{3} b_i\vec{k}_i\,, \quad \hat{\vec{x}}_1 = \vec{x}_0 + h\sum_{i=1}^{4} \hat{b}_i\vec{k}_i$$

Formula for the increments:

$$\vec{k}_i = \vec{f}(\vec{x}_0 + h\sum_{j=1}^{i-1} a_{ij}\vec{k}_j)\,, \quad i = 1, \ldots, 4$$

Adaptive Runge-Kutta-Fehlberg 2(3) Method

- The coefficients of the method read as follows:

a_{21}=1/4			
a_{31}=-189/800	a_{32}=729/800		
a_{41}=214/891	a_{42}=1/33	a_{43}=650/891	
b_1=214/891	b_2=1/33	b_3=650/891	
$\hat{b}_1$=533/2106	$\hat{b}_2$=0	$\hat{b}_3$=800/1053	$\hat{b}_4$=-1/78

- The local error is estimated by:

$$\vec{e} = \vec{x}_1 - \hat{\vec{x}}_1 = O(3)$$

- Formula for step size prediction:

$$h_{opt} = h_{used} \cdot \sqrt[3]{\frac{1}{\mathrm{ERR}}}$$

Adaptive Runge-Kutta-Fehlberg 2(3) Method

- Error norm:

$$\mathrm{ERR} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{e_j}{\mathrm{atol} + \max\{|x_{0,j}|, |x_{1,j}|\} \cdot \mathrm{rtol}}\right)^2}$$

- The new step-size is scaled with a safety factor ρ. It must hold:

$$\sigma \cdot h_{used} < h_{new} < \theta \cdot h_{used}$$

- Values of the parameters:

$$\rho = 0.9\,, \quad \sigma = 0.2\,, \quad \theta = 5$$

Adaptive Runge-Kutta-Fehlberg 4(5) Method

- The Runge-Kutta-Fehlberg 4(5) Method uses two methods of order 4 and 5 to get an estimated error of order 5:

$$\vec{x}_1 = \vec{x}_0 + h \sum_{i=1}^{5} b_i \vec{k}_i \,, \quad \hat{\vec{x}}_1 = \vec{x}_0 + h \sum_{i=1}^{6} \hat{b}_i \vec{k}_i$$

with increments:

$$\vec{k}_i = \vec{f}(\vec{x}_0 + h \sum_{j=1}^{i-1} a_{ij} \vec{k}_j) \,, \quad i = 1, \ldots, 6$$

Velocity Verlet Method

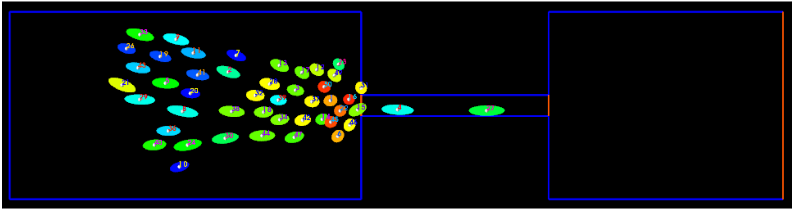
- The Velocity Verlet Method is used in Molecular Dynamics and allows bigger time-steps for dissipative systems.
- The method reads as follows:

$$\vec{R}_i(t+h) = \vec{R}_i(t) + \vec{V}_i(t)h + \frac{1}{2}\vec{F}_i(t)h^2$$

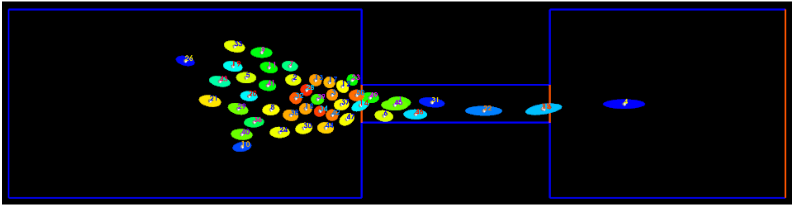
$$\vec{V}_i(t+h) = \vec{V}(t) + \frac{1}{2}\left[\vec{F}_i(t) + \vec{F}_i(t+h)\right] h$$

Test cases

- Bottleneck with 90cm width:



- Bottleneck with 160cm width:



Results - 90cm bottleneck, N=20

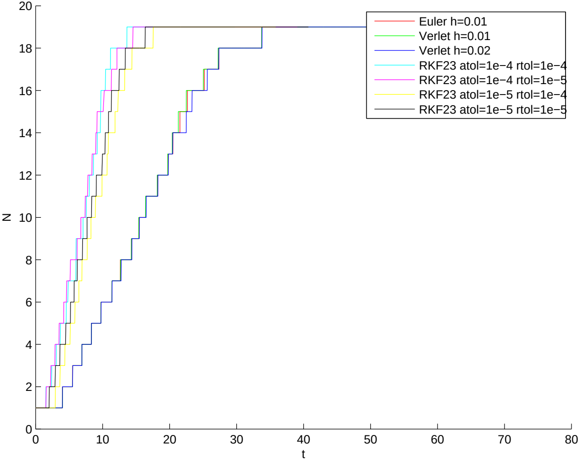


Figure: Pedestrians entered the bottleneck (Euler,Verlet,RKF 2(3))

Results - 90cm bottleneck, N=20

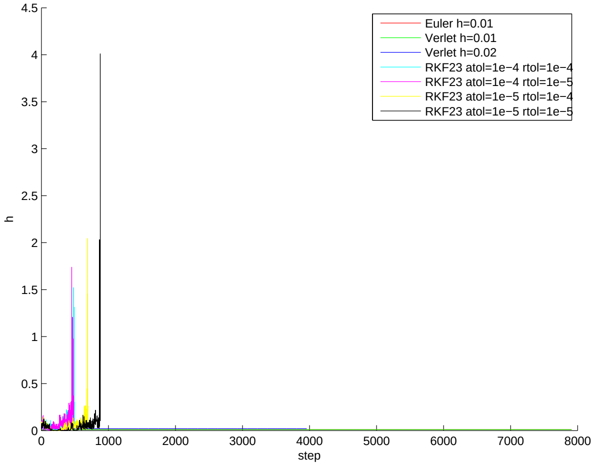


Figure: Step-sizes (Euler,Verlet,RKF 2(3))

Results - 90cm bottleneck, N=20

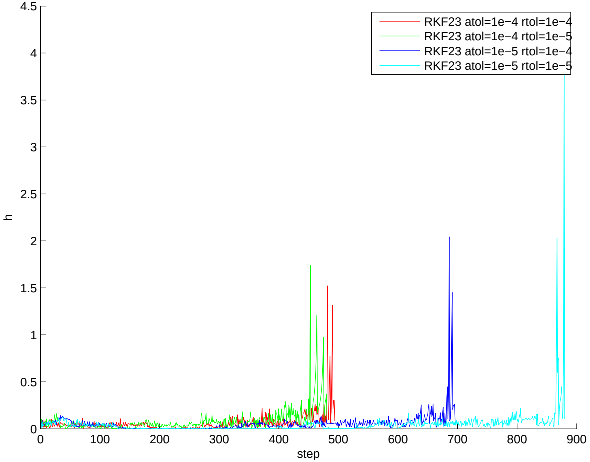
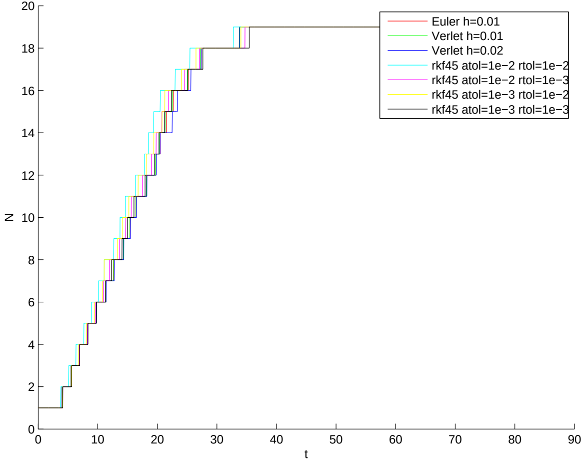


Figure: Step-sizes (RKF 2(3))

Results - 90cm bottleneck, N=20



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Results - 90cm bottleneck, N=20

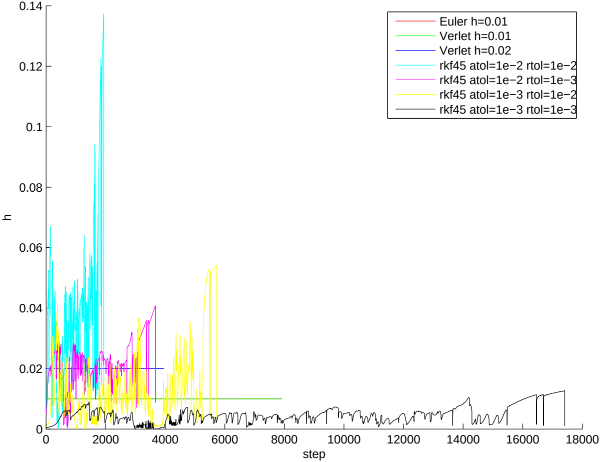


Figure: Step-sizes (Euler,Verlet,RKF 4(5))

Results - 160cm bottleneck, N=50

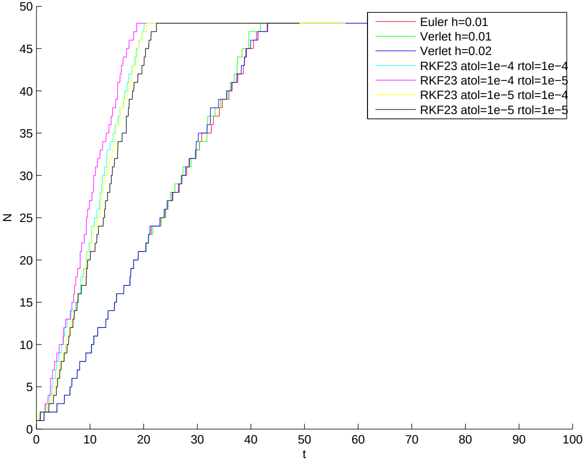
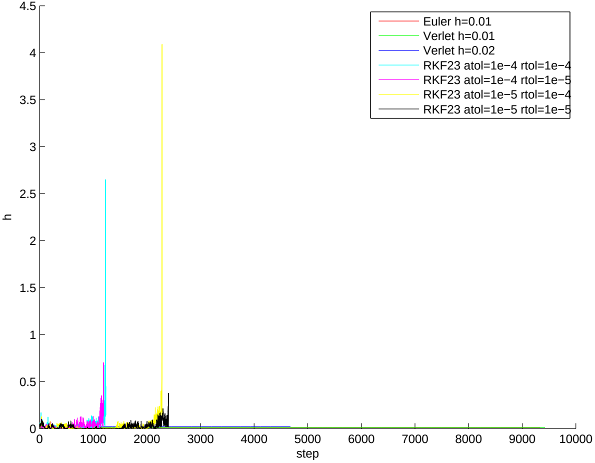


Figure: Pedestrians entered the bottleneck

Results - 160cm bottleneck, N=50



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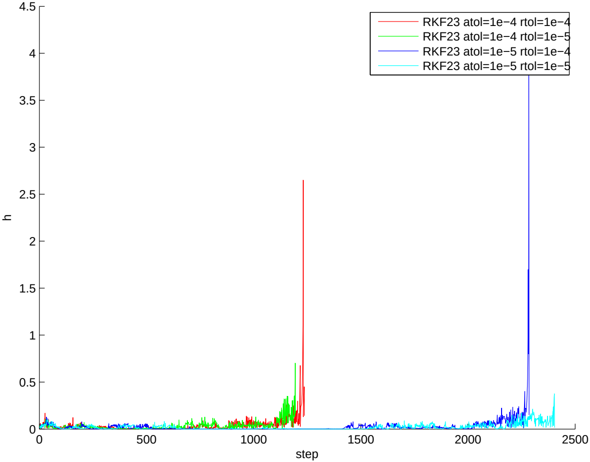
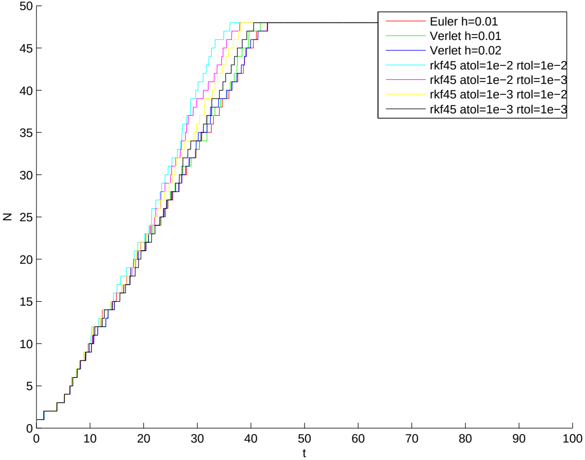


Figure: Step-sizes (RKF 2(3))

Results - 160cm bottleneck, N=50



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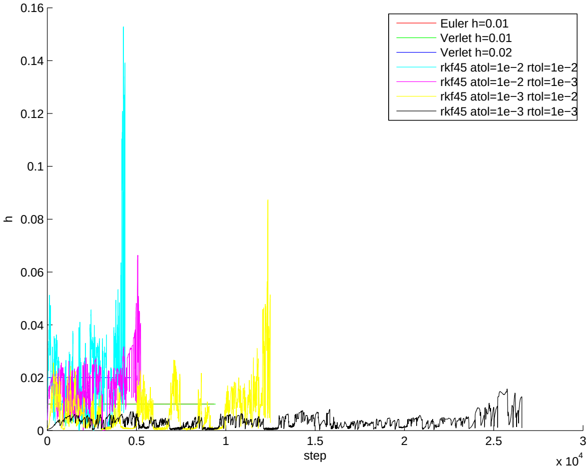

Figure: Step-sizes (Euler,Verlet,RKF 4(5))

Conclusion

- Velocity Verlet allows doubling the step-size and has similar N/t-diagram as Euler-Method.
- Smallest amount of time-steps needed with $atol = 10^{-4}$, $rtol = 10^{-5}$ for RKF 2(3) Method and with $atol = 10^{-2}$, $rtol = 10^{-2}$ for RKF 4(5) Method.
- RKF 2(3) method allows bigger step-sizes but N/t-diagram differs much more from the “reference” N/t-diagram of the Euler-Method than the RKF 4(5) Method.

Problems

- New methods are not stable enough to allow comparison with experimental data (for this simulations with about 180 Pedestrians are needed).
- Some pedestrians are missing in the N/t diagram. This is due to unrealistic increase of forces.

Thank you for your attention!

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