Advanced hydrodynamics

Improved SPH and mesh free hydrodynamical methods

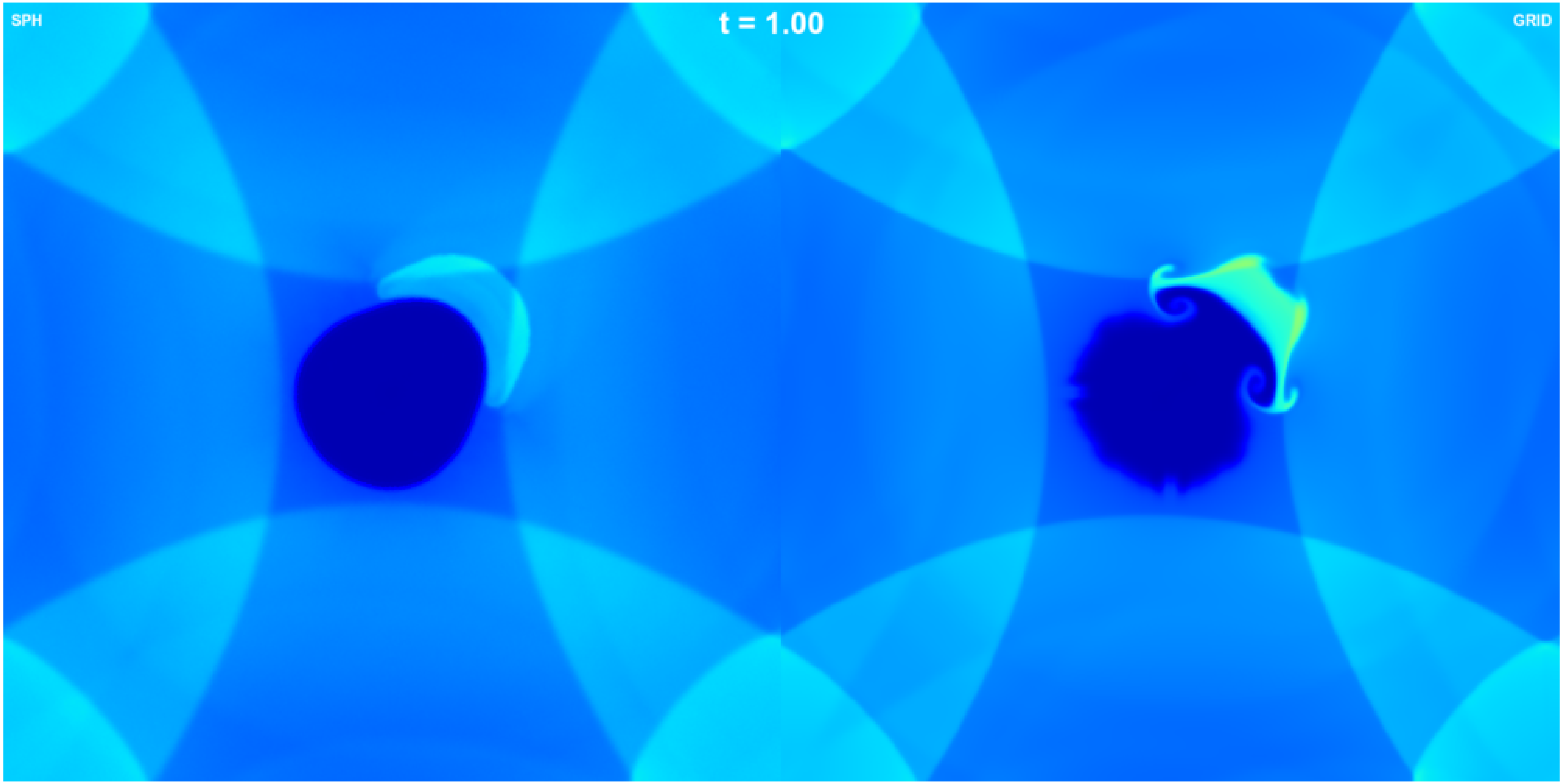
UNIVERSITEIT GENT

Bert Vandenbroucke (bert.vandenbroucke@ugent.be)

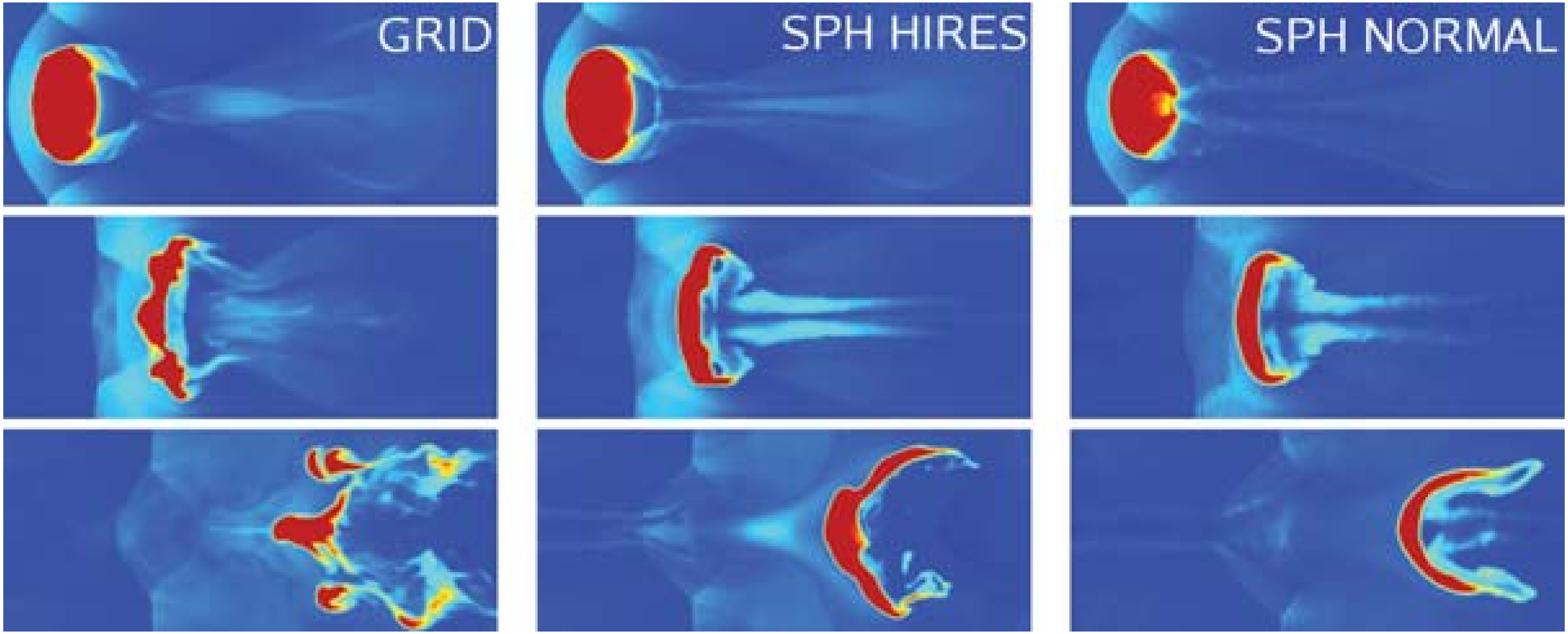
Overview

- Failures and fixes of SPH
- Finite volume methods
- Meshless finite volume
- GIZMO implementation in SWIFT

Failures of SPH

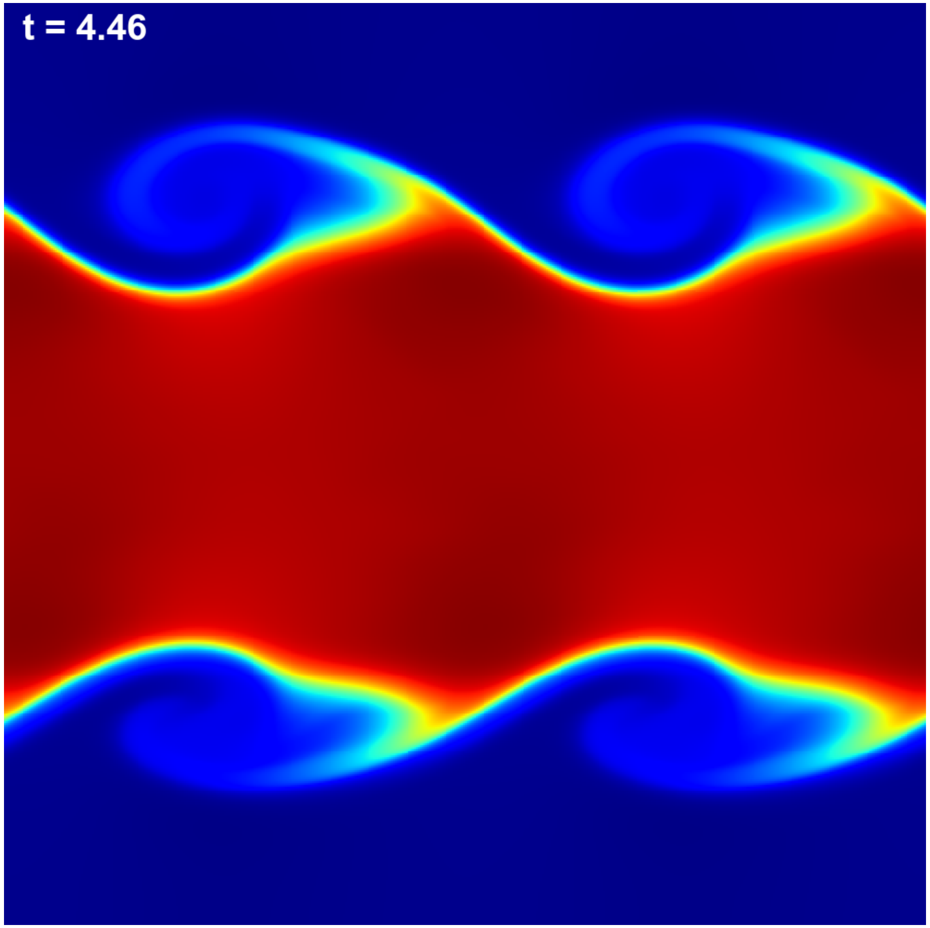


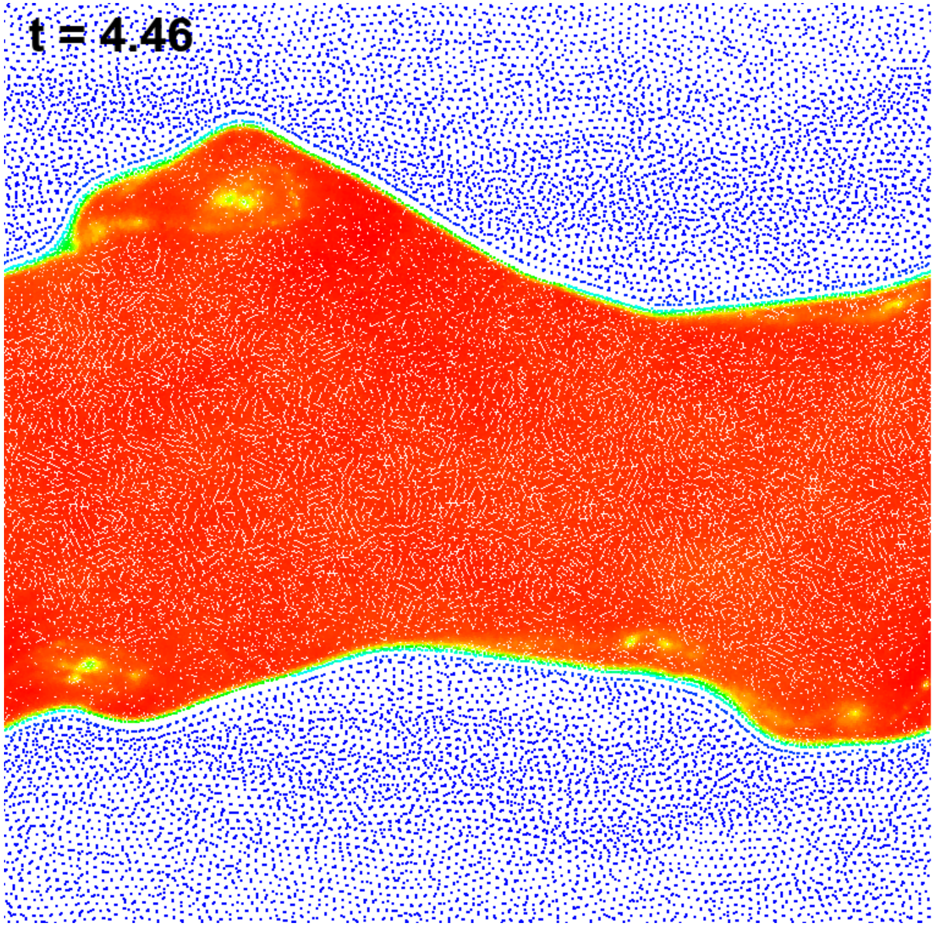
Failures of SPH



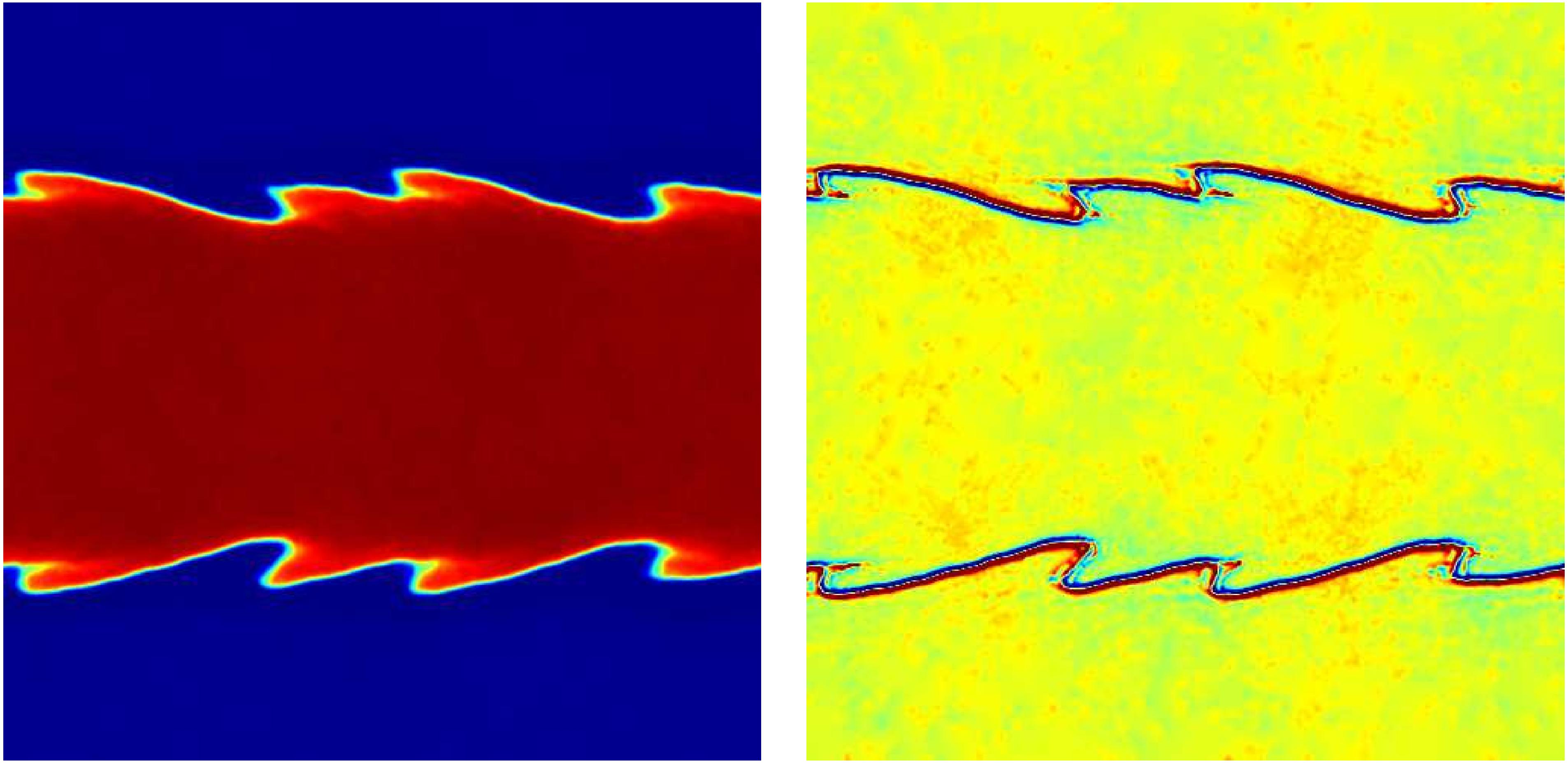
Agertz et al., 2007

Failures of SPH





Failures of SPH



Price, 2008

Failures of SPH

Continuity equation:

$$\int \frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{v}) = 0 \quad W(|\vec{r} - \vec{r}\,'|, h)\, dV$$

$$\frac{d}{dt} \sum_j m_j W_{ij} = \sum_j m_j (\vec{v}_i - \vec{v}_j).\vec{\nabla} W_{ij} - \boxed{\int (\rho' \vec{v}\,' W).d\vec{S}}$$

$$\frac{d\rho_i}{dt} = \sum_j m_j (\vec{v}_i - \vec{v}_j).\vec{\nabla} W_{ij}$$

Surface terms neglected: invalid assumption at discontinuities

Improved SPH

Correction terms:

Artificial viscosity

$$\left(\frac{d\vec{v}_i}{dt}\right)_{\mathrm{corr}} = \sum_j m_j \frac{\alpha(\vec{v}_i - \vec{v}_j).\vec{r}_{ij}}{\rho_{ij}|\vec{r}_{ij}|} \vec{\nabla} W_{ij}$$

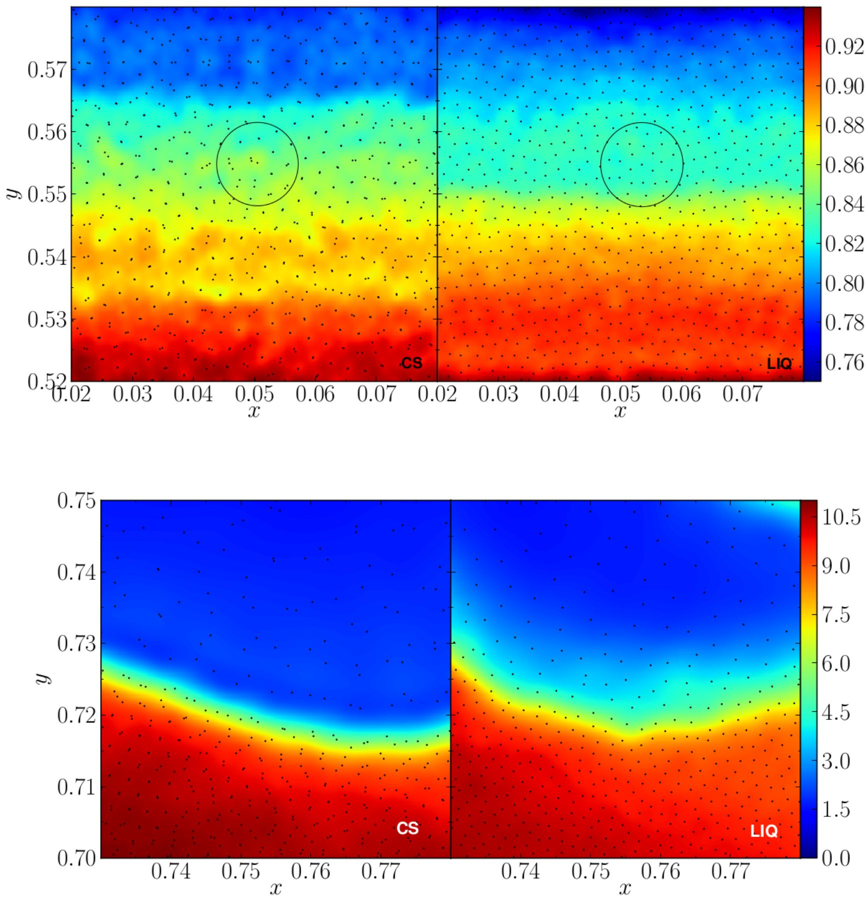
Artificial conductivity

$$\left(\frac{du}{dt}\right)_{\mathrm{corr}} = -\sum_j \frac{m_j}{\rho_{ij}} \left[\frac{1}{2}\alpha(\vec{v}_{ij}.\vec{r}_{ij}) + \beta(u_i - u_j)\right] \frac{\vec{r}_{ij}}{|\vec{r}_{ij}|}.\vec{\nabla} W_{ij}$$

Problem: need a *switch* to find out when correction terms need to be added

Monaghan, 1997, *Price*, 2008

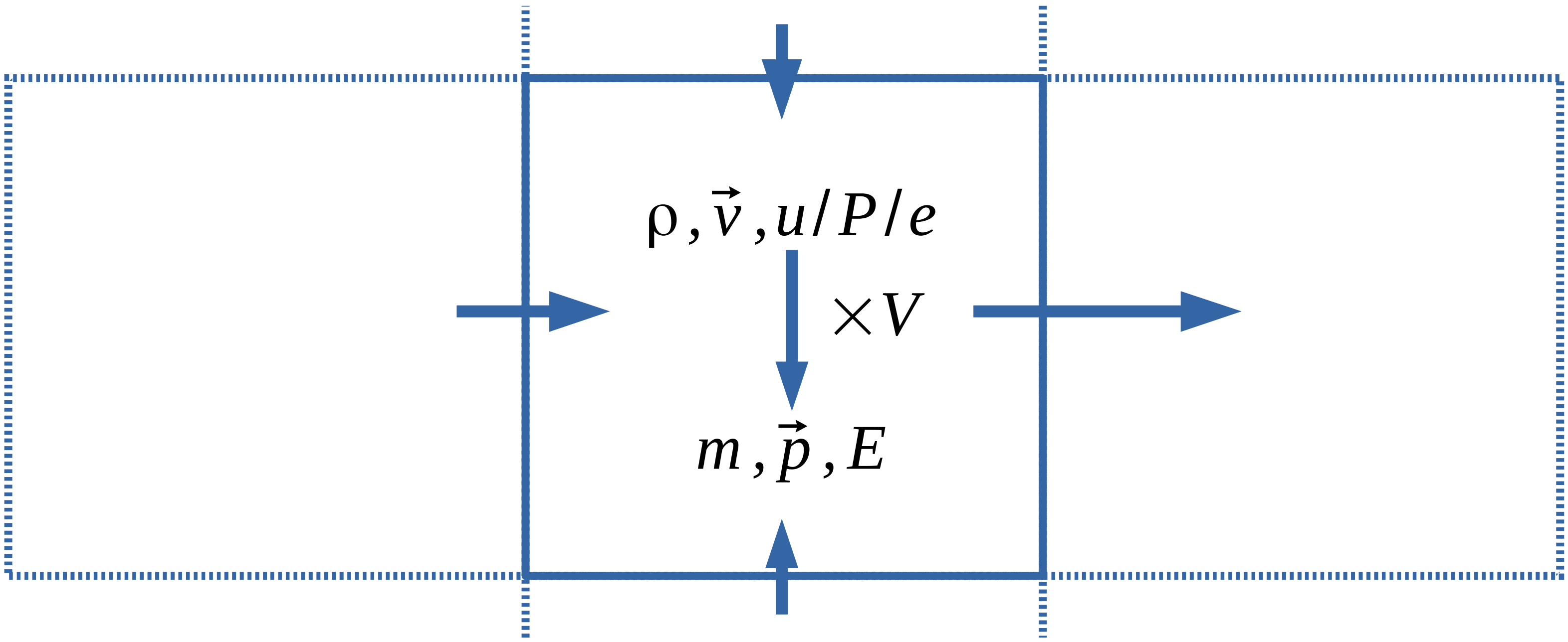
Kernels



- All quantities are smoothed out
- Particles tend to cluster together for some kernels (pair-instability)
- Poisson noise

Valcke et al., 2010

Finite volume methods



Hydrodynamics = flux exchange

Finite volume methods

$$\vec{F}_\rho = \rho_{\text{half}} \left(\vec{w}_{\text{half}} - \vec{v}_{\text{interface}} \right)$$

$$\vec{F}_{w_k} = \rho_{\text{half}} w_{k,\text{half}} \left(\vec{w}_{\text{half}} - \vec{v}_{\text{interface}} \right) + P_{\text{half}} \vec{n}_k$$

$$\vec{F}_P = \rho_{\text{half}} e_{\text{half}} \left(\vec{w}_{\text{half}} - \vec{v}_{\text{interface}} \right) + P_{\text{half}} \vec{w}_{\text{half}}$$

$$e = u + \frac{1}{2} \vec{w}^2 = \frac{P}{(\gamma - 1)\rho} + \frac{1}{2} \vec{w}^2$$

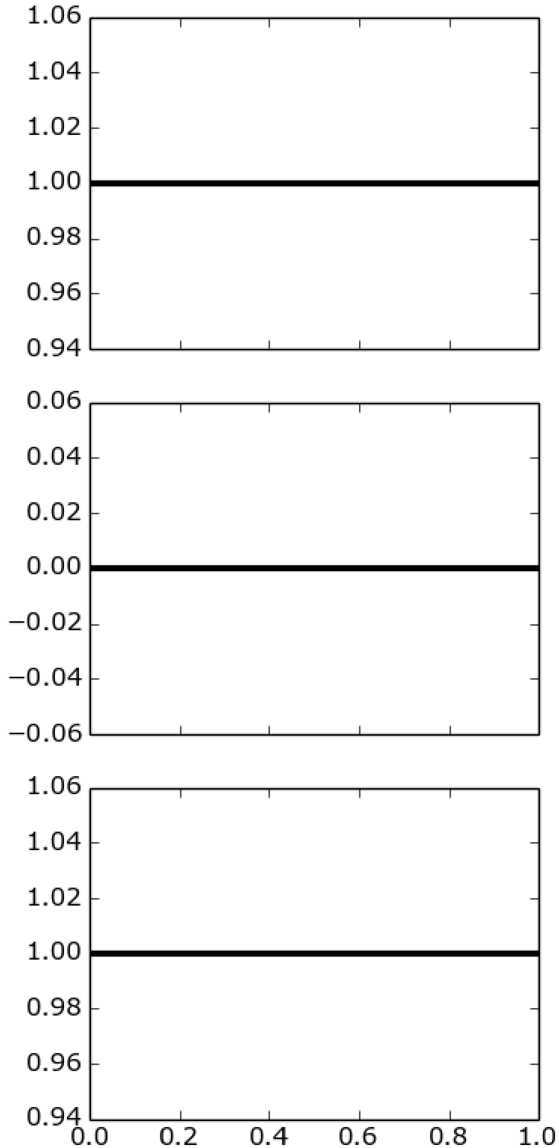
Riemann problem

Left state

ρ_L

w_L

P_L



Right state

ρ_R

w_R

P_R

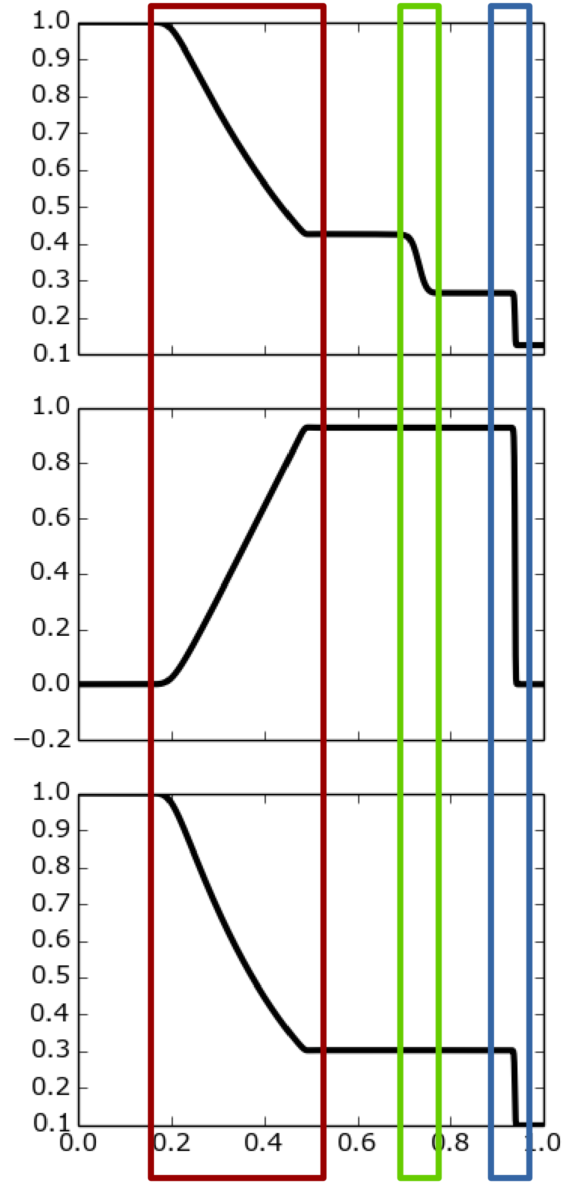
Riemann problem

Left state | Right state

ρ_L | ρ_R

w_L | w_R

P_L | P_R



Rarefaction wave

Shock wave

Contact discontinuity

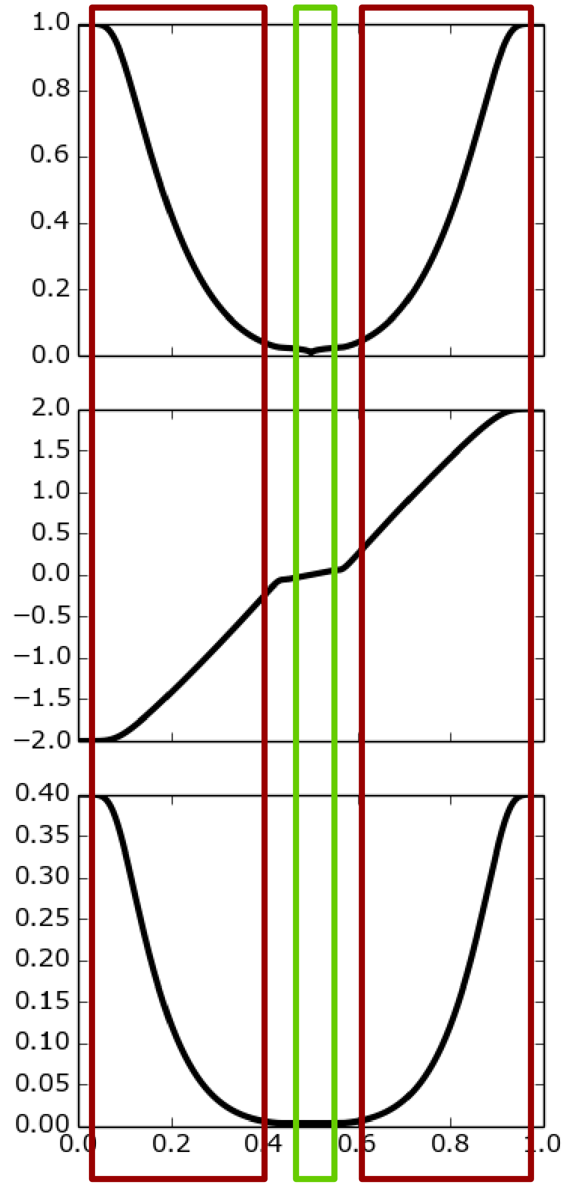
Riemann problem

Left state

ρ_L

w_L

P_L



Right state

ρ_R

w_R

P_R

Rarefaction wave

Shock wave

Contact discontinuity

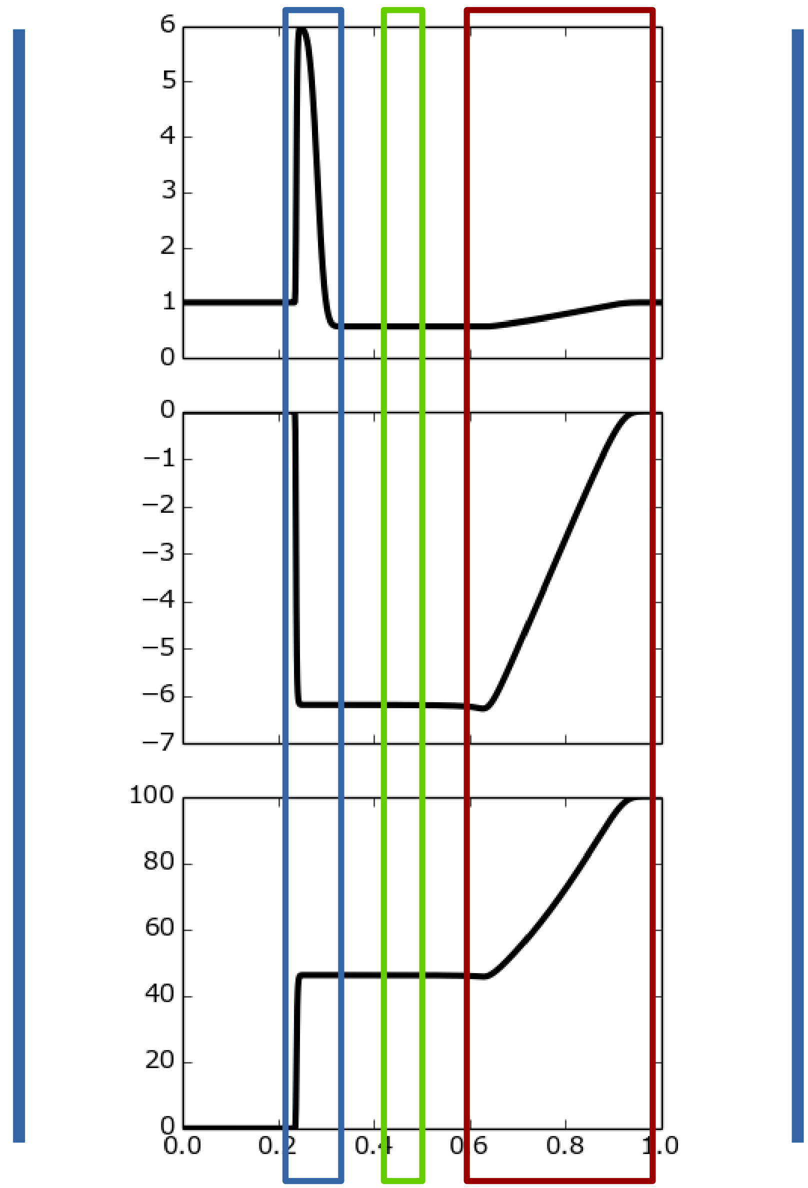
Riemann problem

Left state

ρ_L

w_L

P_L



Right state

ρ_R

w_R

P_R

Rarefaction wave

Shock wave

Contact discontinuity

Riemann problem

Left state

ρ_L

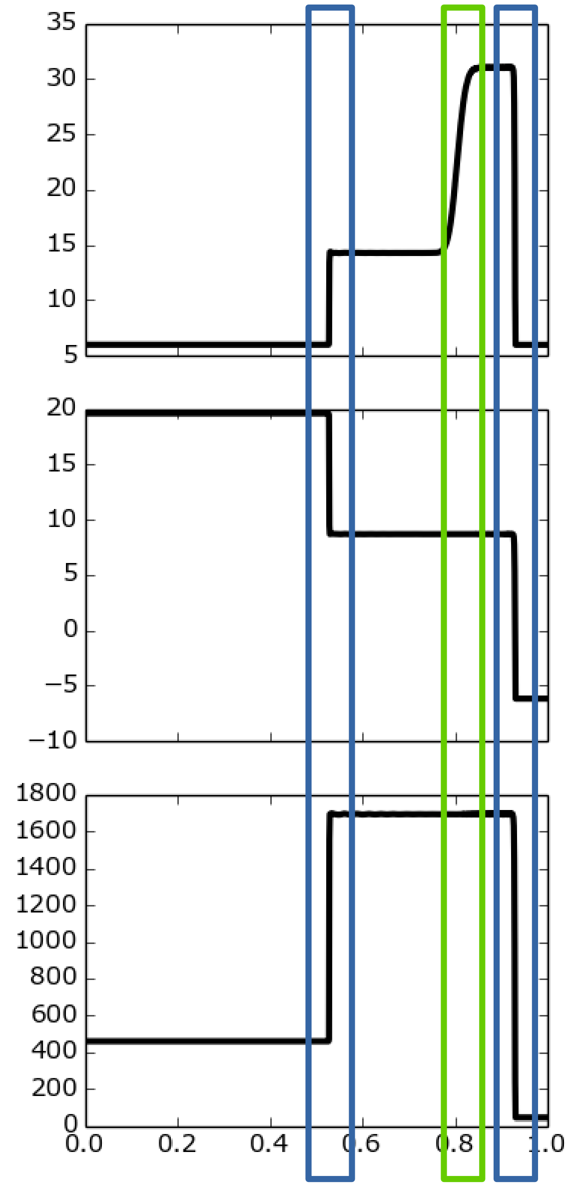
w_L

P_L

Rarefaction wave

Shock wave

Contact discontinuity



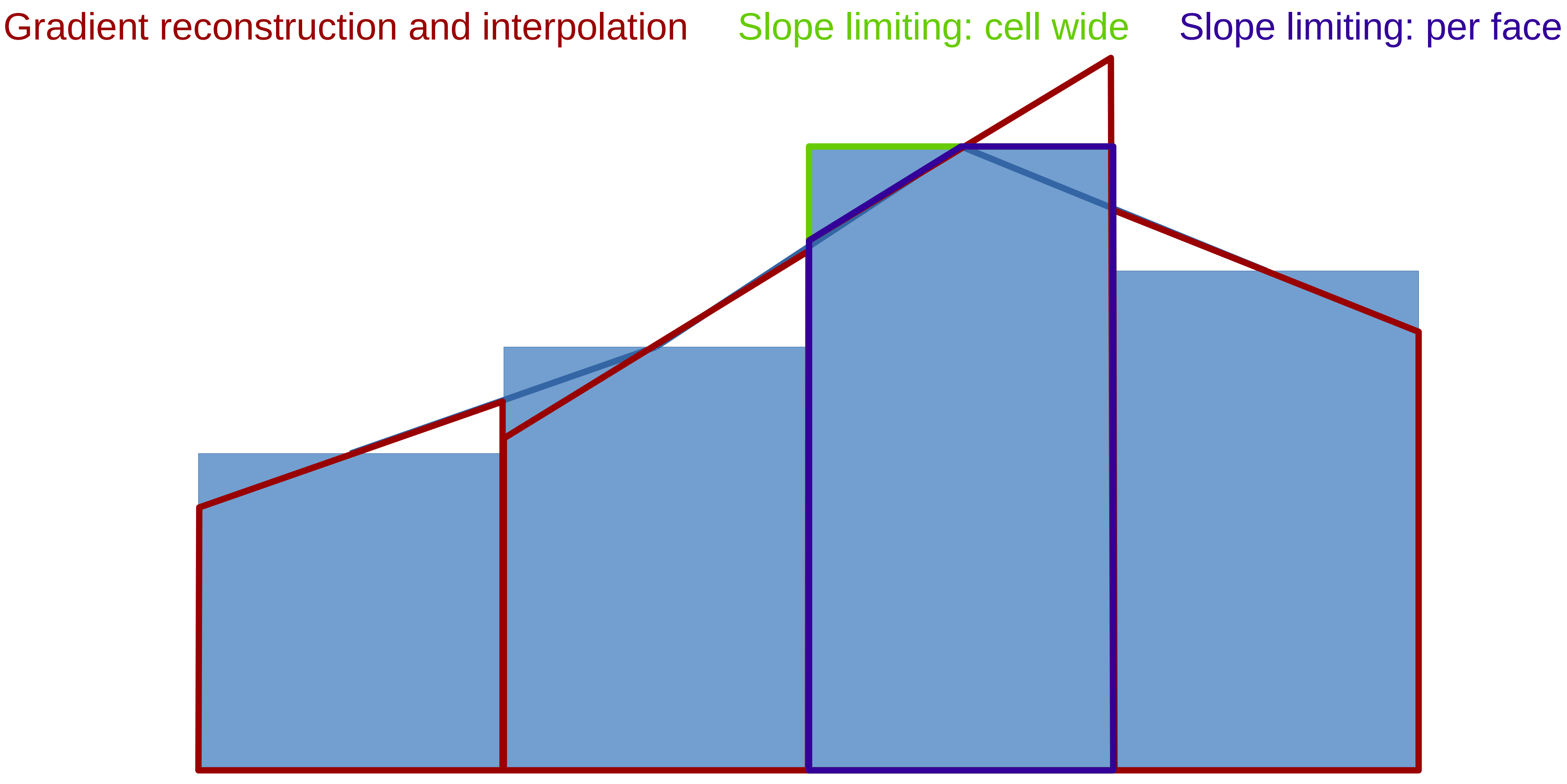
Right state

ρ_R

w_R

P_R

Second order: gradients



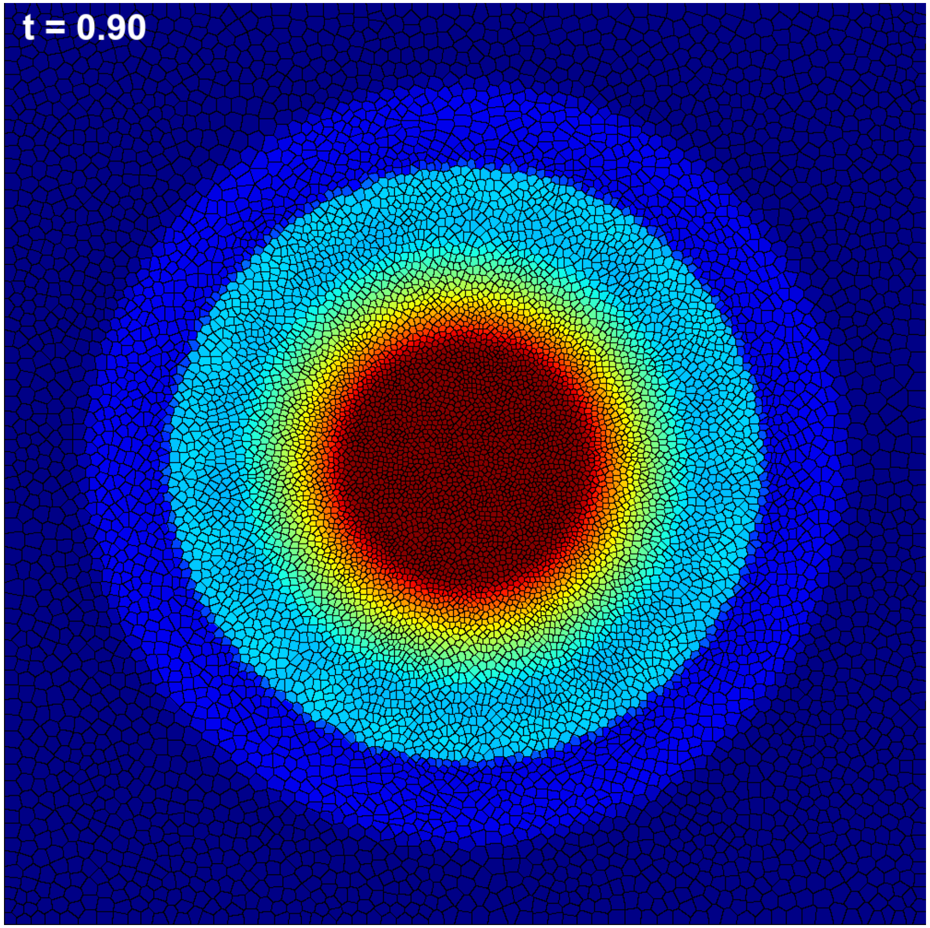
Second order: prediction

$$\boldsymbol{W} \rightarrow \boldsymbol{W} + \frac{\Delta t}{2} \frac{\partial \boldsymbol{W}}{\partial t}$$

$$\frac{\partial \boldsymbol{W}}{\partial t} + \boldsymbol{A}(\boldsymbol{W}) \vec{\nabla} \boldsymbol{W} = 0$$

$$\boldsymbol{A}(\boldsymbol{W}) = \begin{pmatrix} \vec{v} & \rho & 0 \\ 0 & \vec{v} & \frac{1}{\rho} \\ 0 & \gamma P & \vec{v} \end{pmatrix}$$

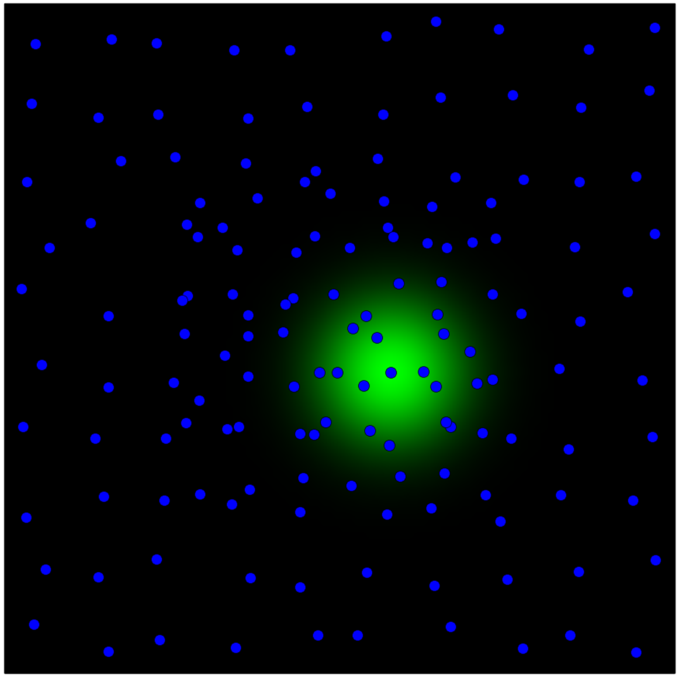
Finite volume methods

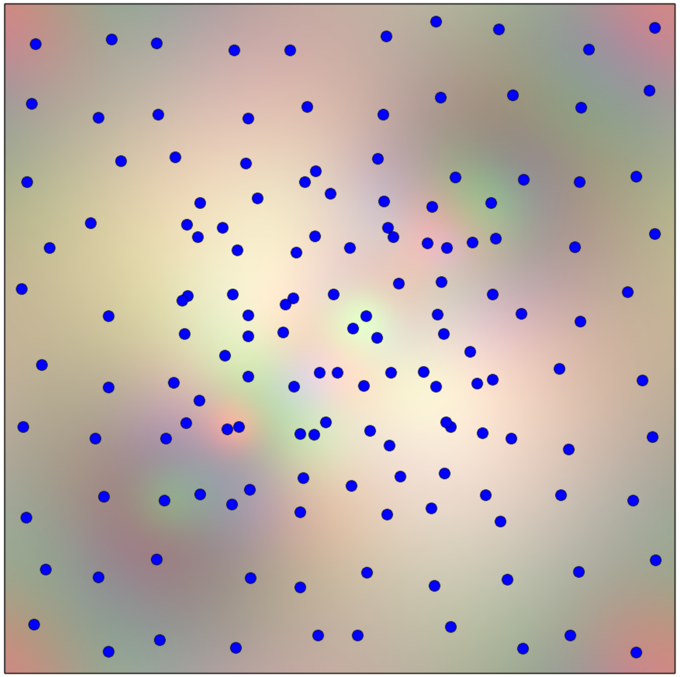


hadowfax

Volume partitioning

SPH: each particle gets part of volume, symmetric around its position

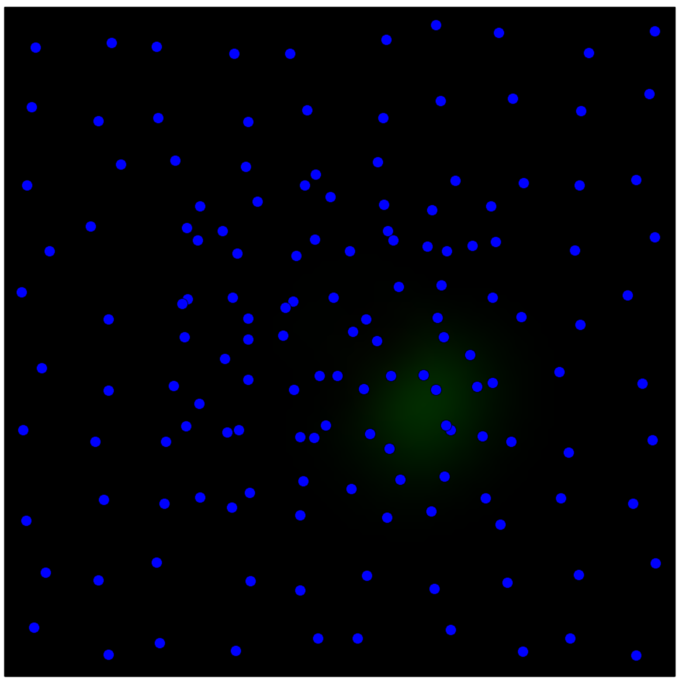


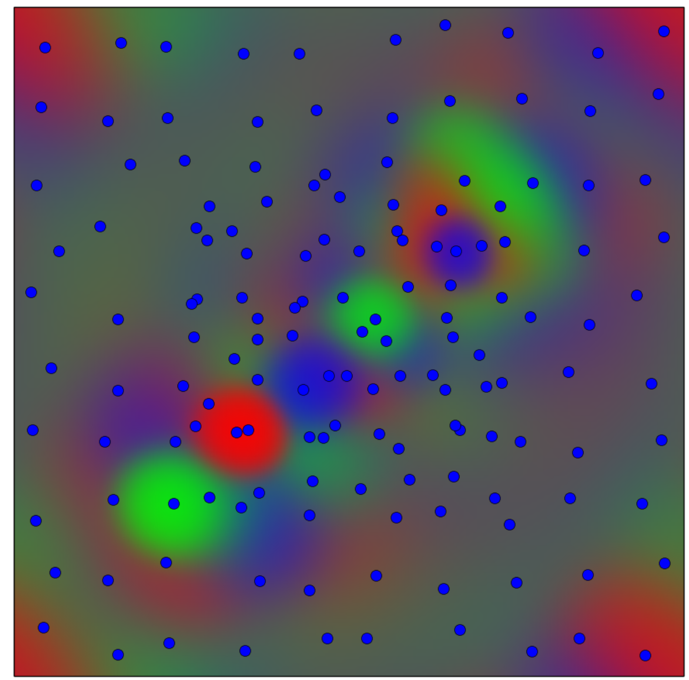


Some parts of the volume are over-sampled, others are under-sampled
Sum of partial volumes not necessarily equal to total volume

Better volume partitioning

GIZMO: volume is distributed over the particles using the kernel





Sum of partial volumes guaranteed to be equal to total volume
Whole volume sampled homogeneously

GIZMO geometry

Particle volume

$$V_i = \frac{1}{\sum_j W(|\vec{r}_i - \vec{r}_j|, h_i)}$$

Hopkins, 2015

Spatial configuration independent gradient estimate

$$\left(\vec{\nabla} X_i\right)^{\alpha} = \sum_j (X_i - X_j) \sum_{\beta} B_i^{\alpha\beta} (\vec{r}_j - \vec{r}_i)^{\beta} W(|\vec{r}_i - \vec{r}_j|, h_i)$$

where $B = E^{-1}$ with $E_i^{\alpha\beta} = \sum_j (\vec{r}_i - \vec{r}_j)^{\alpha} (\vec{r}_i - \vec{r}_j)^{\beta} W(|\vec{r}_i - \vec{r}_j|, h_i)$

Lanson & Vila, 2008

GIZMO geometry

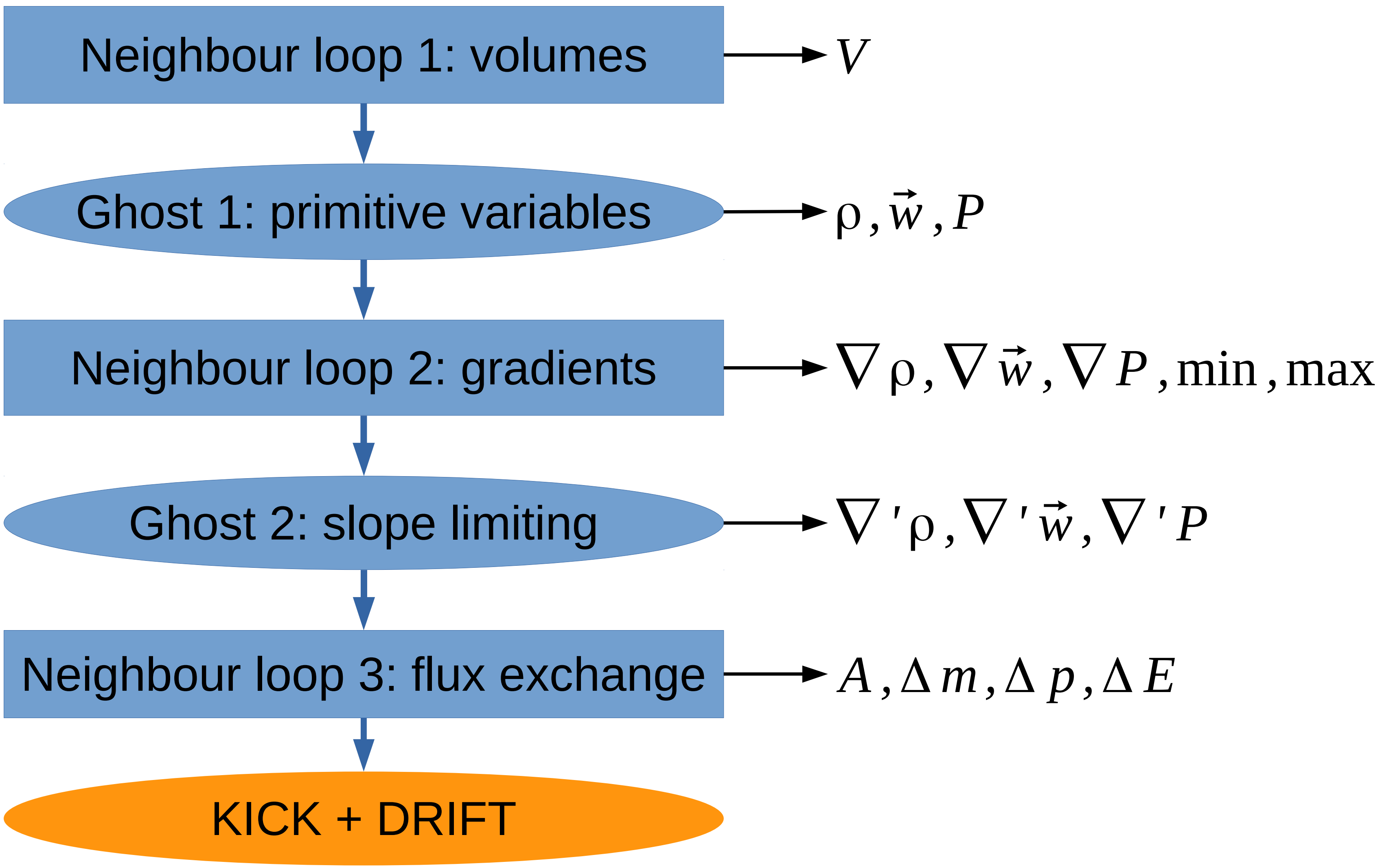
Interface between neighbouring particles

$$\left(\vec{A}_{ij}\right)^{\alpha} = V_i \sum_{\beta} B_i^{\alpha\beta} (\vec{r}_i - \vec{r}_j)^{\beta} W(|\vec{r}_i - \vec{r}_j|, h_i) + V_j \sum_{\beta} B_j^{\alpha\beta} (\vec{r}_j - \vec{r}_i)^{\beta} W(|\vec{r}_i - \vec{r}_j|, h_j)$$

$$\vec{x}_{ij} = \vec{x}_i + \frac{h_i}{h_i + h_j} (\vec{x}_j - \vec{x}_i)$$

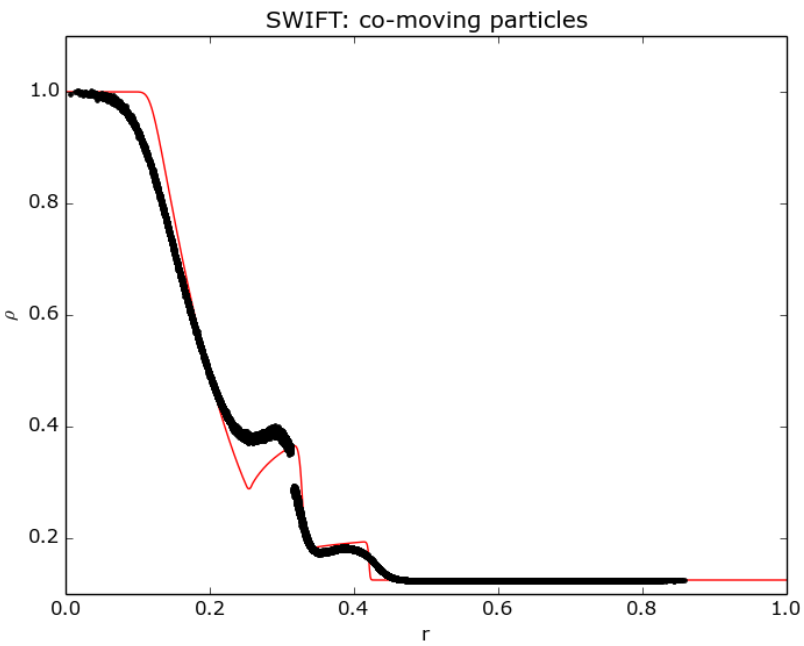
$$\vec{w}_{ij} = \vec{w}_i + (\vec{w}_j - \vec{w}_i) \left[\frac{(\vec{x}_{ij} - \vec{x}_i) . (\vec{x}_j - \vec{x}_i)}{|\vec{x}_i - \vec{x}_j|^2} \right]$$

SWIFT GIZMO

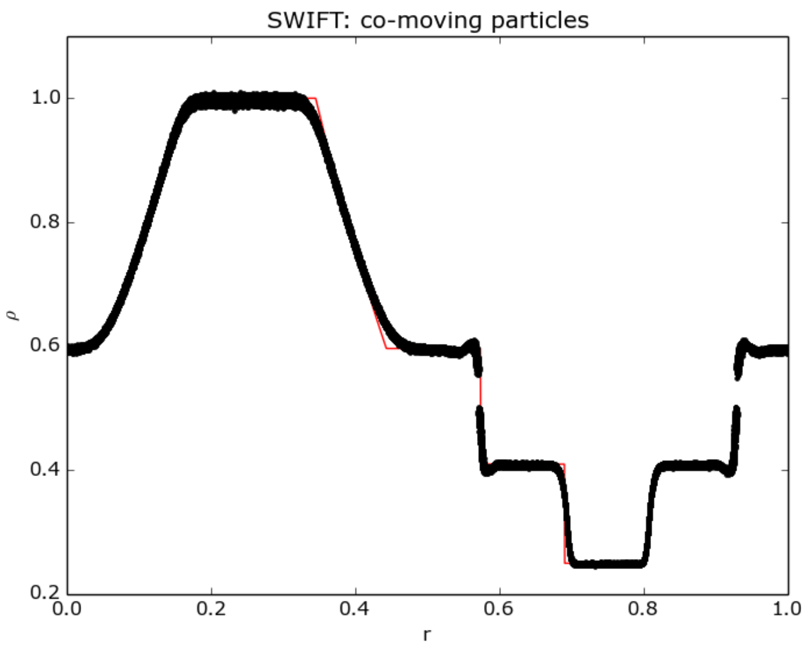

Neighbour loop 1: volumes
V
Ghost 1: primitive variables
$\rho, \vec{w}, P$
Neighbour loop 2: gradients
$\nabla\rho, \nabla\vec{w}, \nabla P, \min, \max$
Ghost 2: slope limiting
$\nabla'\rho, \nabla'\vec{w}, \nabla' P$
Neighbour loop 3: flux exchange
$A, \Delta m, \Delta p, \Delta E$
KICK + DRIFT

SWIFT GIZMO

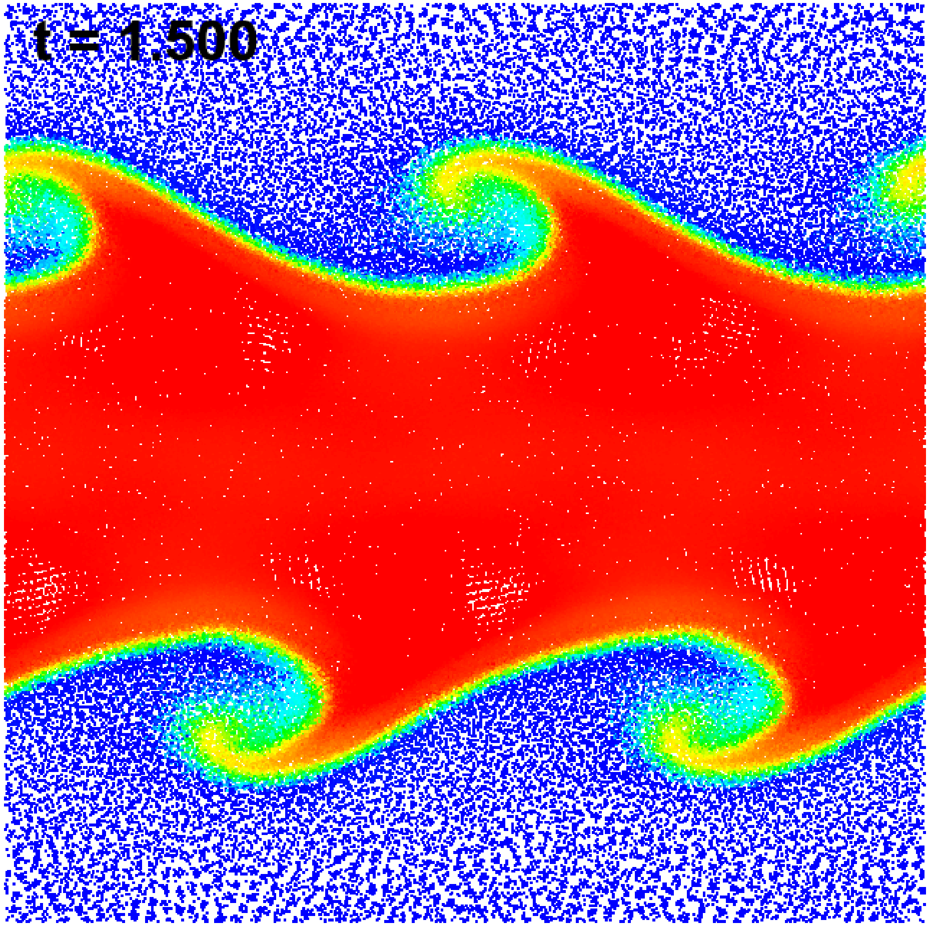
Spherical overdensity



Sod shock

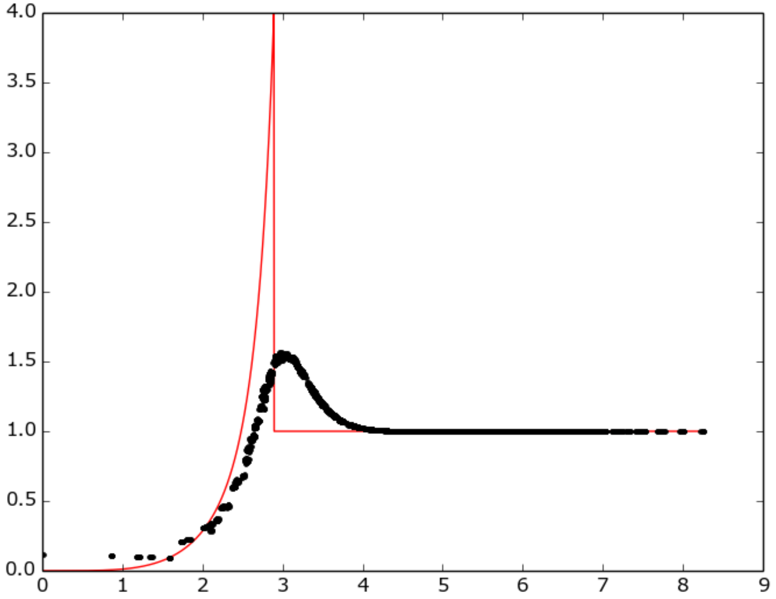


SWIFT GIZMO

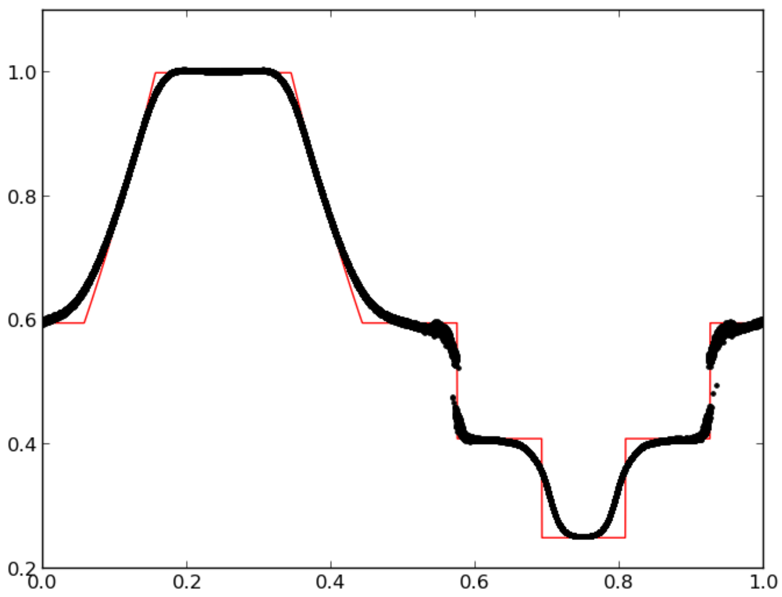


Hands on problems

SedovBlast

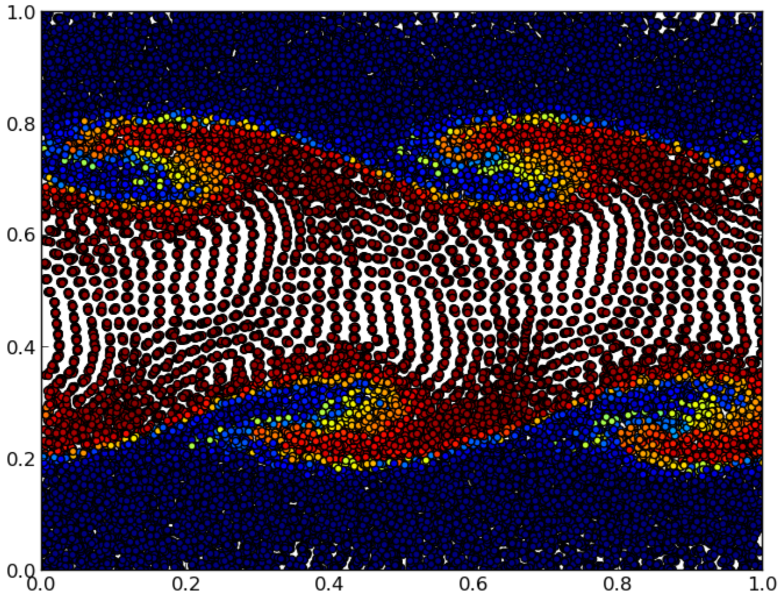


SodShock

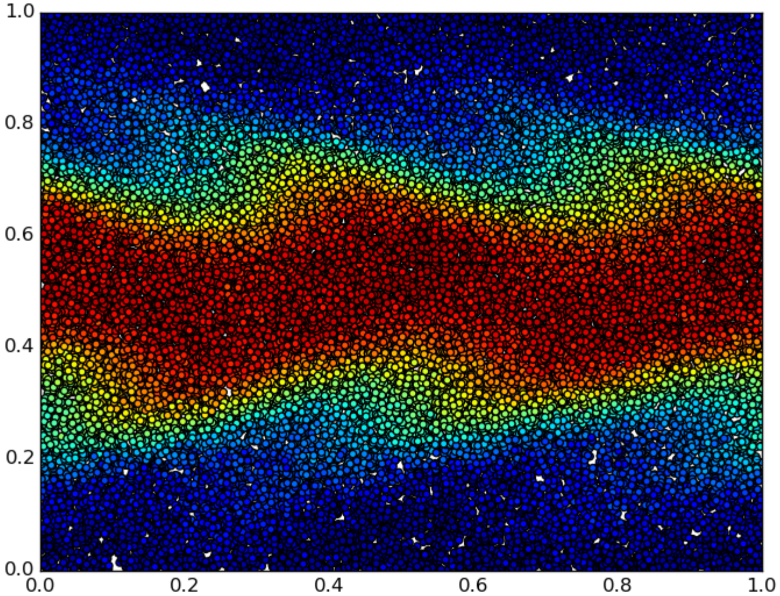


Hands on problems

KelvinHelmholtz



GIZMO



SPH

References

Agertz et al., 2007, MNRAS, 380, 963

Hopkins, 2015, MNRAS, 450, 53

Lanson & Vila, 2008, SIAM J. Numer. Anal., 46, 1935

Monaghan, 1997, J. Comp. Phys., 136, 298

Price, 2008, J. Comp. Phys., 227, 10040

Valcke et al., 2010, MNRAS, 408, 71