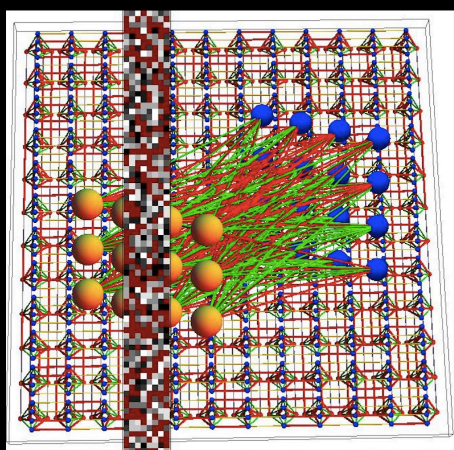
Explorations on Quantum Annealers: *Past, Present, and Future*

- **Adiabatic Quantum Computers**
- **Testing D-Wave: Spanning Trees**
- **Testing D-Wave: Answer Checking**
- **AQC + Small-World (SW)**
- **Boltzmann machines + AQC**
- **Conclusions and Provocations**



Mark A. Novotny

MISSISSIPPI STATE UNIVERSITY

Supported in part by NSF and PNNL

NOTES
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Hans De Raedt

New Tools are never optimal: Who has done computations …

Computer CPU based on vacuum tubes?

Data storage on paper tape?

RAM from coil-wound solenoids?

Programmed in Assembly language?

Disruptive to computing

Been doing computational physics since
PhD in Physics from Stanford University in 1978

In 35+ years, ***two*** disruptive computing innovations:

Disruptive to computing

Been doing computational physics since
PhD in Physics from Stanford University in 1978

In 35+ years, ***two*** disruptive computing innovations:

Gordon Moore
1965

Redacted Figure

Disruptive to computing

Been doing computational physics since
PhD in Physics from Stanford University in 1978

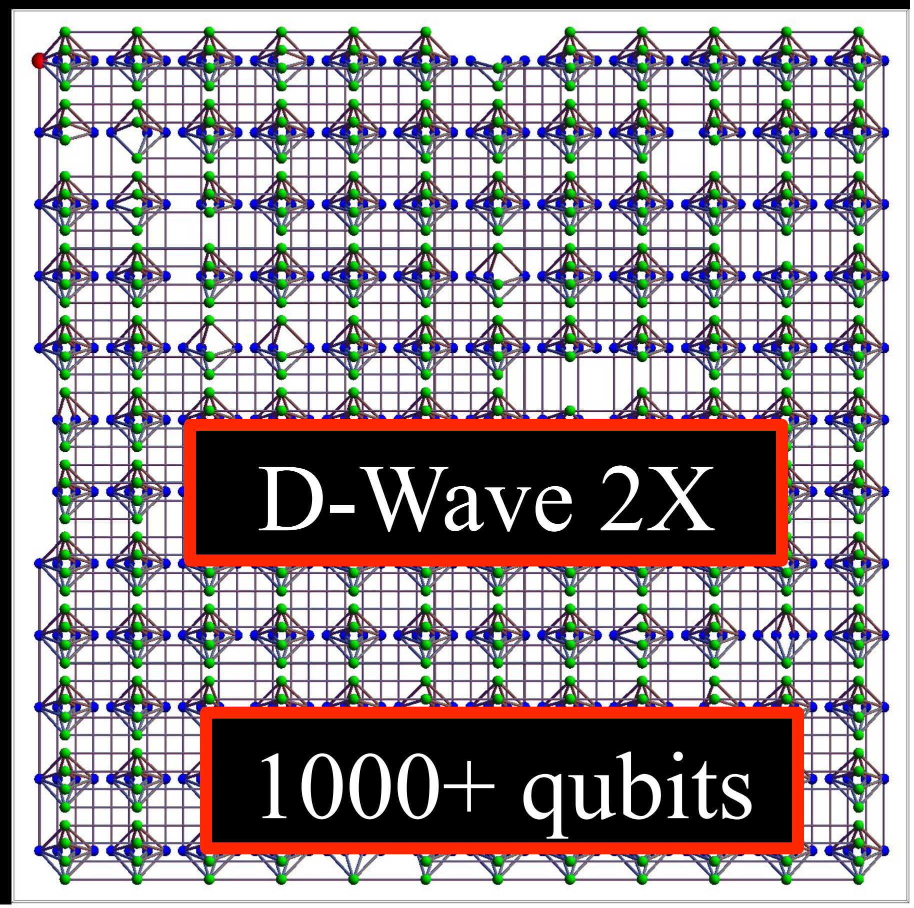
In 35+ years, ***two*** disruptive computing innovations:

Quantum Computing

"Perhaps the quantum computer will change our everyday lives in this century in the same radical way as the classical computer did in the last century."

Nobel press release (Oct. 2012)

David J. Wineland
Serge Haroche



New Tools are never optimal:

How to approach a new tool

- Test the tool to understand how well it works
- Work to improve subsequent tool generations
- Understand how the current and subsequent tool generations can be applied to enable novel applications.

Adiabatic Quantum Computer (AQC)

Testing D-Wave: Spanning Trees

AQC

You *give* AQC:

bias h_j on each qubit

coupling J_{ij} between qubit pairs

You *get* (probably)

Ground state Ising spin configuration

Oracle

$$\mathcal{H}_0 = -\sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \sum_{i=1}^{N} h_i^z\sigma_i^z$$

AQC Guarantees:

You will not always get the correct answer

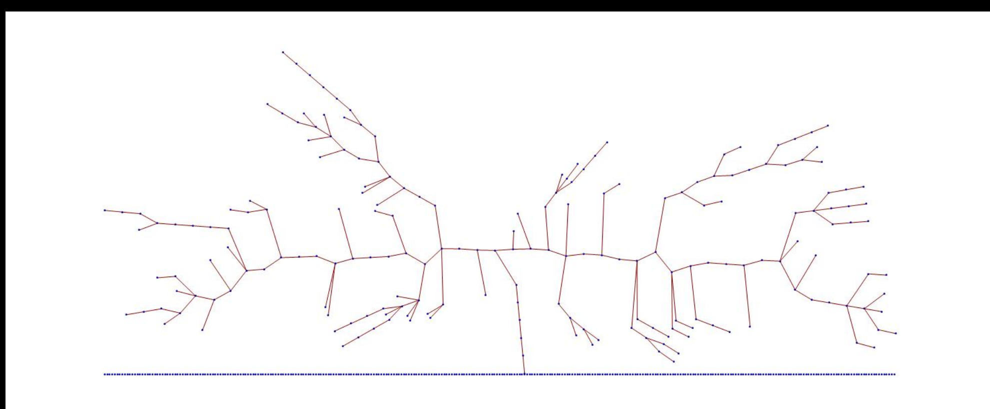
Test D-Wave

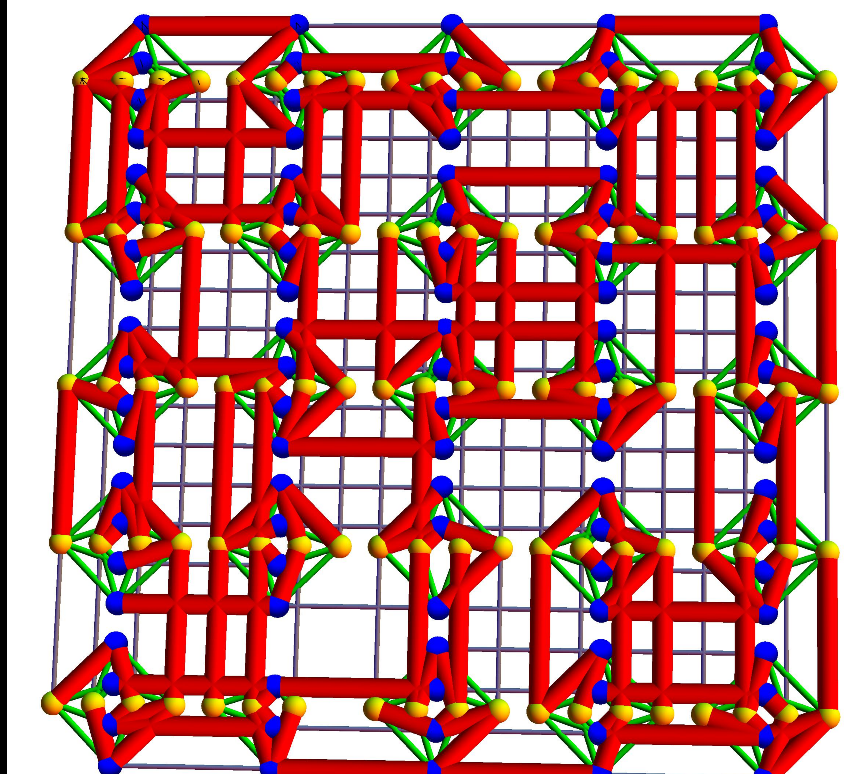
Give problems you know:

Spanning trees

(from ensemble of all spanning trees)

Oracle





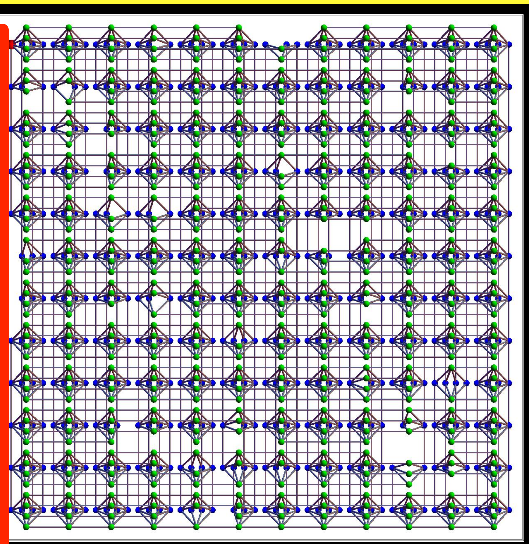
$$E_0=(N-1)\ |J|$$

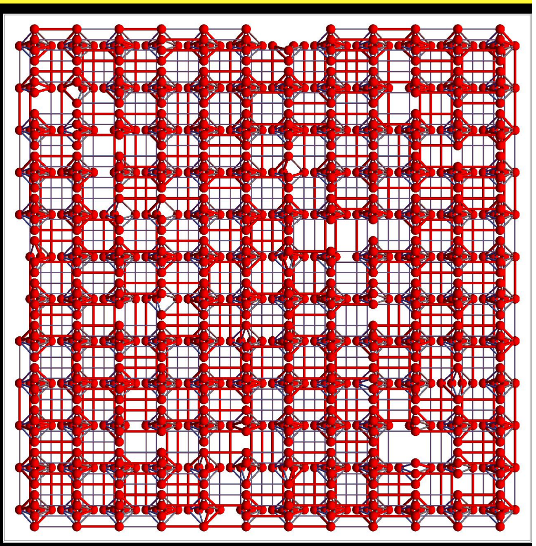
Spanning Trees: algorithm

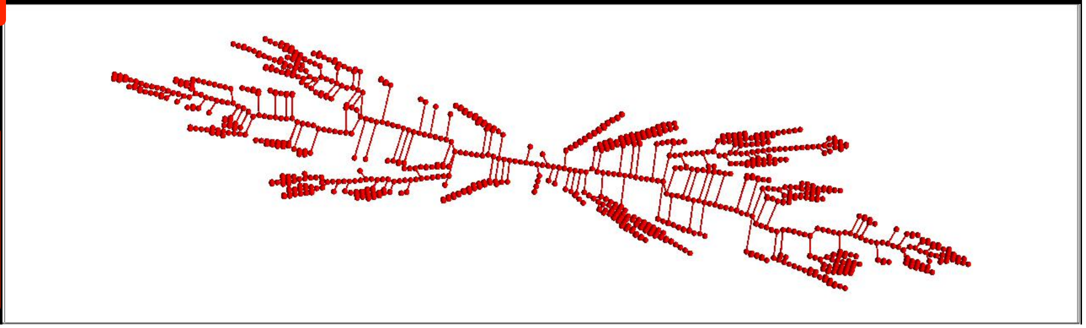
A spanning tree uniformly drawn from the ensemble of all spanning trees

Spanning tree advantage:

- Each tree includes all qubits
- Ensemble covers all bonds
- Ensemble is well defined
- Works on any graph
- Known groundstate
- Known spin arrangements
- Hard problem for AQC







$J_{i,j} = \pm 1 \qquad h_j = 0$

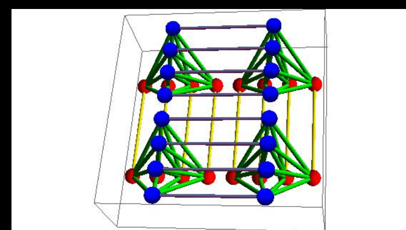
$E_{ground} = N_{qubits} - 1$

D-Wave 2 8×8 $K_{4,4}$ Chimera

Spanning trees
496 qubit
D-Wave 2

L x L square

100 spanning trees
up to100 submissions
each 10^3 anneals
$J_{ij} = \pm 1$



Good
Oracle?

Redacted Figure

**J.S. Hall, et al,
Physics Procedia
vol 68, 56-60 (2015)**

D-Wave 2X

12×12 $K_{4,4}$ Chimera

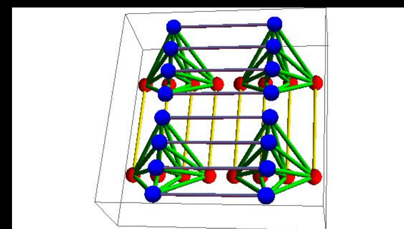
Spanning trees
496 qubit
D-Wave 2

1097 qubit
D-Wave 2X

L x L square

100 spanning trees
up to100 submissions
each 10^3 anneals
$J_{ij} = \pm 1$

M.A. Novotny, et al, J. Phys. Conf. Proc. (2016)



Good
Oracle?

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Adiabatic Quantum Computer (AQC)

Testing D-Wave: Answer Checking

Test D-Wave Answer Checking

Give problems with symmetry

Good Oracle?

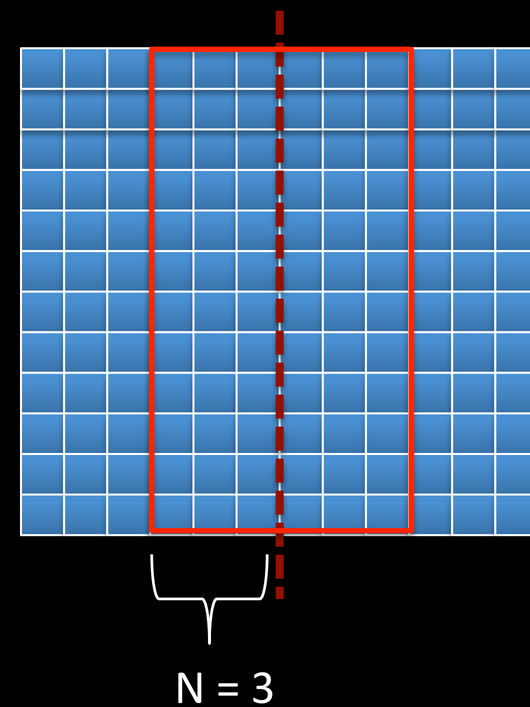
Redacted Figure

- Define the mirror axis.
- Mirror the missing qubits and couplers onto both sides.
- Embed the desired problem onto one side of the mirror.
- Embed the mirror image of the problem onto the other side.
- Connect a selected set of qubits with their corresponding mirror qubits via strong ferro-/ antiferromagnetic couplers.

D-Wave 2X: Answer Checking

Redacted Figure

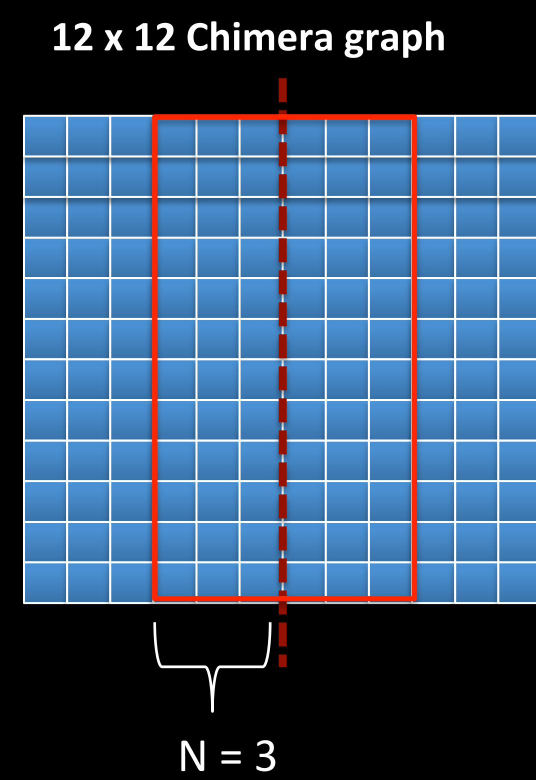
12 x 12 Chimera graph



***P**$_{sym}$* – Probability of obtaining at least a single symmetrical state as the lowest energy solution among 1000 annealing runs

D-Wave 2X: Answer Checking

Redacted Figure

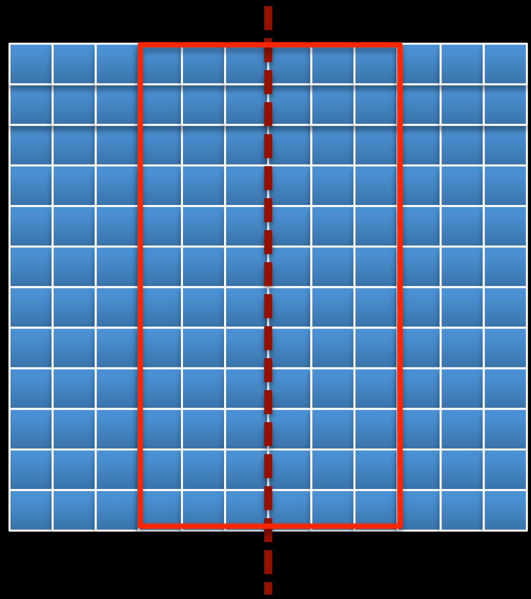


P_{sym} – Probability of obtaining at least a single symmetrical state as the lowest energy solution among 1000 annealing runs

D-Wave 2X: Answer Checking

Redacted Figure

12 x 12 Chimera graph



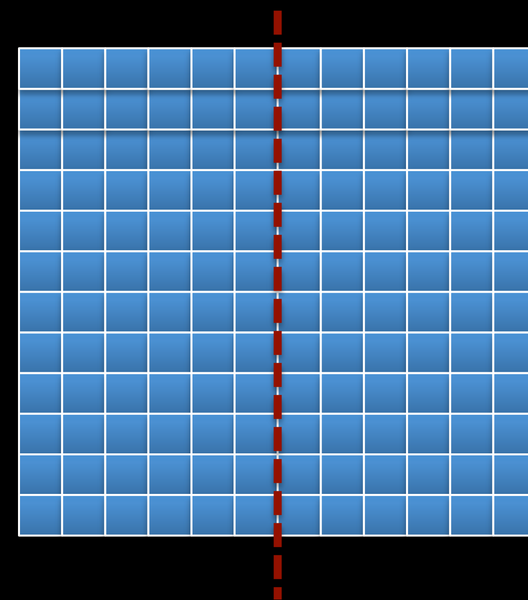
Hamming Distance

S_{28}

D-Wave 2X: Answer Checking

Redacted Figure

12 x 12 Chimera graph

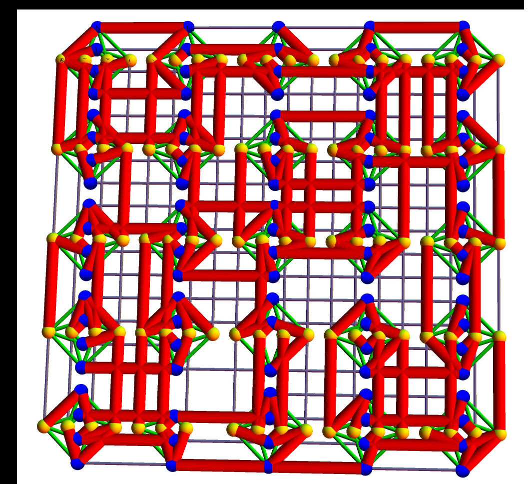


Hamming Distance

S$_{28}$

D-Wave 2X: Answer Checking

Redacted Figure



75% of the data points are above the diagonal

Error mitigation

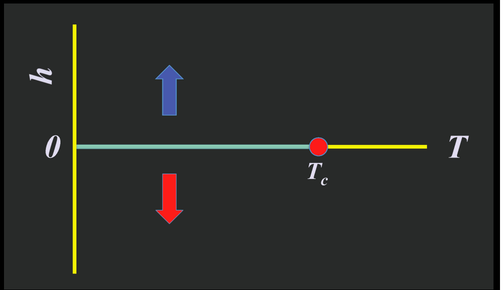
Adiabatic Quantum Computer (AQC)

Small-World (SW) Connections

Ising Ferromagnet or Spin Glass

y_T=1/ν critical exponents depend on dimension d

$$\mathcal{H}_0 = -\sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \sum_{i=1}^{N} h_i^z \sigma_i^z$$



***d*=1+ε expansions**

***ν*=1 exact *d*=2**

Redacted Figure

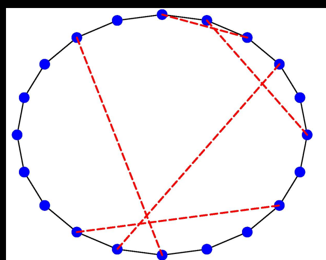
resummed *d*=4-ε expansions

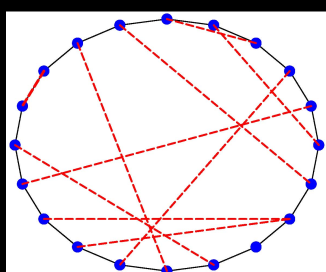
***ν*=1/y_T**

Numerical transfer matrix Novotny PRB 1992

Intro. to Small World Networks

Construct Small-World Networks

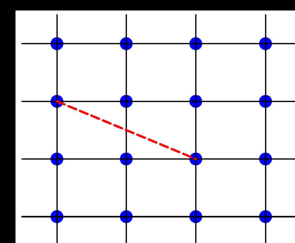


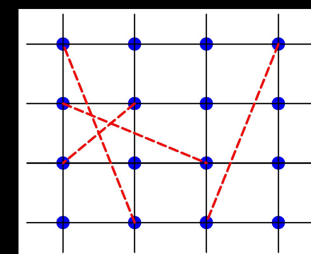


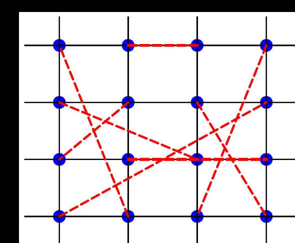
d=1 **to *SW***

or

d=2 **to *SW***



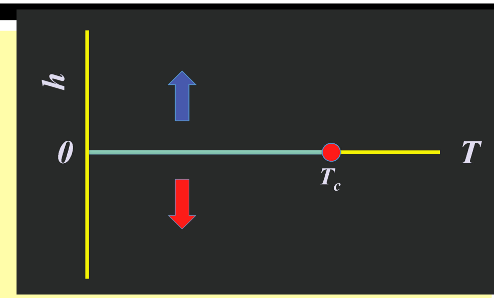




- ***p*** = Fraction of bonds added
- ***L*** (or ***L***^{2}) *lattice nodes*
- Average distance between nodes ***d~ln(L)***

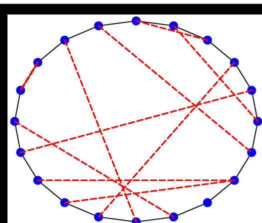
Small World (z-model)

- Start with $d=1$ chain of spins
- Randomly connect pairs of spins
- All spins have z interactions (strength 1)



Ising Ferromagnet

$z=3$



Redacted Figure

Introduced by Scalettar 1991

Small-world (*z*-model) y_T mean field

- Binder 4^{th}-order cumulant

Ising Ferromagnet

$$t = \left|(T - T_c)/T_c\right|$$

$$U_4(t) = f_U\left(tN^{\frac{1}{2}}\right)$$

$$\frac{\partial U_4(t)}{\partial t} = N^{\frac{1}{2}}\, f_U'\left(tN^{\frac{1}{2}}\right)$$

Brézin

Zinn-Justin

1985

Redacted Figure

Exact N=∞ value at T_c

D-Wave: improve the AQC?

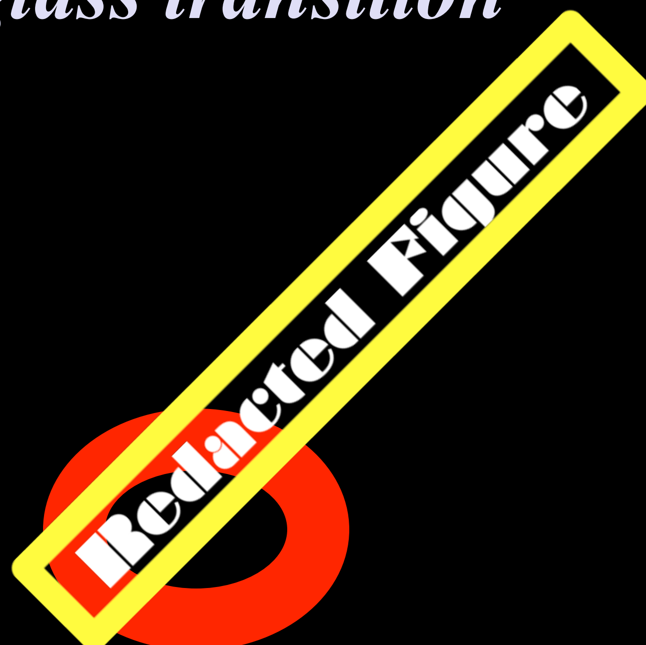
D-Wave 2X with 1000+ qubits does …

D-Wave 2X is NOT ideal adiabatic quantum computer

No finite-temperature spin-glass transition

Chimera $K_{4,4}$ lattice

Katzgraber, et al, PRX 2014



D-Wave: improve the AQC?

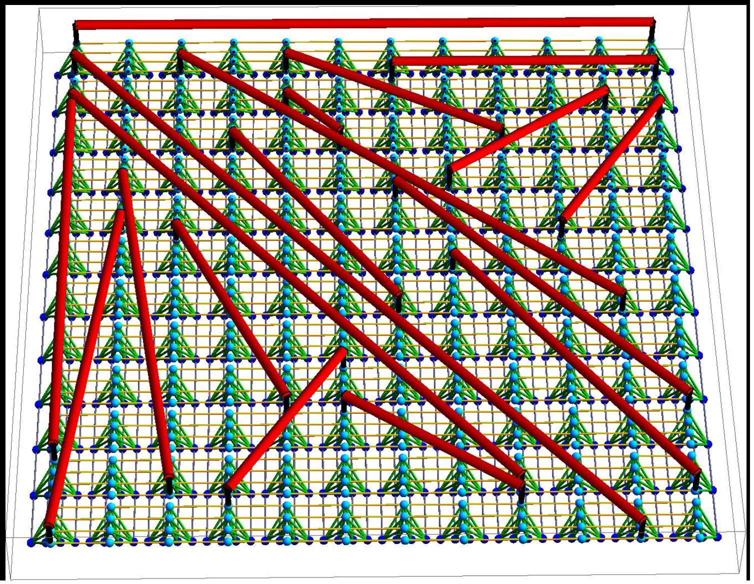
Adding SW connections gives finite-temperature spin-glass transition with mean-field exponents

Redacted Figure

$\nu = d_u / 2$

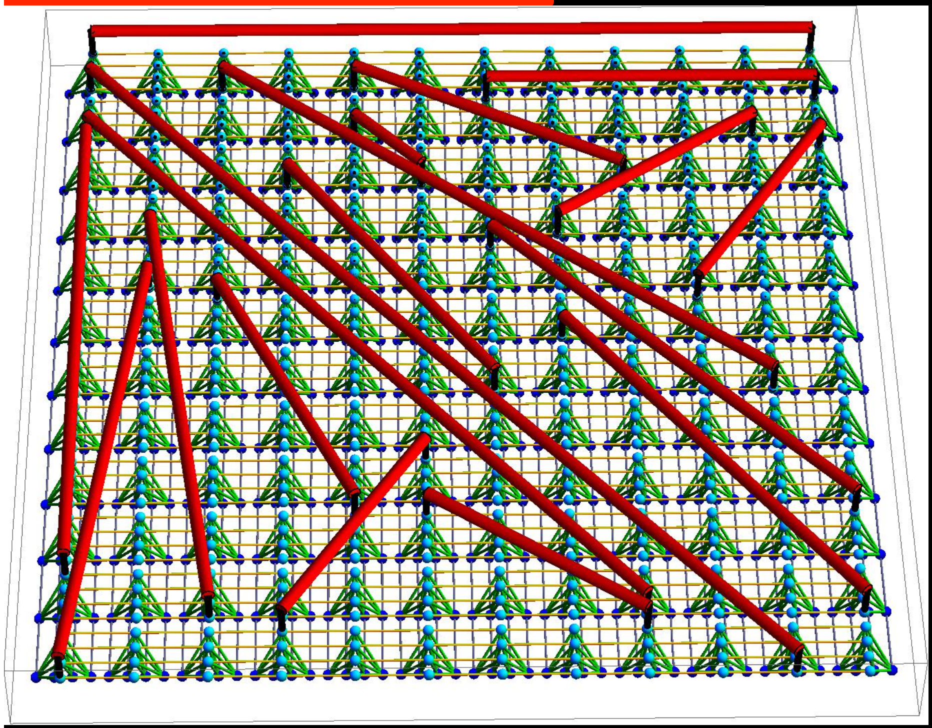
Ferromagnet $d_u=4$

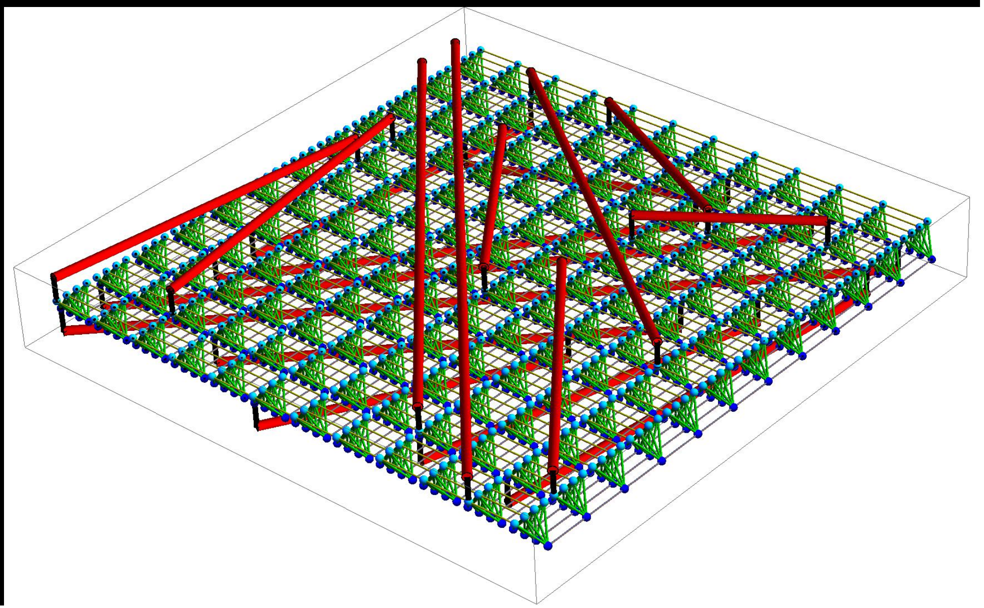
Spin Glass $d_u=6$



How to improve D-Wave AQC?

Change graph
to SW graph
d=∞





Better
Oracle?

Requires only 1-2
additional chip layers

US Patent. + US Patent Pending

How to *NOT* improve D-Wave AQC?

Change graph
to hierarchical graph

(19) **United States**

(12) **Patent Application Publication** (10) **Pub. No.: US 2009/0321720 A1**

Rose (43) **Pub. Date: Dec. 31, 2009**

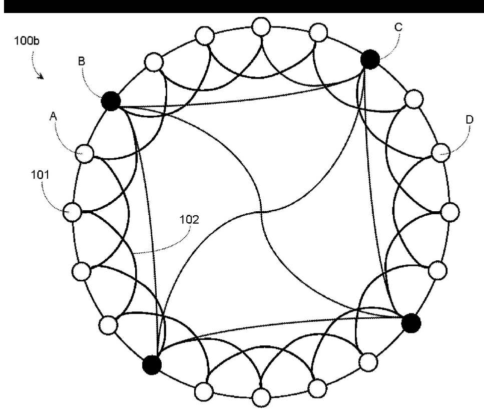
(54) **SYSTEMS AND DEVICES FOR QUANTUM PROCESSOR ARCHITECTURES**

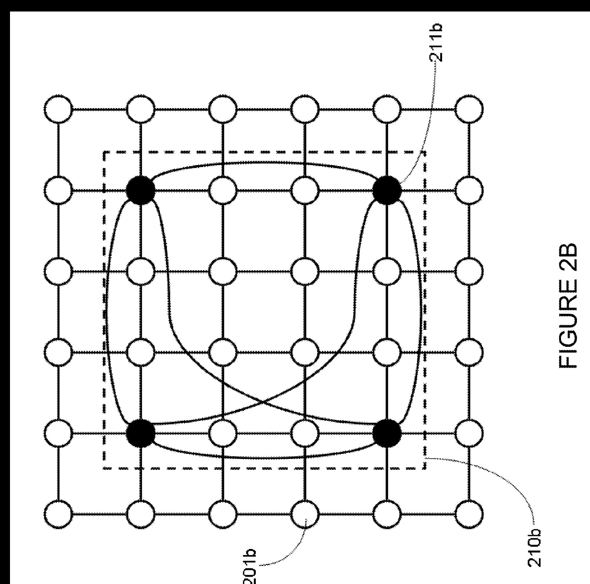
(76) Inventor: **Geordie Rose**, Vancouver (CA)

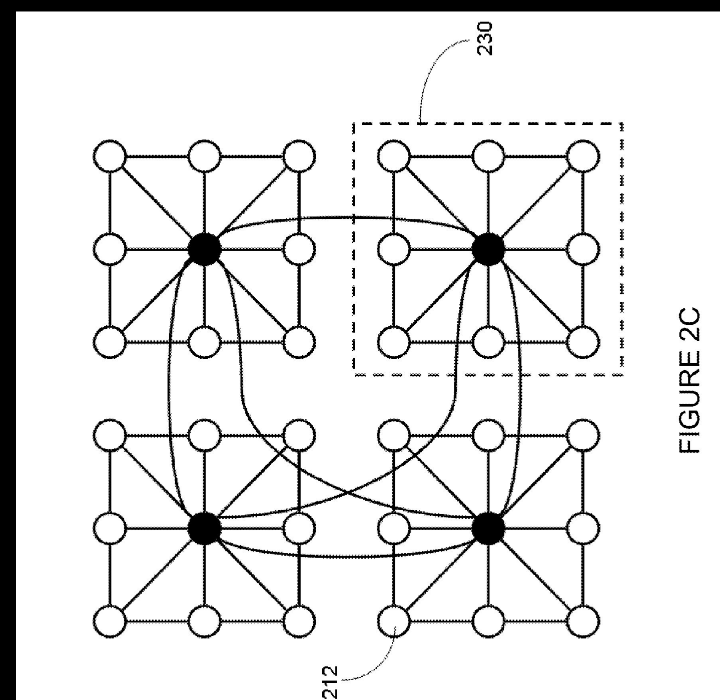
Publication Classification

(51) **Int. Cl.**
H01L 39/22 (2006.01)
H01L 21/02 (2006.01)

NOT Better
Oracle?







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Physical Review Letters **45**,
855-858 (1980)

NOT Better
Oracle?

Critical Phenomena on Fractal Lattices

Yuval Gefen
Department of Physics and Astronomy, Tel Aviv University, Ramat Aviv, Israel

and

Benoit B. Mandelbrot[(a)]
Department of Mathematics, Harvard University, Cambridge, Massachusetts 02138

and

Amnon Aharony[(b)]
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

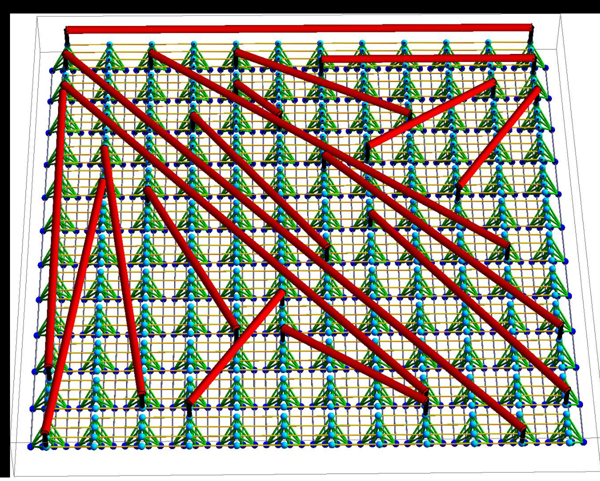
Renormalization-group techniques are applied to Ising-model spins placed on the sites of several self-similar fractal lattices. The resulting critical properties are shown to vary with the (noninteger) fractal dimensionality D, but also with several topological factors: ramification, connectivity, lacunarity, etc. For any $D \geq 1$, there exist systems with both $T_c = 0$, and $T_c > 0$; hence a lower critical dimensionality is not defined. The nonvanishing values of T_c and the critical exponents depend on all these factors.

How to improve D-Wave AQC?

Change graph
to SW graph
$d=\infty$

Redacted Figure

Better
Oracle?



Requires only 1-2
additional chip layers

US Patent. + US Patent Pending

How to improve D-Wave AQC?

p=0.20 *p*=0.38≈p_c *p*=0.42

Change graph
to SW graph
$d=\infty$

Chimera SW graphs
increase p_c
More robust against missing qubits

Better
Oracle?

Redacted Figure

p_c (*N*) p_c (*N*)

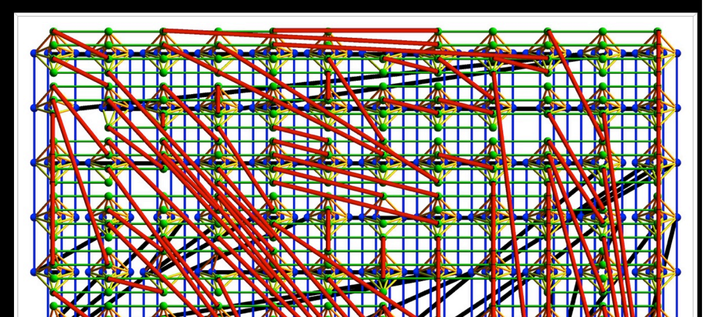
O. Melchert, H.G. Katzgraber, MAN, PRE **93**, 042128 (2016)

How to improve D-Wave AQC?

Change graph
to SW graph
$d=\infty$

Redacted Figure

Better
Oracle?



Requires only 1-2
additional chip layers

US Patent. + US Patent Pending

Finite spin-glass T_c
Mean-field exponents

How to test SW on D-Wave 2X?

Change graph
to SW graph
$d=\infty$

$K_{2,2}$ Chimera + SW

Better
Oracle?

SW algorithm (with answer-checking)

- Randomly choose top or bottom layer, and 2 $K_{2,2}$ qubits in that layer
- Determine shortest path via unused $K_{4,4}$–$K_{2,2}$ couplings
- If shortest path exists:
    - Add this short SW bond to $K_{2,2}$ + SW
    - Remove used couplers and qubits from available $K_{4,4}$–$K_{2,2}$
    - Go to first step if more SW bonds desired
  - else
    - Go to first step

How to improve D-Wave AQC?

Change graph
to SW graph
$d=\infty$

$K_{2,2}$ Chimera + SW

U_1 $J_{ij} \in \{-1,+1\}$

SW bonds: 15 top, 15 bottom

Redacted Figure

Better
Oracle?

For $K_{2,2}$ $p_{svm}=0.141$

For $K_{4,4}$ $p_{svm}=0.003$ ($N=6$)

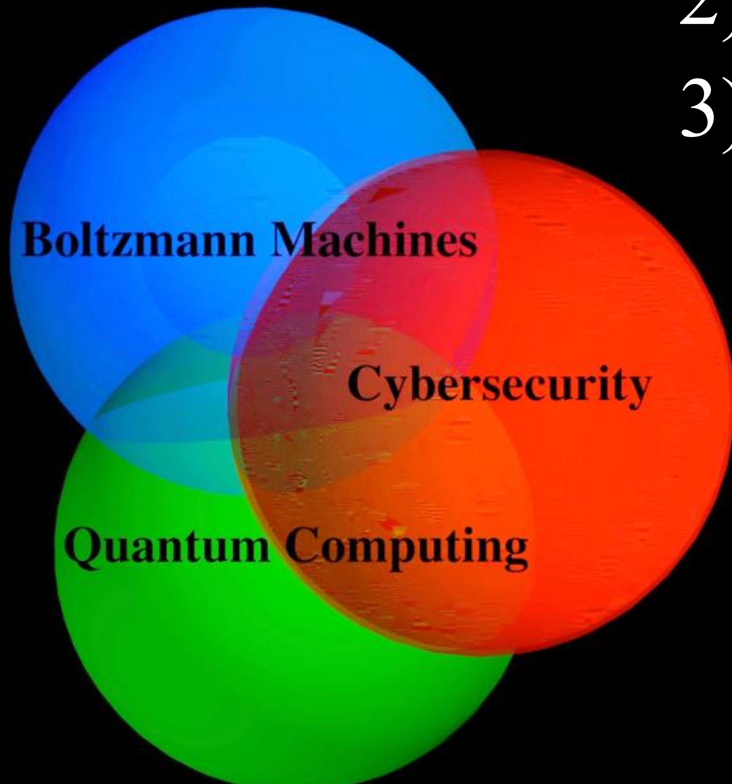
Adiabatic Quantum Computer (AQC)

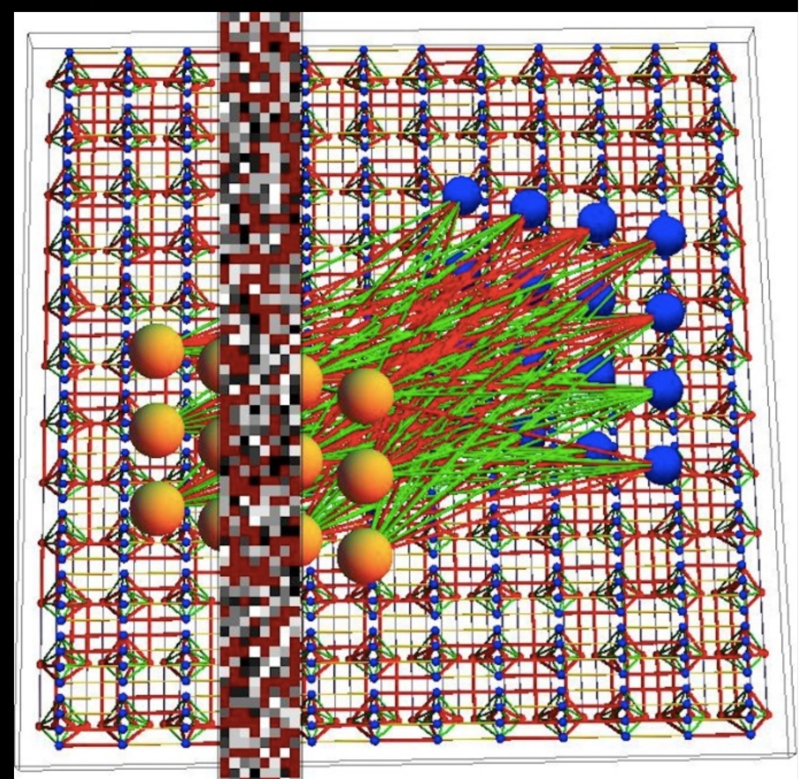
Boltzmann Machine Applications

My current application of AQC
Boltzmann Machines (Deep Belief)

Intersection of 3 fields:

1) Cybersecurity
2) Boltzmann machines
3) Quantum computing

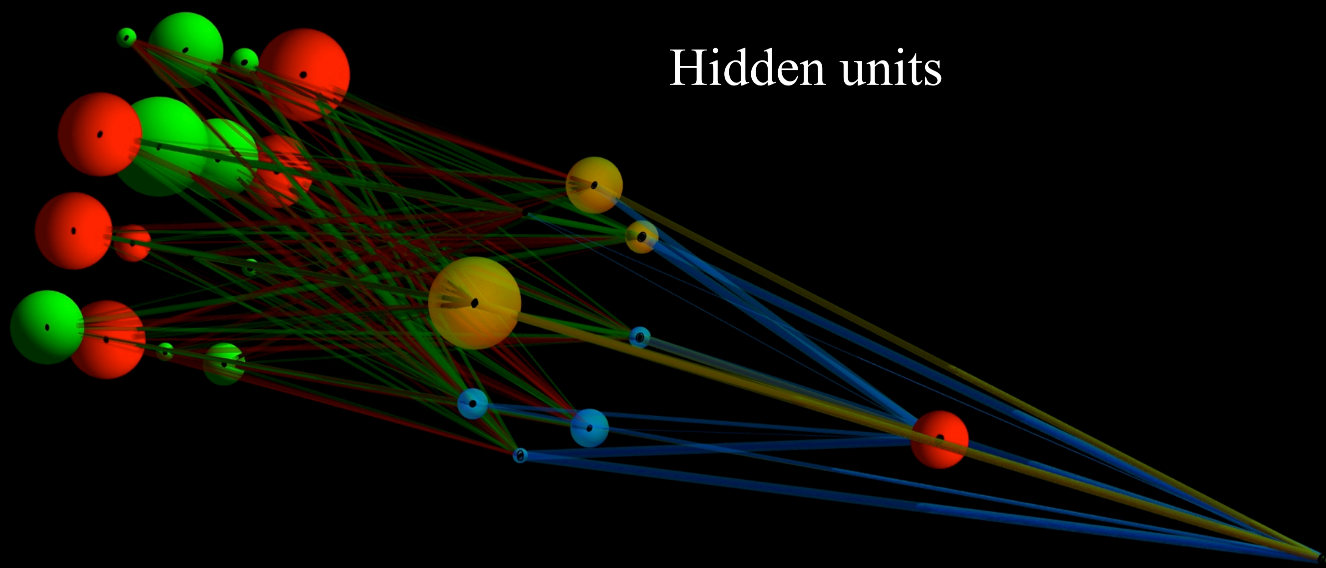




Boltzmann Machines

Run and test Restricted Boltzmann Machines

Hidden units

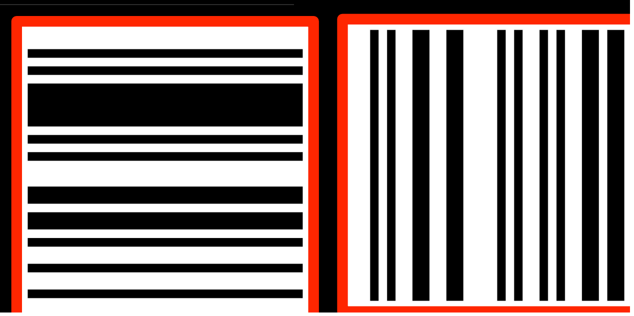


Visible units

(Bars **and** Stripes)

Visible units

(Bars **or** Stripes)



Boltzmann Machines

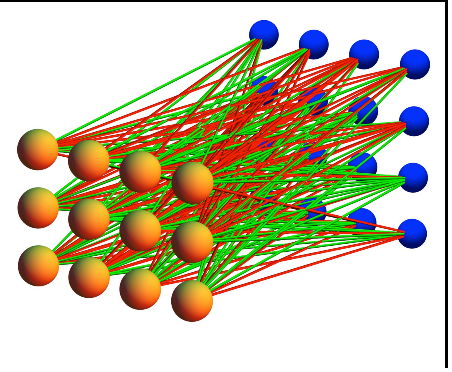
Technical Approach: Cybersecurity

- Use network data sets at Caida.org
  - Center for Applied Internet Data Analysis
  - Active Real-time Macroscopic Topology
- Test data sets:
  - Bars and Stripes model
  - Handwritten digits (MNIST)
  - KDD network traffic
  - ISCX network attack data set

Boltzmann Machines (BM)

Technical Approach: Boltzmann machines

- Restricted Boltzmann machines (RBM)
  - k-step contrastive divergence
  - parallel tempering
- More complicated graphs for BM
  - Chimera graph (D-Wave II)
  - Chimera + cloned graph (D-Wave II)



Boltzmann Machines (BM) with AQC

Technical Approach: Boltzmann machines

Redacted Figure

Conclusions

- D-Wave machines are first generation AQC
- D-Wave 2X works better than D-Wave 2 (spanning trees)
- Answer-checking gives indication of AQC abilities
- Small-World Connections should significantly enhance next generations of AQC
  - Finite spin-glass T_c
  - Mean-field exponents
  - Need only a few more fabrication layers
  - Better robustness (percolation)

Provocation

- Can another machine instruction (other than the current D-Wave instruction QMI) be useful?
  - Machine instruction to find # of ground states (QMI#)
    - #SAT type of problems
    - Maybe use results from recent PRA paper

Redacted Figure

$$\sigma(t) = \sqrt{\sum_{i=1}^{D_S-1} \sum_{j=i+1}^{D_S} \left|\hat{\rho}_{ij}(t)\right|^2}$$

$$\lim_{\beta\to\infty} \mathcal{E}\left(\sigma^2\right) \approx \frac{(g_S-1)(g_S g_E-1)}{2 g_S^2 g_E^2}$$

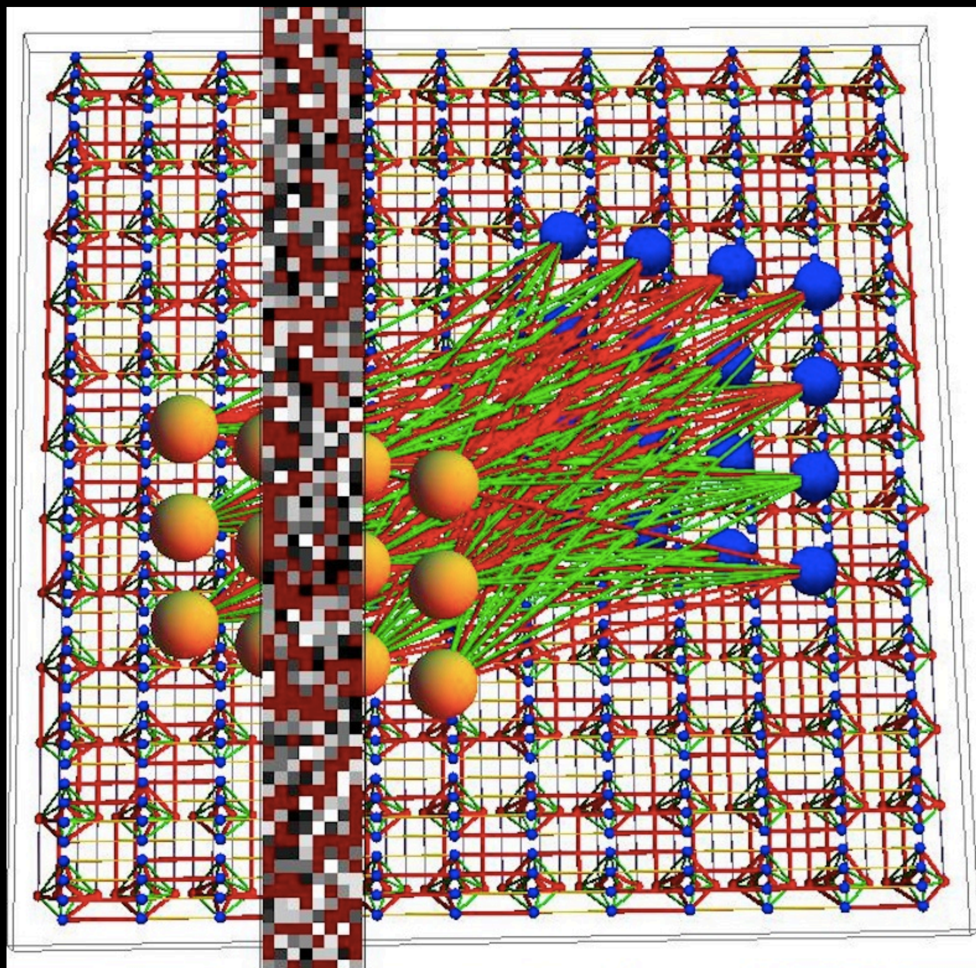
Phys Rev A **93**, 032110 [46 pages] (2016)

Thank You!

AQC past

AQC present

AQC future



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