Calculation of turbulence intensity

In general terms turbulence for a quasi steady state flow can be described as a mean velocity plus a fluctuating component. $\mathrm{u} = \bar{\mathrm{u}} + \mathrm{u}'$

The turbulent kinetic energy k below and to characterise the level of turbulence the figure turbulence intensity (I) is used. U is the velocity at given time.

$$I \equiv \frac{u'}{\bar{U}}$$

$$u' = \sqrt{\frac{2}{3}k}$$

$$U = \sqrt{u^2 + v^2 + w^2}$$

$$\mathrm{k} = \frac{1}{2}\left(\overline{\mathrm{u}'^2} + \overline{\mathrm{v}'^2} + \overline{\mathrm{w}'^2}\right)$$

$$\mathrm{I} = \frac{\left(\frac{2}{3} \cdot \mathrm{k}\right)^{1/2}}{\bar{\mathrm{U}}}$$

$$\Downarrow$$

$$\mathrm{I} = \frac{\left(\frac{2}{3} \cdot \frac{1}{2} \cdot \left(\overline{\mathrm{u}'^2} + \overline{\mathrm{v}'^2} + \overline{\mathrm{w}'^2}\right)\right)^{1/2}}{\bar{\mathrm{U}}}$$

$$\Downarrow$$

$$\mathrm{I} = \frac{\sqrt{\frac{1}{3} \cdot \left(\overline{\mathrm{u}'^2} + \overline{\mathrm{v}'^2} + \overline{\mathrm{w}'^2}\right)}}{\bar{\mathrm{U}}}$$

Practical considerations for calculating the turbulent intensity

$\overline{\mathrm{u}'^2}$ $\overline{\mathrm{v}'^2}$ $\overline{\mathrm{w}'^2}$ are the root mean square of the velocity fluctuations in each direction. This is the same as the square of the standard deviation of the velocity fluctuations, also called the variance. Further the mean time averaged velocity $\bar{\mathrm{U}}$ is the mean of the total velocity and not the individual components in the vector.

$$\bar{\mathrm{U}} = \frac{1}{\mathrm{n}} \cdot \sum_{\mathrm{t}=1}^{\mathrm{n}} \left(\sqrt{\mathrm{u(t)}^2 + \mathrm{v(t)}^2 + \mathrm{v(t)}^2}\right)$$

This gives the following formula to calculate for calculation of turbulence intensity:

$$\mathrm{I} = \frac{\sqrt{\frac{1}{3} \cdot \left(\mathrm{Var(u)} + \mathrm{Var(v)} + \mathrm{Var(w)}\right)}}{\bar{\mathrm{U}}} = \frac{\sqrt{\frac{1}{3} \cdot \left(\sigma(\mathrm{u})^2 + \sigma(\mathrm{v})^2 + \sigma(\mathrm{w})^2\right)}}{\bar{\mathrm{U}}}$$

The above calculation can easily be done in excel for a chosen time period, e.g. a period where the flow is quasi steady state. For example 5 seconds of the simulation time where the conditions do not change. Note that the values for either variance or standard deviation should be calculated on the basis of a sample (and not the entire population), so the formula used is the one where there is divided with (n-1) instead of n.

Including turbulence on the sub-grid

The above example is for a fluid, where all the turbulence are resolved and a part of the total turbulent energy. Assuming that a LES model is used, - only some of the turbulent energy will be on the sub grid scale, k_{sgs} , turbulent energy on the sub grid.

$$k_{total} = k_{resolved} + k_{sgs}$$

$$k_{total} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) + k_{sgs}$$

$$I = \frac{\left(\frac{2}{3} \cdot k_{total}\right)^{1/2}}{\bar{U}}$$

$$\Downarrow$$

$$I = \frac{\left(\frac{2}{3} \cdot \frac{1}{2} \cdot \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) + \frac{2}{3} \cdot k_{sgs}\right)^{1/2}}{\bar{U}}$$

$$\Downarrow$$

$$I = \frac{\sqrt{\frac{1}{3} \cdot \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) + \frac{2}{3} \cdot k_{sgs}}}{\bar{U}}$$

Using the same method as previously the intensity can be written as:

$$I = \frac{\sqrt{\frac{1}{3} \cdot \left(\mathrm{Var(u)} + \mathrm{Var(v)} + \mathrm{Var(w)}\right) + \frac{2}{3} \cdot k_{sgs}}}{\bar{U}} = \frac{\sqrt{\frac{1}{3} \cdot \left(\sigma(u)^2 + \sigma(v)^2 + \sigma(w)^2\right) + \frac{2}{3} \cdot k_{sgs}}}{\bar{U}}$$

Relation between the turbulence on grid and on sub grid

Turbulence resolution:

$$\text{Turbulence resolution} = \frac{k_{sgs}}{k_{total}} = \frac{k_{sgs}}{k_{resolved} + k_{sgs}} = \frac{k_{sgs}}{\frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) + k_{sgs}}$$

Expressed in terms for evaluation in Excel:

$$\text{Turbulence resolution} == \frac{k_{sgs}}{\frac{1}{2}\left(\mathrm{Var(u)} + \mathrm{Var(v)} + \mathrm{Var(w)}\right) + k_{sgs}}$$

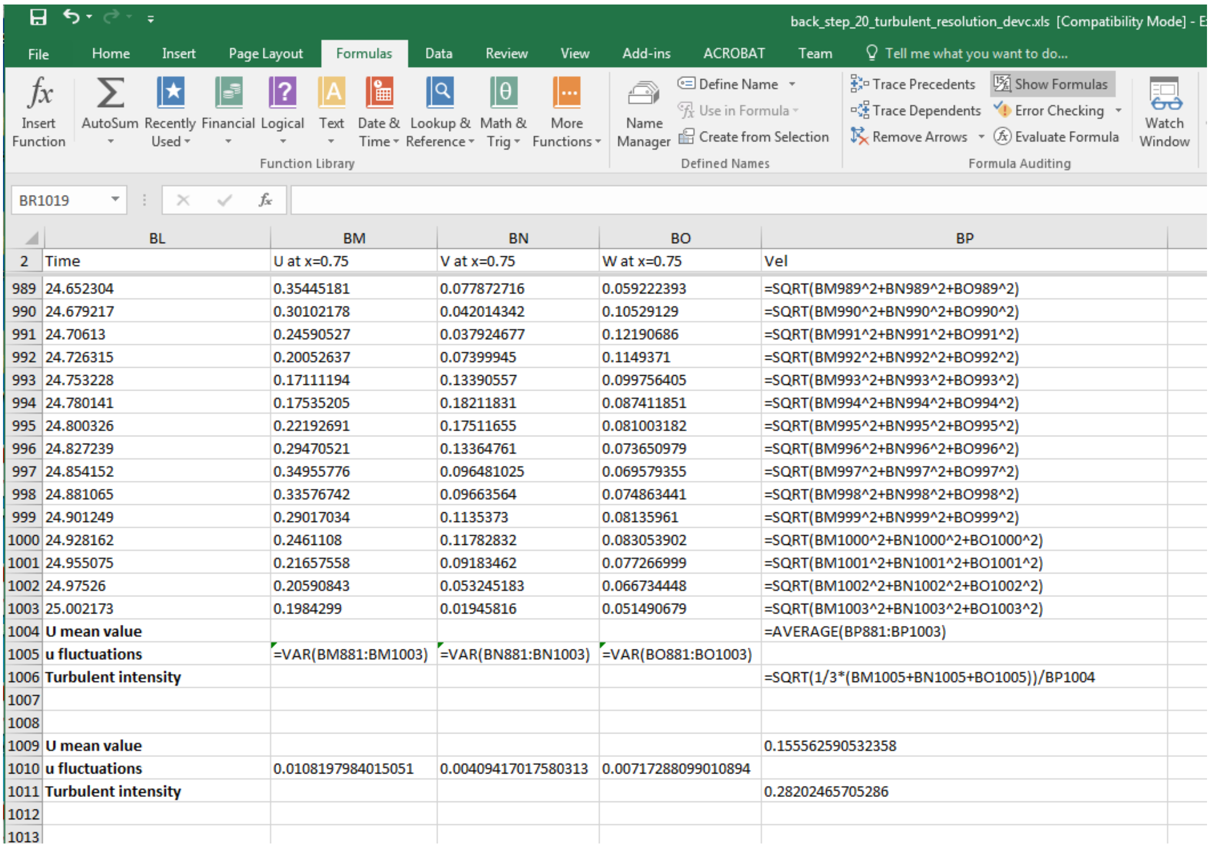
General about turbulence levels

(copied from http://www.cfd-online.com/Wiki/Turbulence_intensity

1. High-turbulence case: High-speed flow inside complex geometries like heat-exchangers and flow inside rotating machinery (turbines and compressors). Typically the turbulence intensity is between 5% and 20%
2. Medium-turbulence case: Flow in not-so-complex devices like large pipes, ventilation flows etc. or low speed flows (low Reynolds number). Typically the turbulence intensity is between 1% and 5%
3. Low-turbulence case: Flow originating from a fluid that stands still, like external flow across cars, submarines and aircrafts. Very high-quality wind-tunnels can also reach really low turbulence levels. Typically the turbulence intensity is very low, well below 1%.

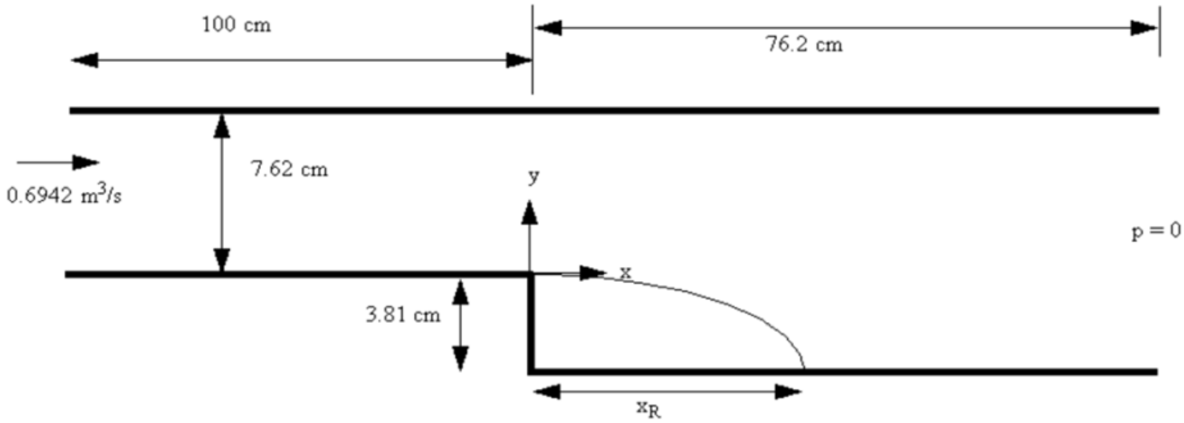
Practical example using excels to calculate the turbulent resolution.

This example gives a turbulent intensity as shown in field BP1011 of 28% for the position at x=0.75 m (the example on the next side is from another calculation, hence the value at x=0.75 m is different). Note that velocity has to be calculated first for every time step and then this list of velocities is averaged.

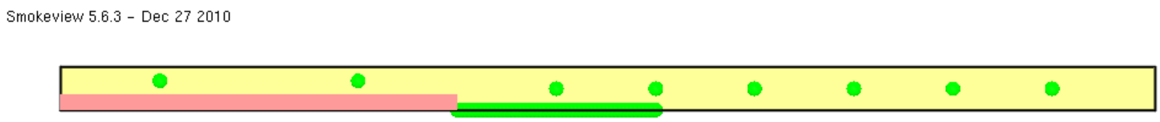


	BL	BM	BN	BO	BP
2	Time	U at x=0.75	V at x=0.75	W at x=0.75	Vel
989	24.652304	0.35445181	0.077872716	0.059222393	=SQRT(BM989^2+BN989^2+BO989^2)
990	24.679217	0.30102178	0.042014342	0.10529129	=SQRT(BM990^2+BN990^2+BO990^2)
991	24.70613	0.24590527	0.037924677	0.12190686	=SQRT(BM991^2+BN991^2+BO991^2)
992	24.726315	0.20052637	0.07399945	0.1149371	=SQRT(BM992^2+BN992^2+BO992^2)
993	24.753228	0.17111194	0.13390557	0.099756405	=SQRT(BM993^2+BN993^2+BO993^2)
994	24.780141	0.17535205	0.18211831	0.087411851	=SQRT(BM994^2+BN994^2+BO994^2)
995	24.800326	0.22192691	0.17511655	0.081003182	=SQRT(BM995^2+BN995^2+BO995^2)
996	24.827239	0.29470521	0.13364761	0.073650979	=SQRT(BM996^2+BN996^2+BO996^2)
997	24.854152	0.34955776	0.096481025	0.069579355	=SQRT(BM997^2+BN997^2+BO997^2)
998	24.881065	0.33576742	0.09663564	0.074863441	=SQRT(BM998^2+BN998^2+BO998^2)
999	24.901249	0.29017034	0.1135373	0.08135961	=SQRT(BM999^2+BN999^2+BO999^2)
1000	24.928162	0.2461108	0.11782832	0.083053902	=SQRT(BM1000^2+BN1000^2+BO1000^2)
1001	24.955075	0.21657558	0.09183462	0.077266999	=SQRT(BM1001^2+BN1001^2+BO1001^2)
1002	24.97526	0.20590843	0.053245183	0.066734448	=SQRT(BM1002^2+BN1002^2+BO1002^2)
1003	25.002173	0.1984299	0.01945816	0.051490679	=SQRT(BM1003^2+BN1003^2+BO1003^2)
1004	**U mean value**				=AVERAGE(BP881:BP1003)
1005	**u fluctuations**	=VAR(BM881:BM1003)	=VAR(BN881:BN1003)	=VAR(BO881:BO1003)	
1006	**Turbulent intensity**				=SQRT(1/3*(BM1005+BN1005+BO1005))/BP1004
1007					
1008					
1009	**U mean value**				0.155562590532358
1010	**u fluctuations**	0.0108197984015051	0.00409417017580313	0.00717288099010894	
1011	**Turbulent intensity**				0.28202465705286
1012					
1013					

The turbulence levels have been calculated for the backward facing step as described[1, 2]:

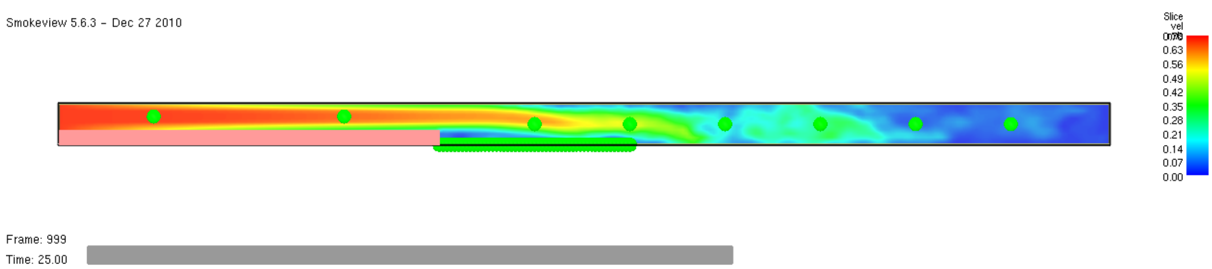


The backward facing step has been modelled in FDS using a step height of 4 cm and a height in the inlet tunnel of 7 cm. The turbulence levels have been measured at 8 point along the centre line of the wind tunnel. The step is places at x=0 m. And the measuring points are placed at -0.75m, -0.25 m, 0.25 m, 0.50 m, 0.75 m, 1.00 m, 1.25 m, 1.50 m.



X position	-0.75	-0.25	0.25	0.5	0.75	1	1.25	1.5
Turbulence intensity	0.18%	0.18%	2.0%	22%	24%	19%	41%	15%

It can be seen that before the step (at x=-0.75 m and x=-25 m) the turbulence level are very low, which is also as expected as no turbulence is introduced at the boundary. Also immediately after the step the intensity is low, as the eddies created by the step, do not disturb the measurement position at x=0.25 m. For the remaining length of the channel the turbulence levels are high, nearly all over 20%, except the last point at x=1.5 m, where the flows seems to stabilize again. The can also be seen at the figure below showing the velocity.



References

1. *CFdesign Technical Reference, version 9.0* 2007, Charlottesville, VA: Blue Ridge Numerics.
2. Vogel and Eaton, *Combined heat transfer and fluid dynamic measurements downstream of a backward-facing step.* Transactions of the ASME. Journal of Heat Transfer, 1985. **107**(4): p. 922-929.