

# Colloidal stability



## Important practical questions:

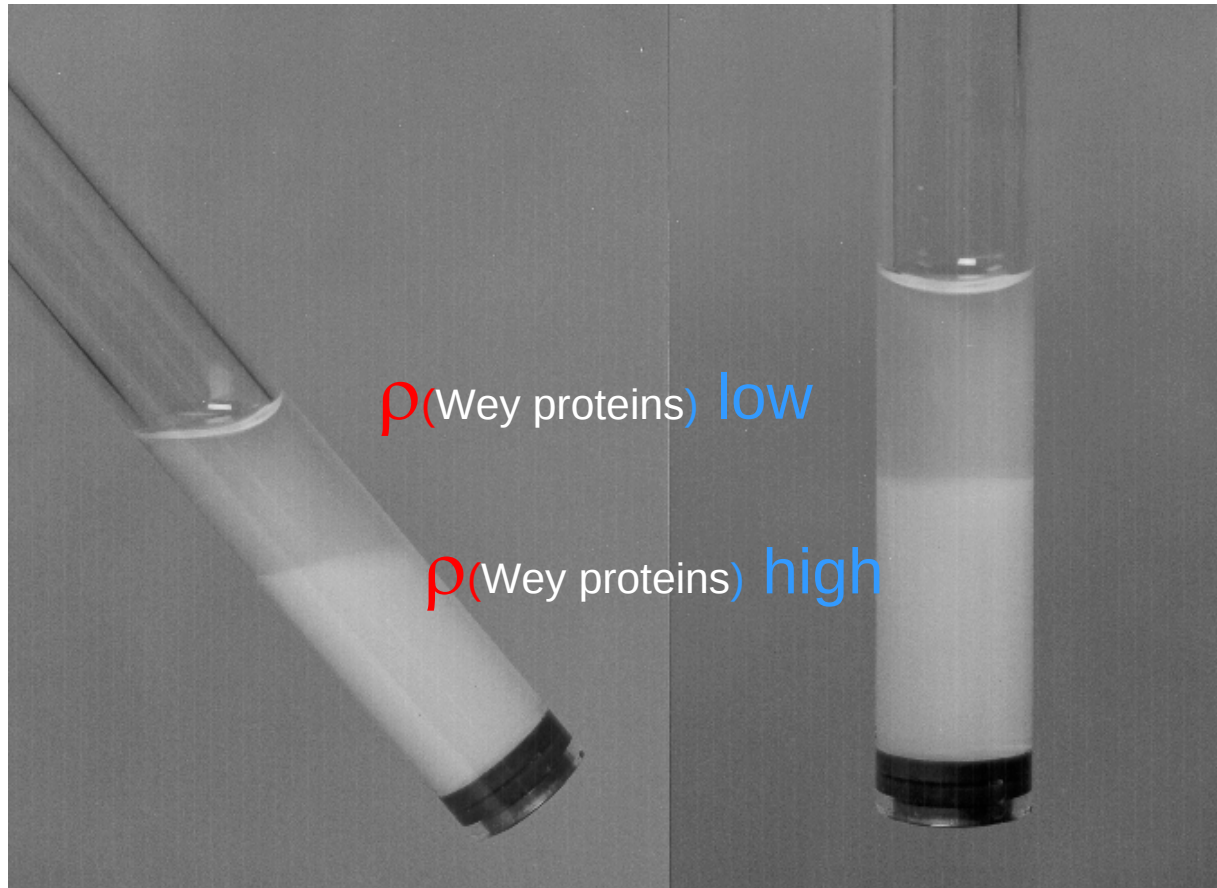
- Does dispersion change when T, P or ... is changed?
- If T, P or ... is changed and the dispersion phase separates, what are then the final products?
- How fast does the phase separation take place?

## Examples:

- Food processing
- Protein crystal growing
- Rain makers
- Cataract (grauer Star)
- Synthesis of monodisperse colloidal particles
- Precipitation of  $\text{CaCO}_3$

## Dutch example: Cheese

Depletion of Wey proteins by Extracellular Poly-Saccharides



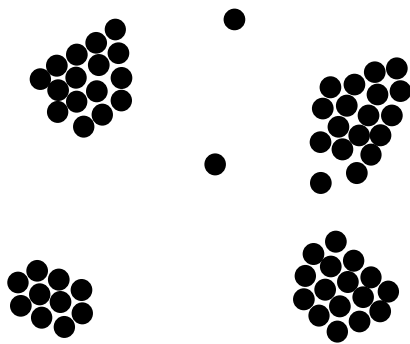


# Phase separation:

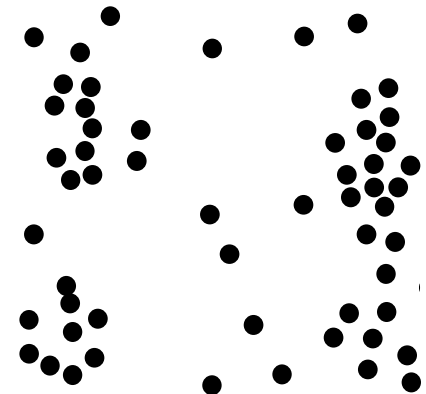
System with concentration  $\rho_0$  decomposes into a phase with a higher concentration  $\rho_+$  and a phase with a lower concentration  $\rho_-$ .

Two types of phase separation, two types of kinetics :

Liquid-solid or Gas-Solid



Gas-liquid



## Molecular systems hard to study:

- Phase separation is too fast
- Particles are too small to be observed with a microscope
- No control on/knowledge of the interaction potential

## Use Colloids:

- Mesoscopic Brownian particles dispersed in solvent instead of molecules dispersed in vacuum → consider  $\Pi$  instead of  $P$ .

## Advantages:

- Potential can be tuned by choice of particles and solvent
- Dynamics is slow
- Particles are detectable by microscopy and light scattering

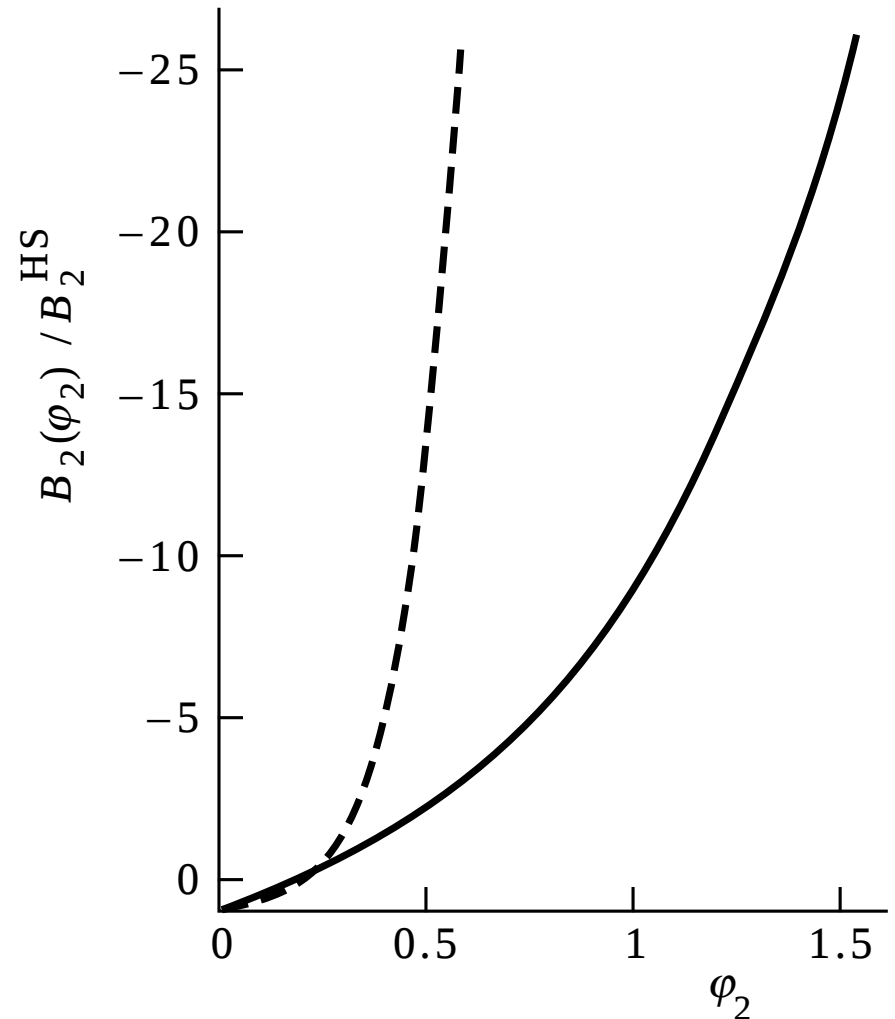
# Simple estimation of spinodal

When  $B_2$  becomes sufficiently negative demixing takes place

$$\frac{\partial(\Pi_c v_c / kT)}{\partial \phi} = 1 + 2(B_2 / v_c)\phi + \dots$$

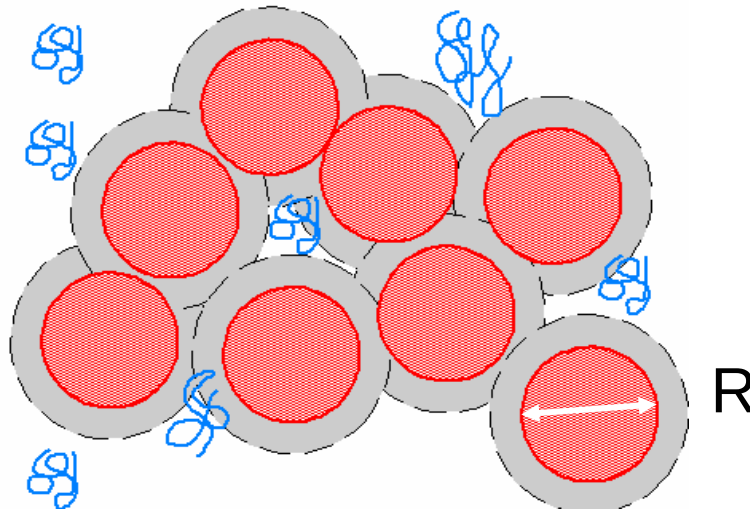
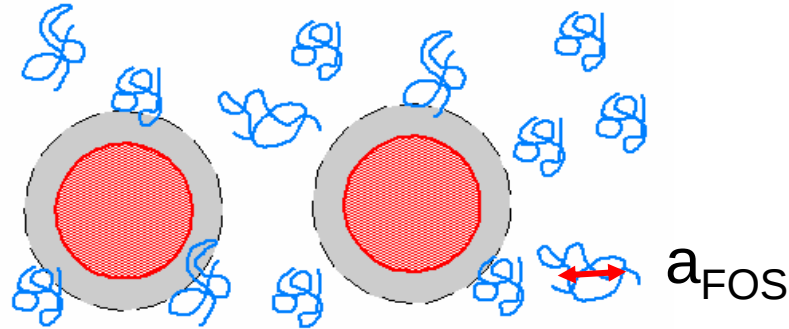
Estimation spinodal:

$$\phi_{spinodal} = \frac{-v_c}{2B_2}$$



# Colloid Dispersion with mediated by depletion forces

$$q = a_{\text{FOS}}/R$$



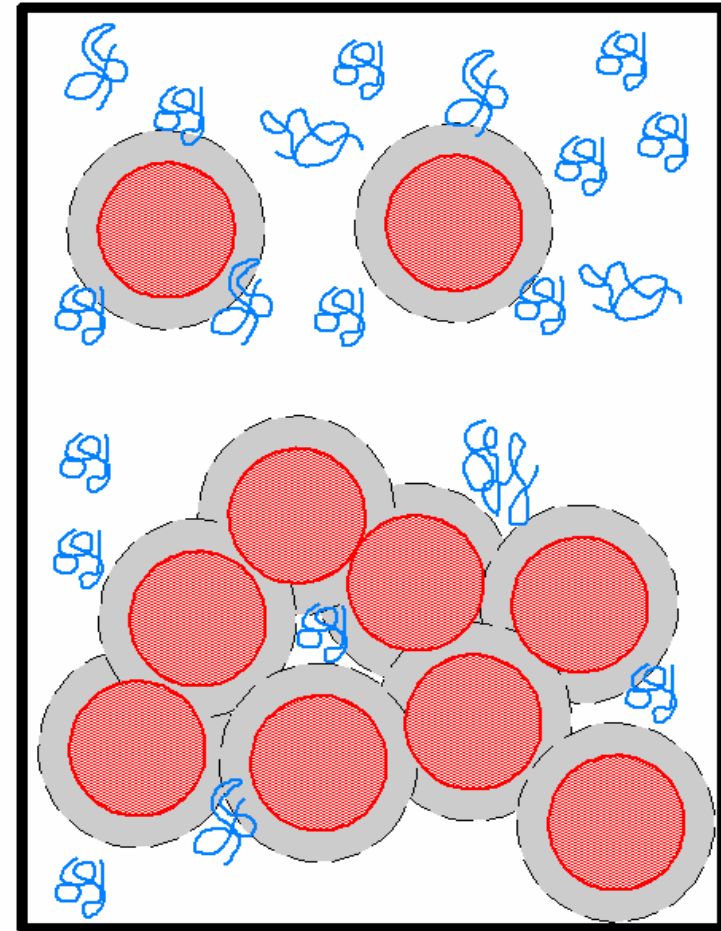
# Second virial coefficient

$B_2$  can be computed from depletion potential:

$$B_2 = 2\pi \int_0^{\infty} r^2 \left[ 1 - e^{\frac{-\omega(r)}{kT}} \right] dr$$

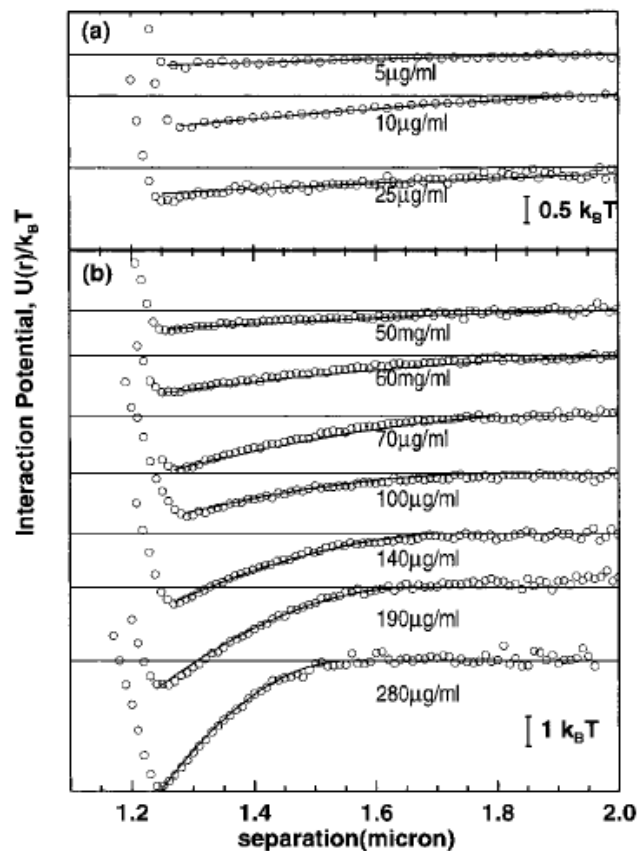
$$\omega(r) = \begin{cases} \infty & \text{for } r < \sigma \\ -\Pi_p V_{\text{overlap}}(r) & \text{for } \sigma < r < \sigma + 2\delta \\ 0 & \text{for } r > \sigma + 2\delta \end{cases}$$

$$B_2 = 4V_{HS} + 2\pi \int_{\sigma}^{\sigma+2\delta} r^2 \left[ 1 - \exp\left( \frac{\Pi_p V_{ov}(r)}{kT} \right) \right] dr$$

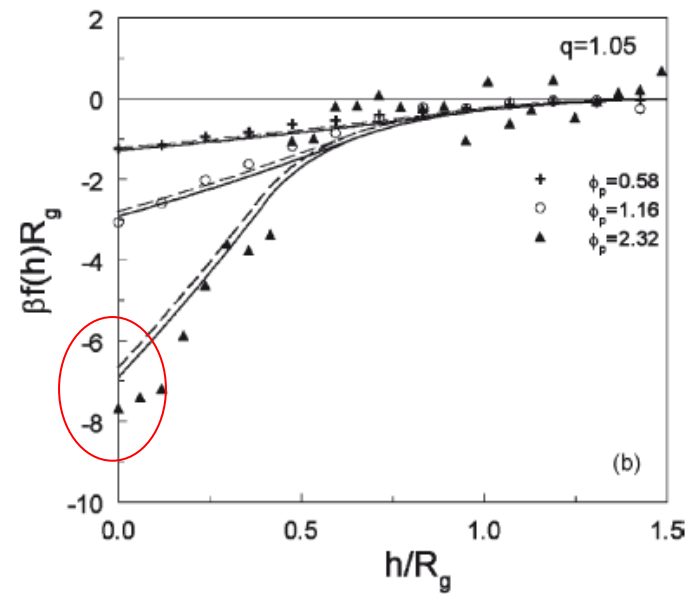
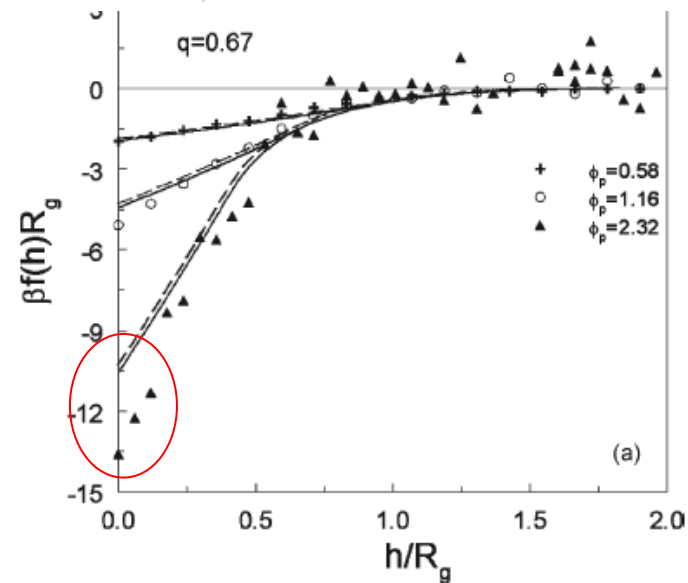


# Attractions between Hard Colloidal Spheres in Semiflexible Polymer Solutions

Ritu Verma, J. C. Crocker, T. C. Lubensky, and A. G. Yodh\*

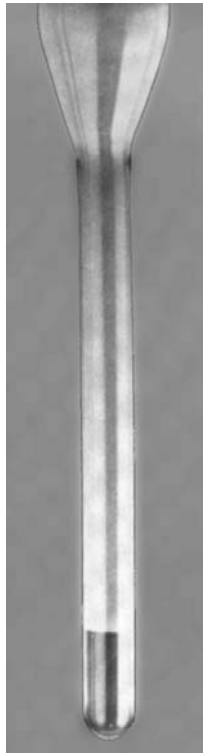


## Pair interaction and phase separation in mixtures of colloids and excluded volume polymers

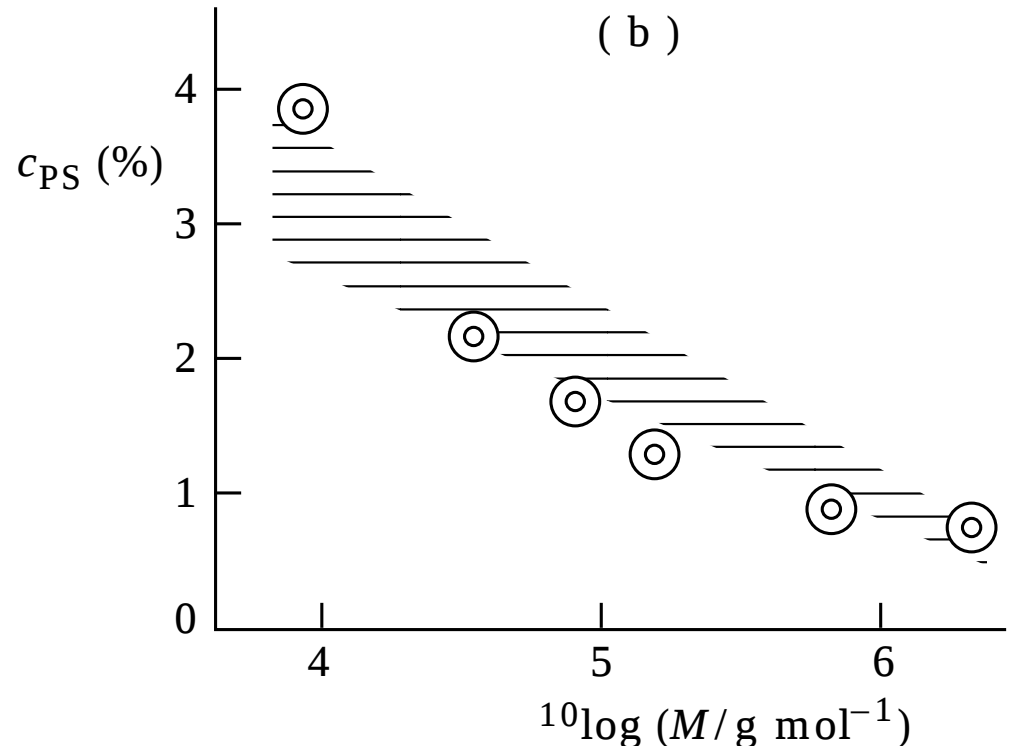
R. Tuinier,<sup>a</sup> D. G. A. L. Aarts,<sup>b</sup> H. H. Wensink<sup>b</sup> and H. N. W. Lekkerkerker<sup>b</sup>

# Demixing concentrations estimated from $B_2$

- De Hek and Vrij, 1980:
- 1 wt% HS-like silica  $a=46$  nm
- mixed with PS in CHX

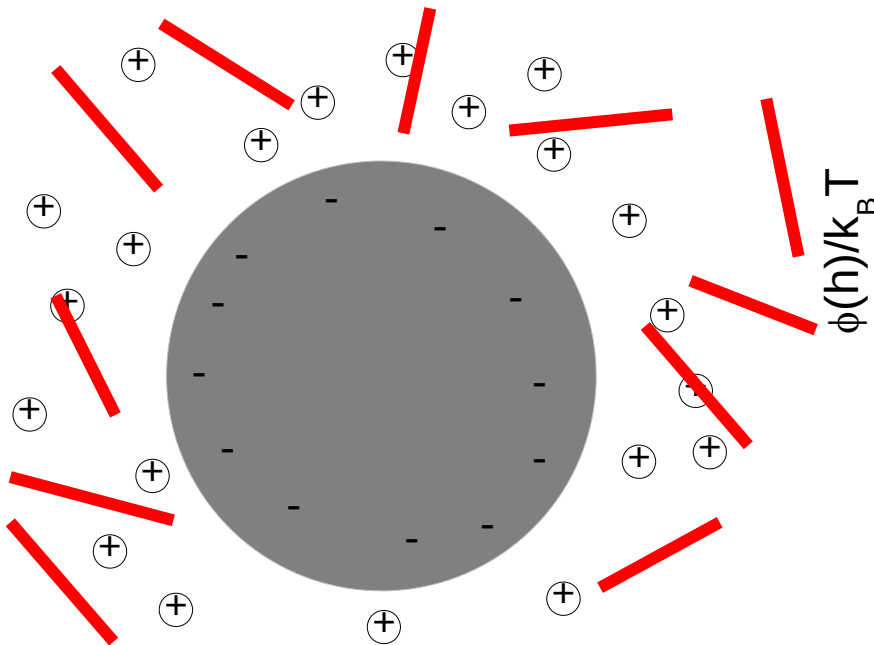


( a )

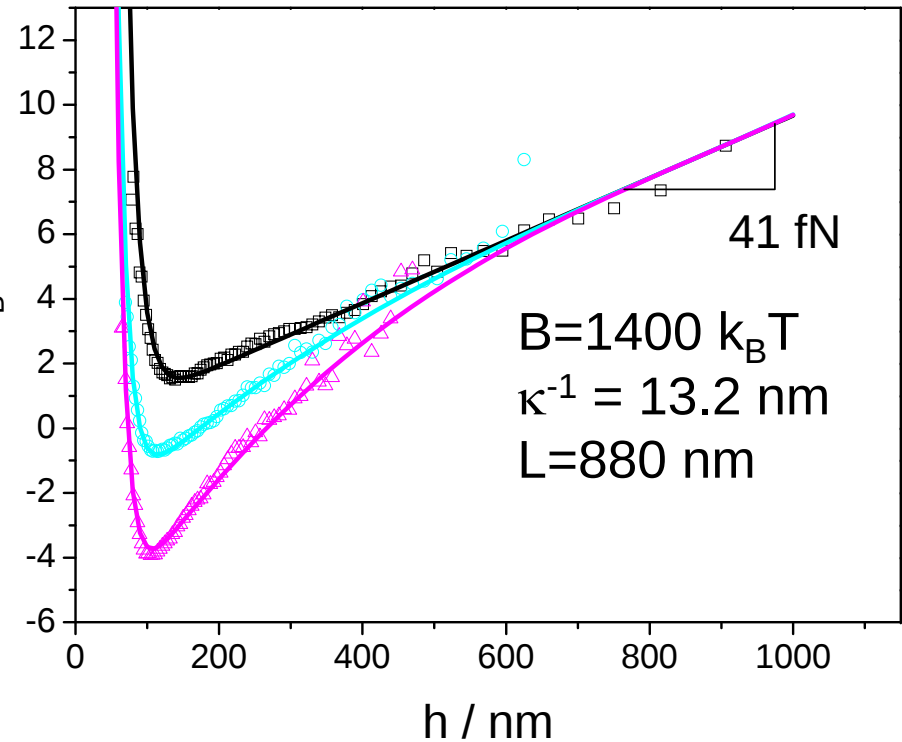


# Sphere Rod Mixtures

$R = 3 \mu\text{m}$   
 $c_{\text{TRIS}} = 1.2 \text{ mM}$   
 $\kappa^{-1} = 12.4 \text{ nm}$



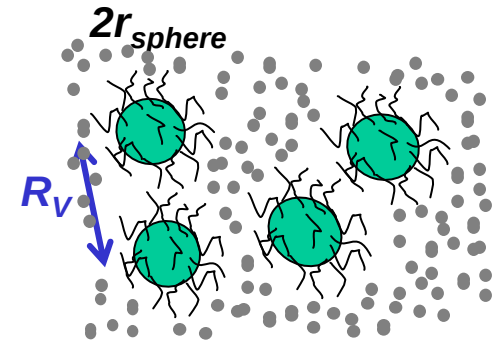
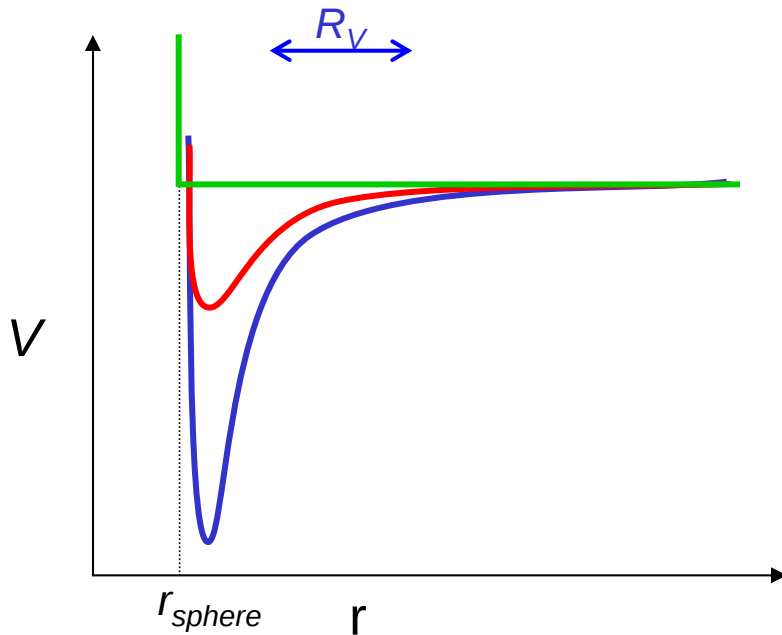
$$\frac{\Delta\phi(h)}{k_B T} = B \exp\{-\kappa h\} + \frac{G_{\text{eff}} h}{k_B T} + \frac{\phi_{\text{depl}}}{k_B T}$$



$$\frac{\phi_{\text{depl}}}{k_B T} = -\frac{\pi}{3} c \frac{N_A}{M_{fd}} R L^2 \left(1 - \frac{h}{L}\right)^3$$



**Example:** *stearyl alcohol-coated silica in cyclohexane*  
Tunable attraction, hard core repulsion

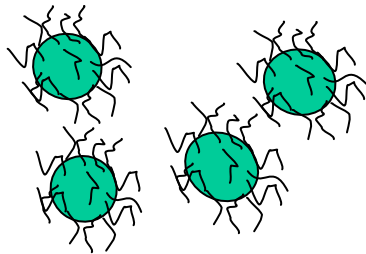
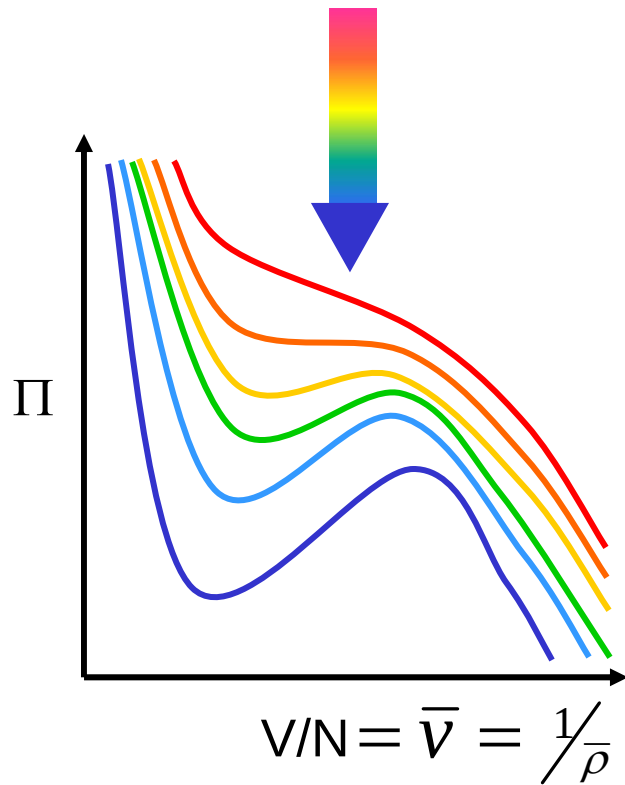


Potential of colloidal particles similar to non-ideal gas →  
van der Waals-like behavior of osmotic pressure  $\Pi$ .

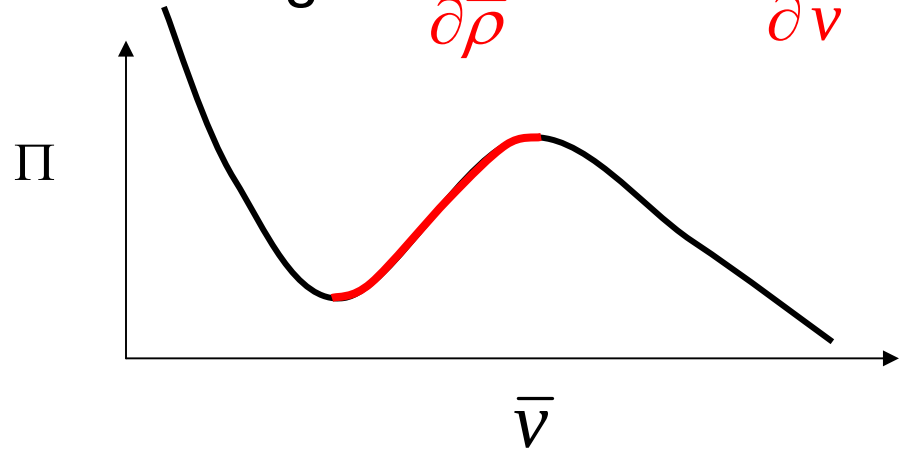


# Unstable region

Temperature quench

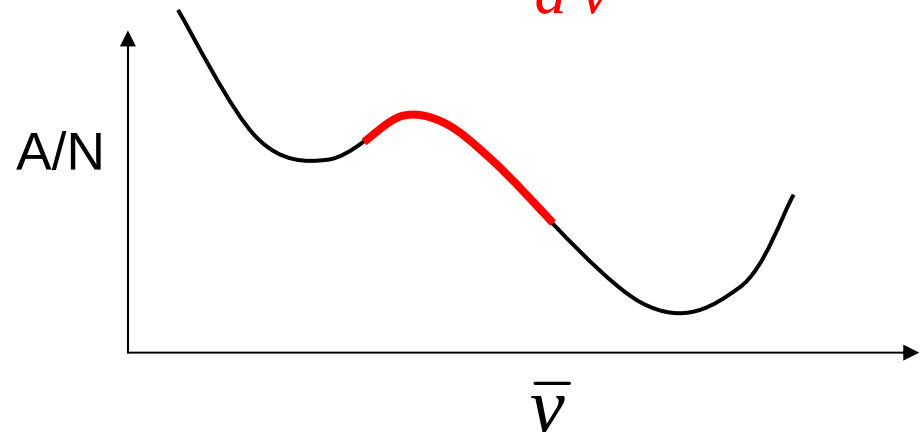


Unstable region:  $\frac{\partial \Pi}{\partial \bar{\rho}} < 0$  or  $\frac{\partial \Pi}{\partial \bar{v}} > 0$



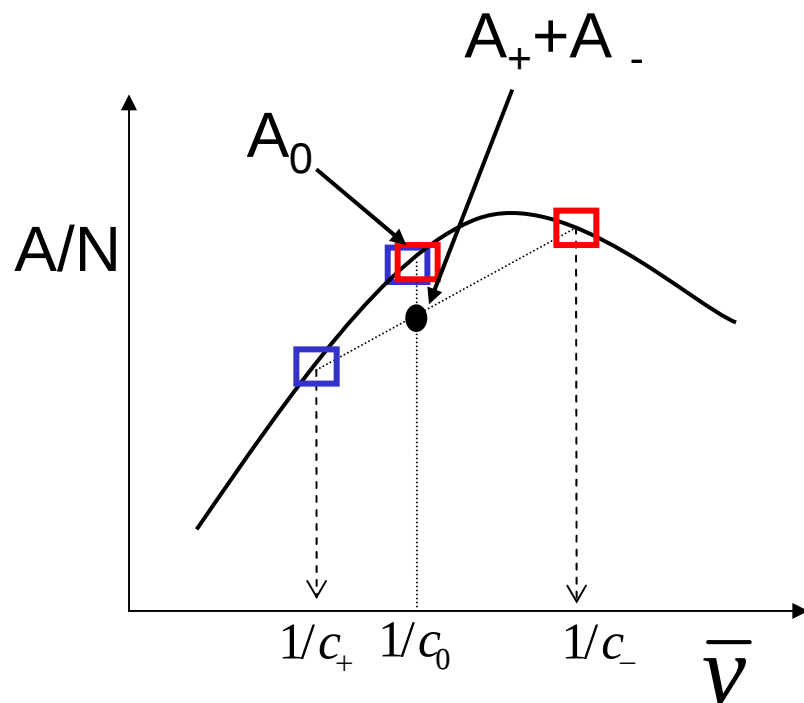
Connection to free energy:  $\Pi = -\partial A / \partial V|_N$

Unstable region:  $\frac{d^2(A/N)}{d\bar{v}^2} < 0$



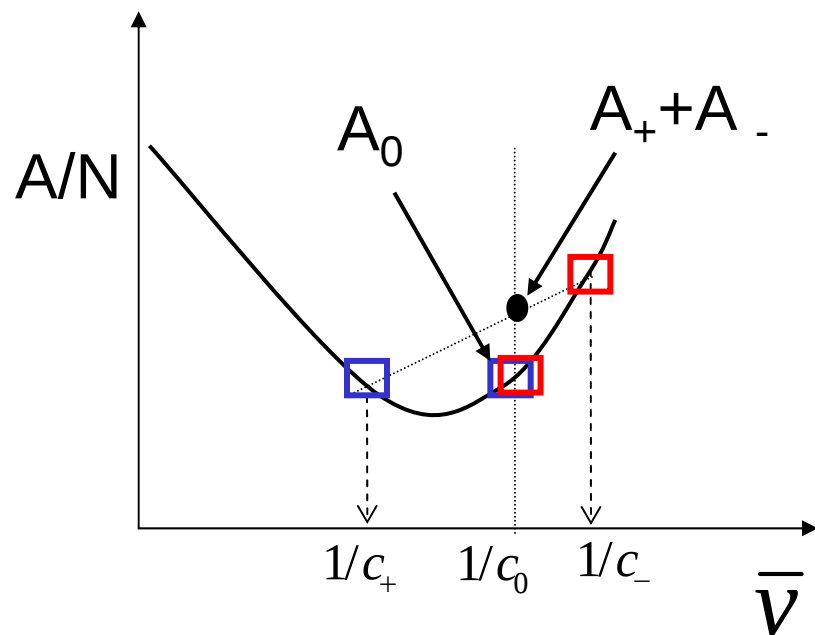


# Phase separation? Minimize Free Energy



$$A_0 > A_+ + A_-$$

**Concave,**  $\frac{d^2(A/N)}{d\bar{v}^2} < 0$ : YES

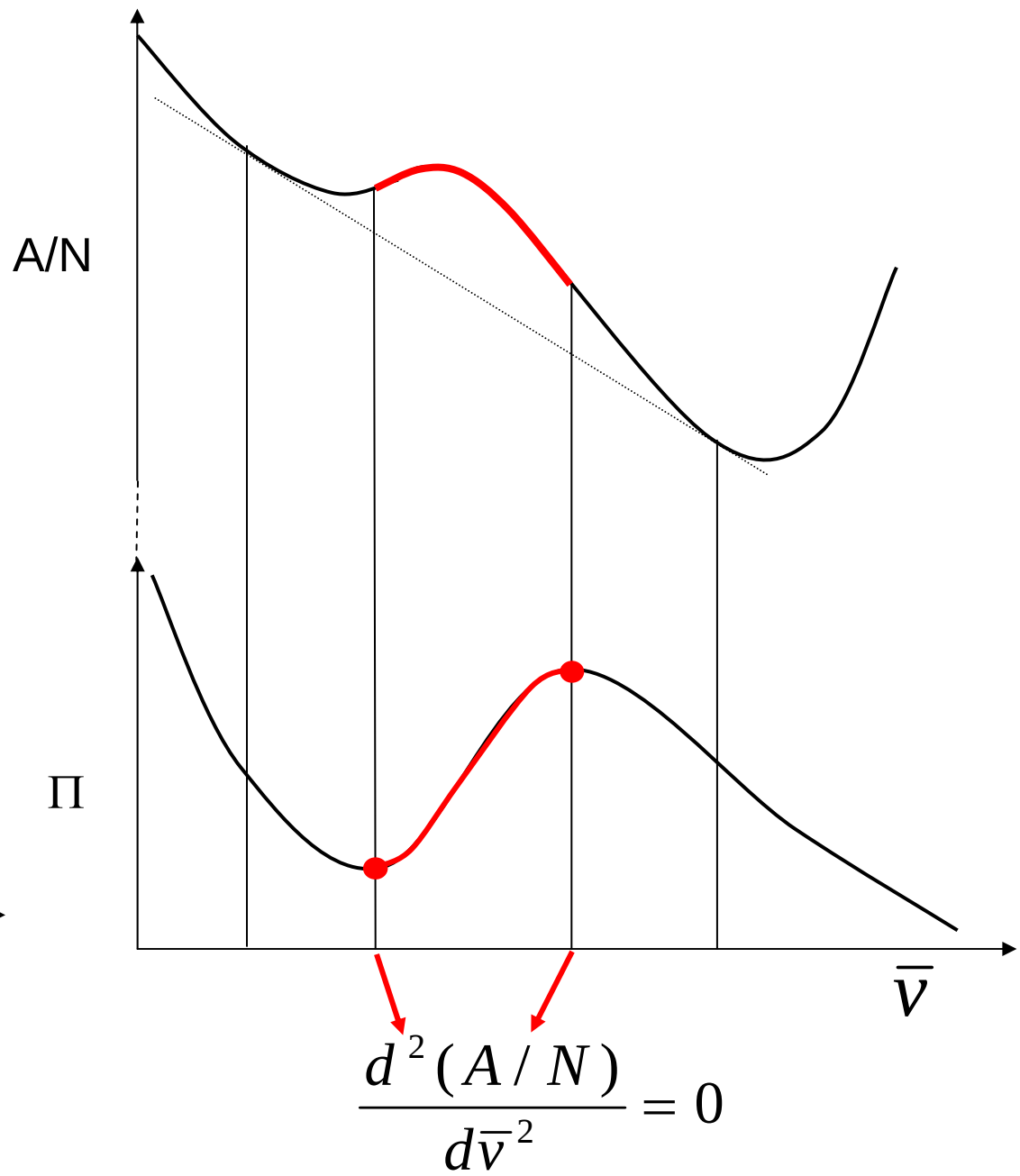
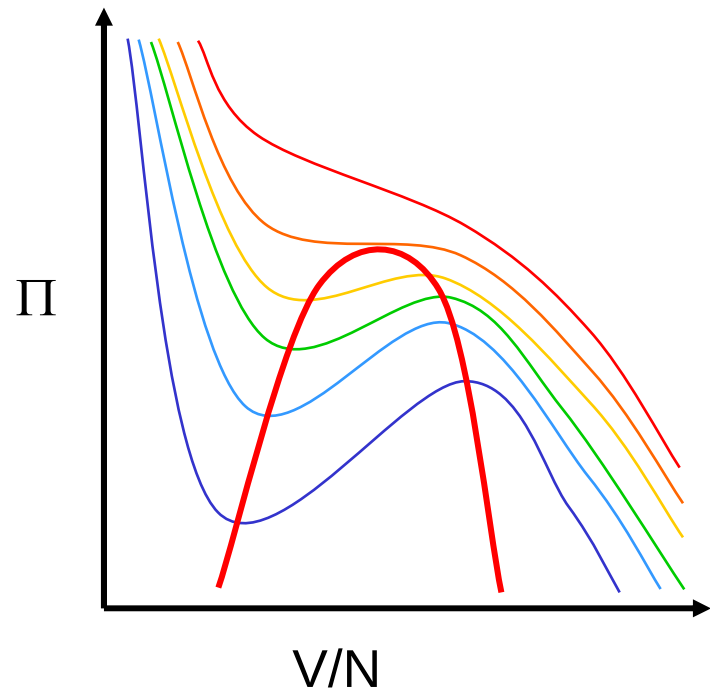


$$A_0 < A_+ + A_-$$

**Convex,**  $\frac{d^2(A/N)}{d\bar{v}^2} > 0$ : NO

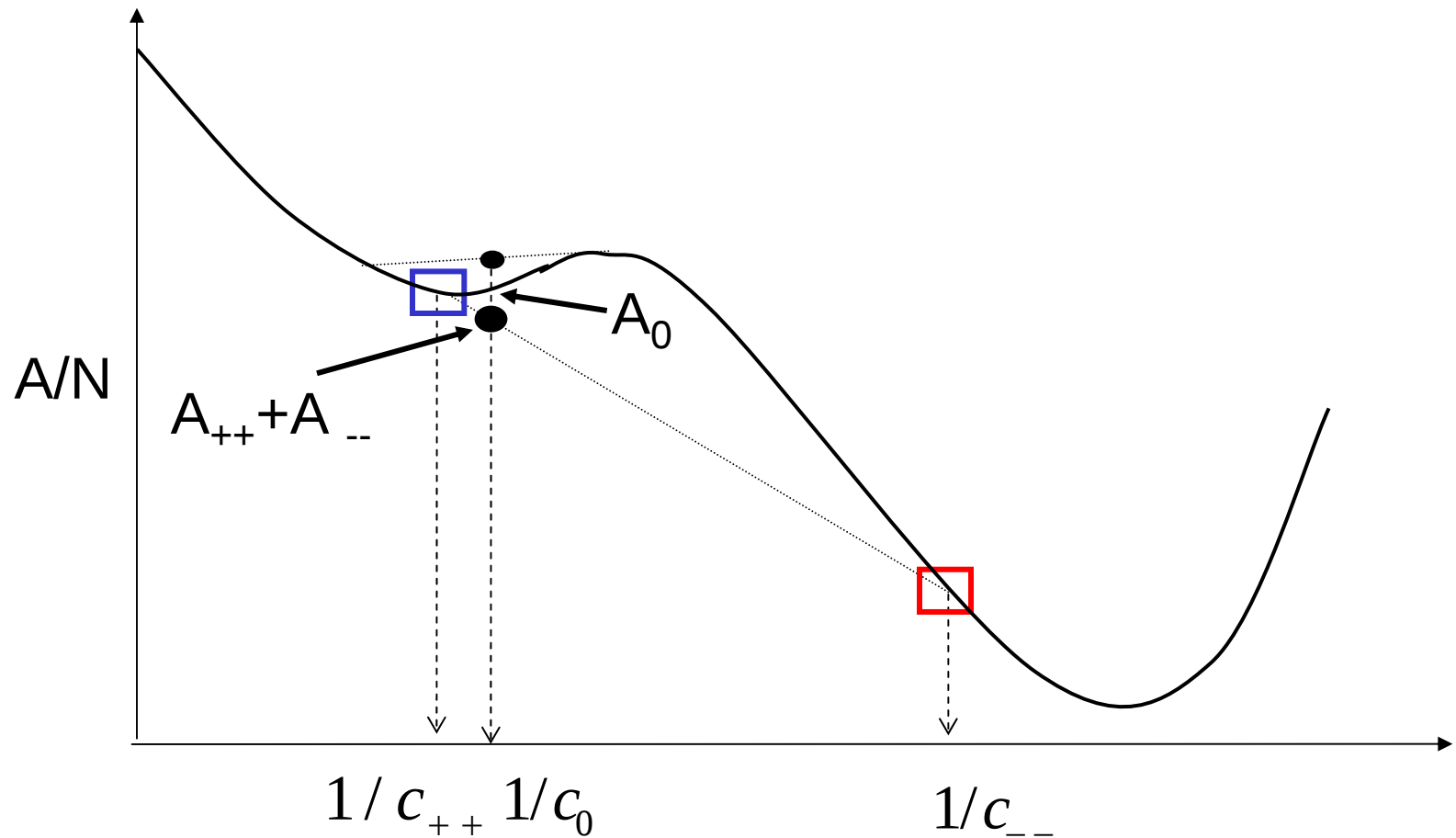


# Spinodal





## Meta-stable region



$$A_{++} + A_{--} < A_0$$

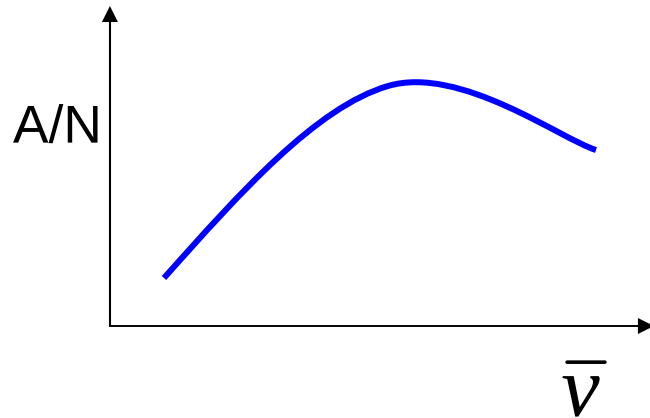
Lower free energy for big fluctuations  
*But* small probability



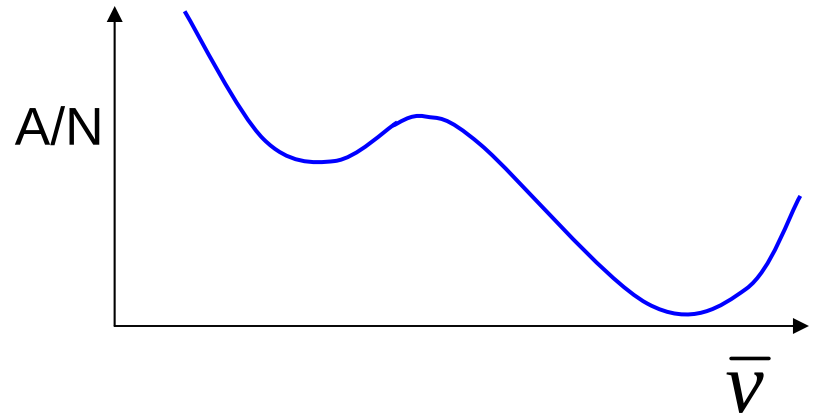
Unstable:  
Spinodal decomposition



Meta-stable:  
Nucleation and growth



- Small fluctuations grow
- Instantaneous start of phase separation
- Phase separation starts throughout the sample

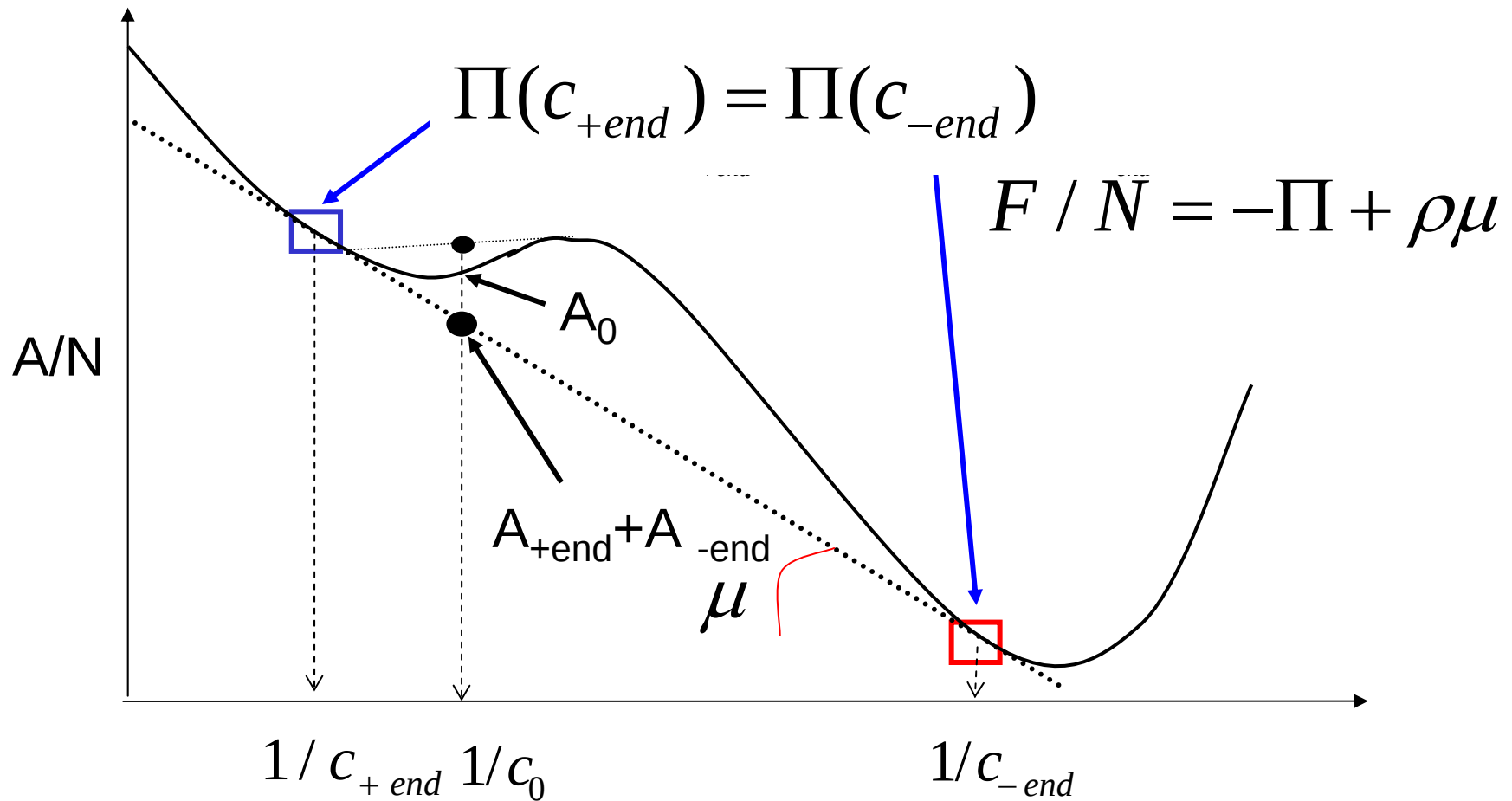


- Only big fluctuations grow
- Induction time before phase separation
- Phase separation only local

Where does the phase separation end up?



Concentrations of lowest energy: Equality of chemical potential



..... The line of minimum energy

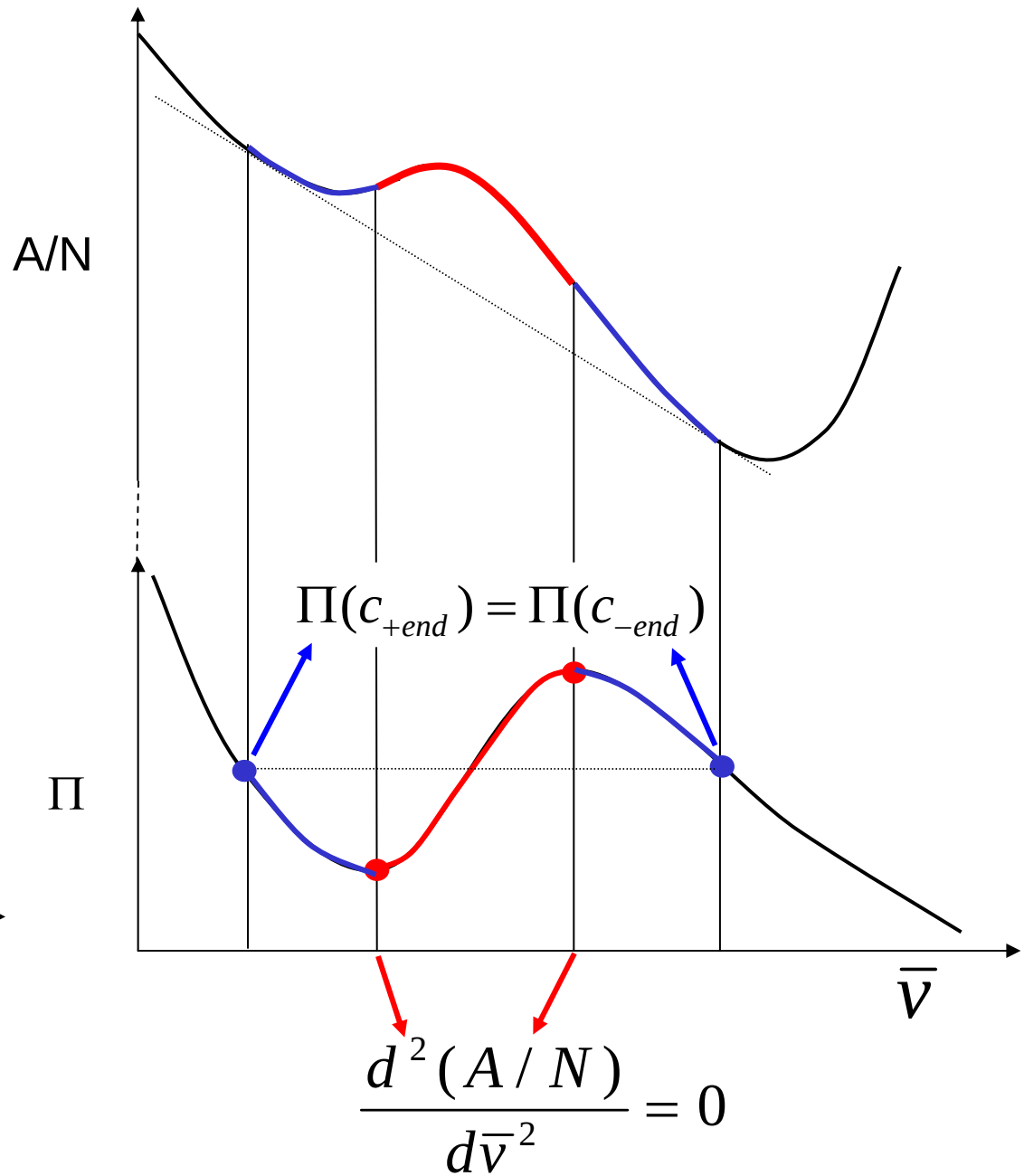
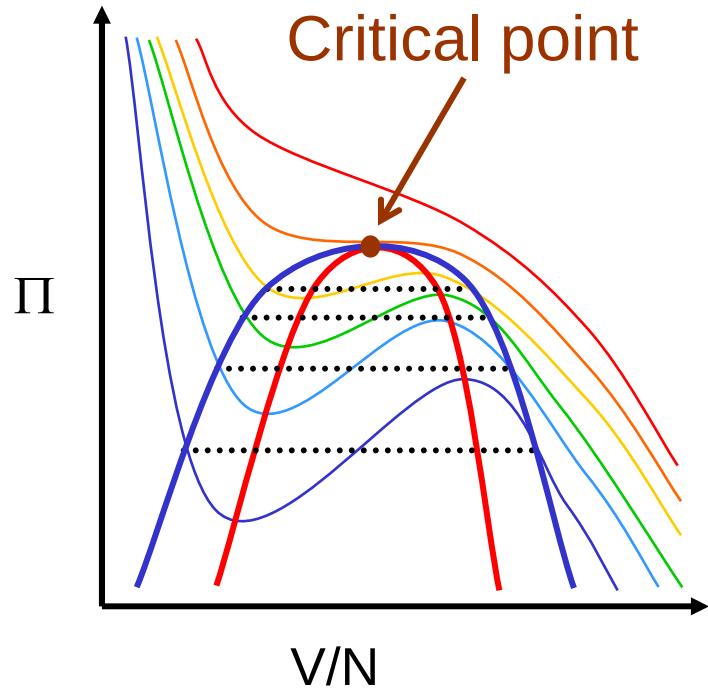
We found the binodal points  
(which depend on  $T$  or ...)



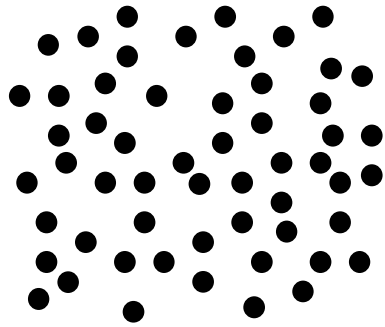
# Spinodal

and

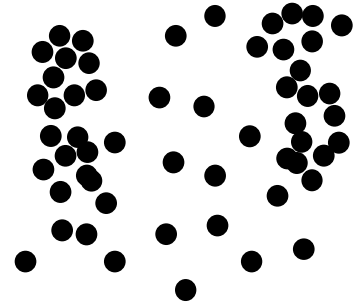
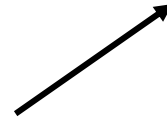
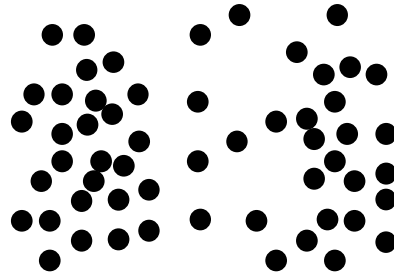
# Binodal



Attractive potential between particles

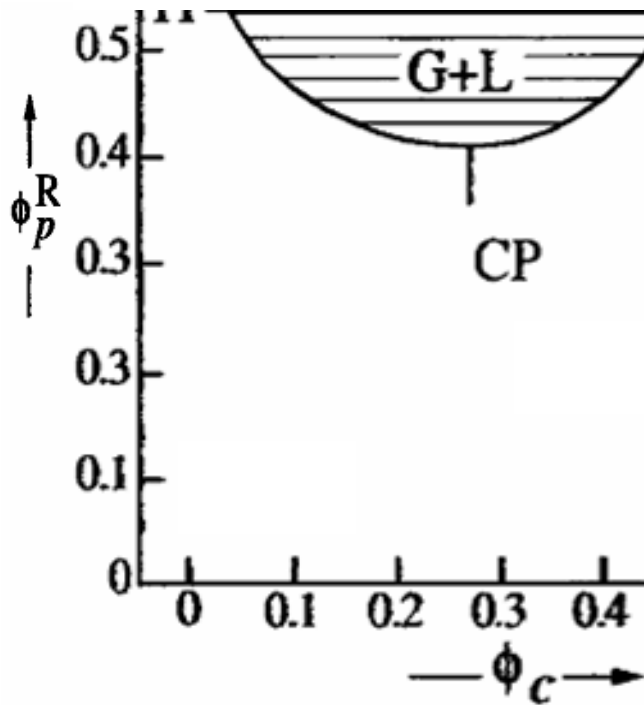


Small density fluctuation  $\delta\rho$

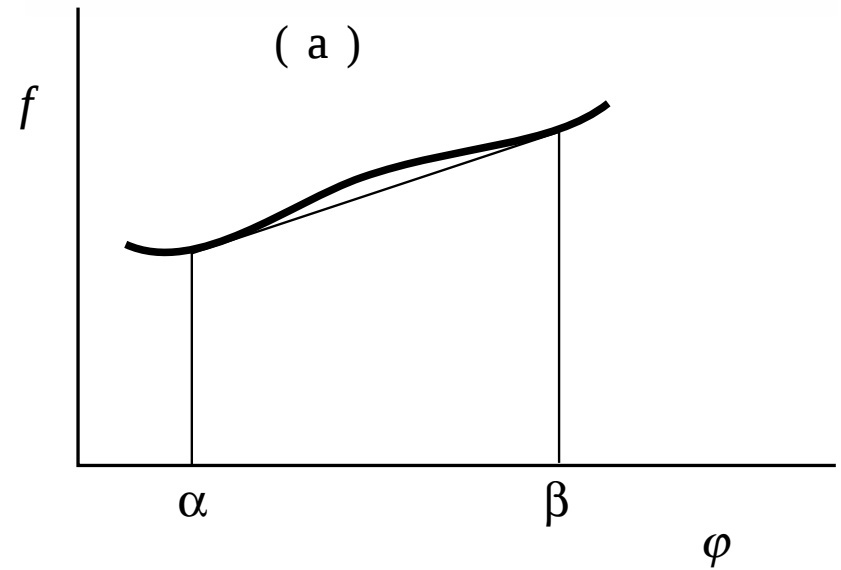
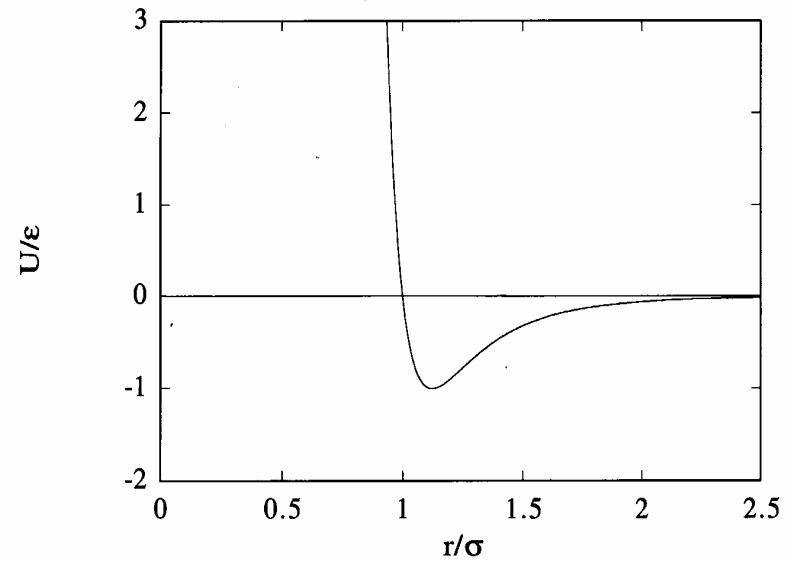


# gas-liquid

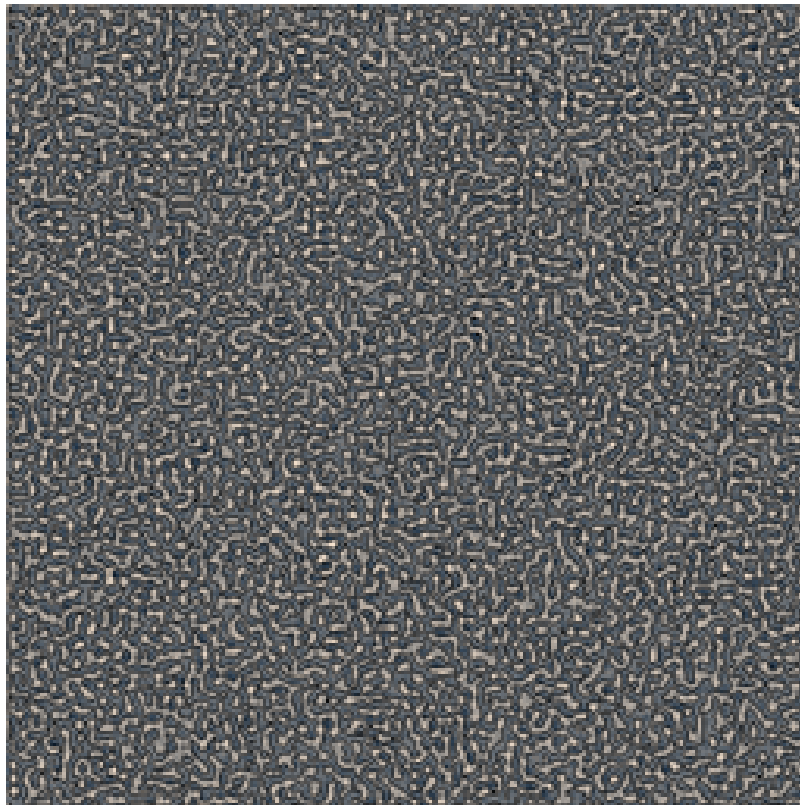
## Lennard-Jones



polymer concentration  
 $\approx$  inversed temperature!

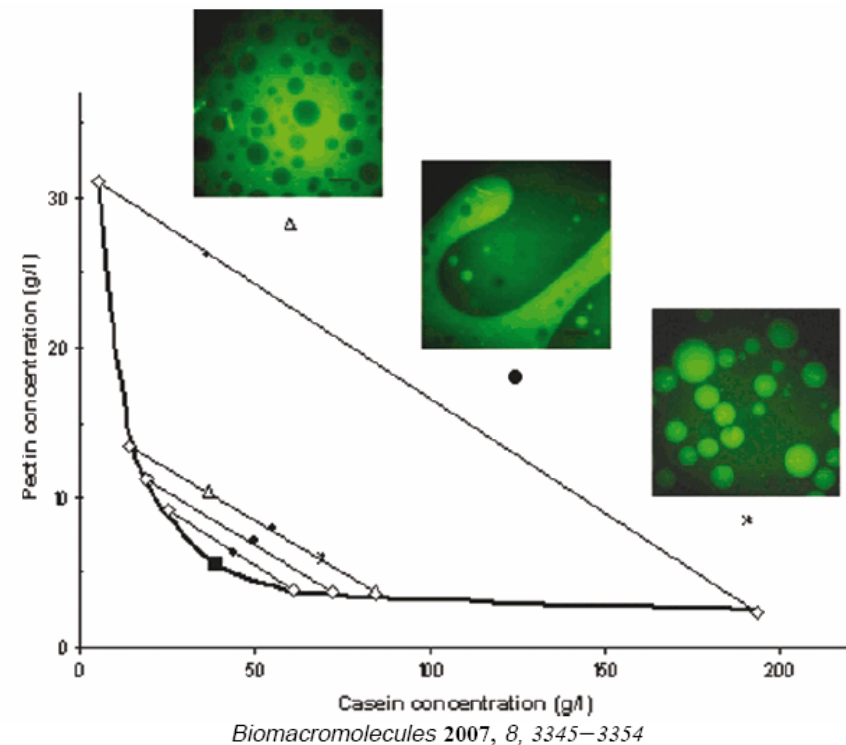


Simulation of the **spinodal**  
decomposition followed by domain  
coarsening



Cates and Wagner, Nov. 2000

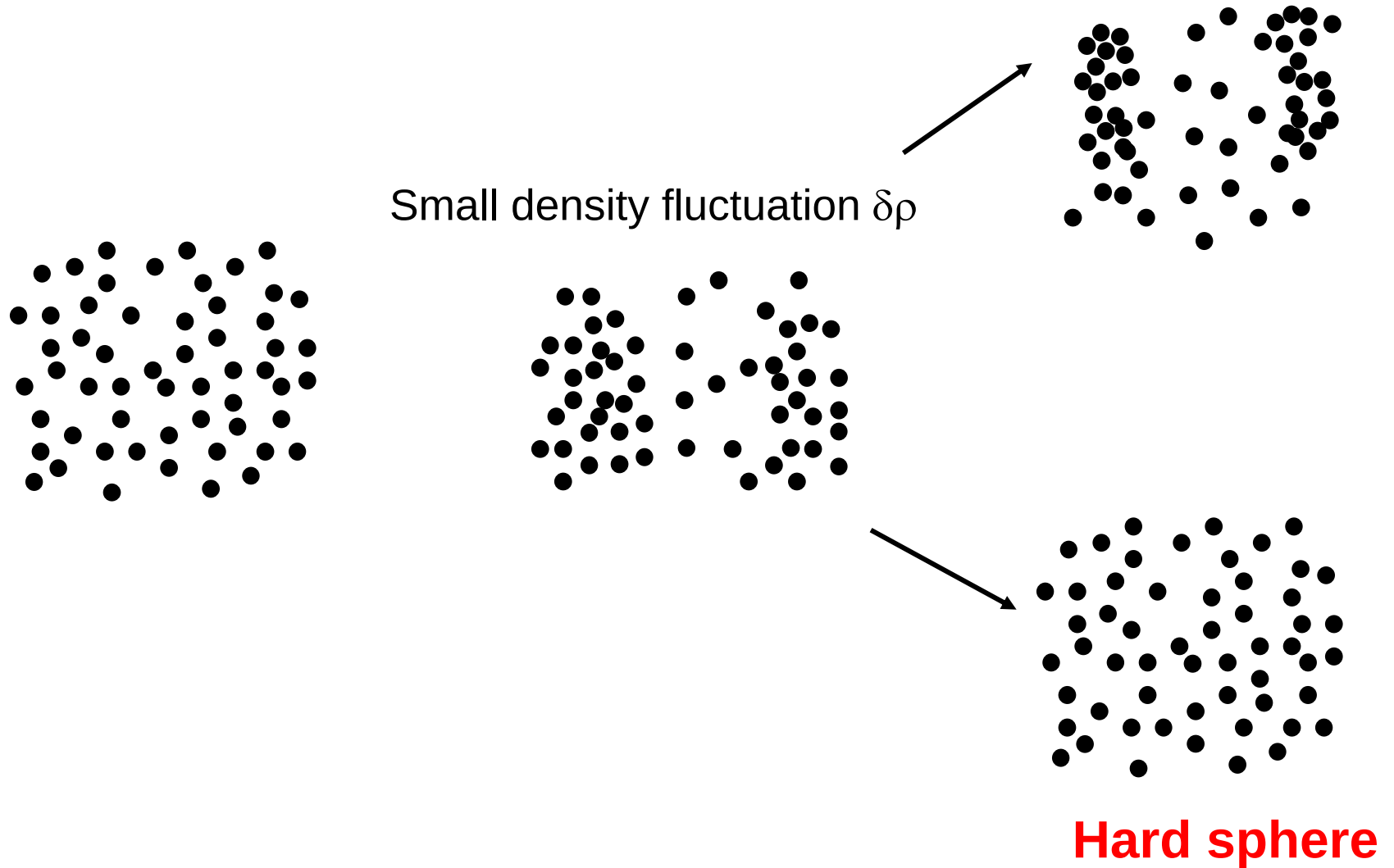
End states after demixing...



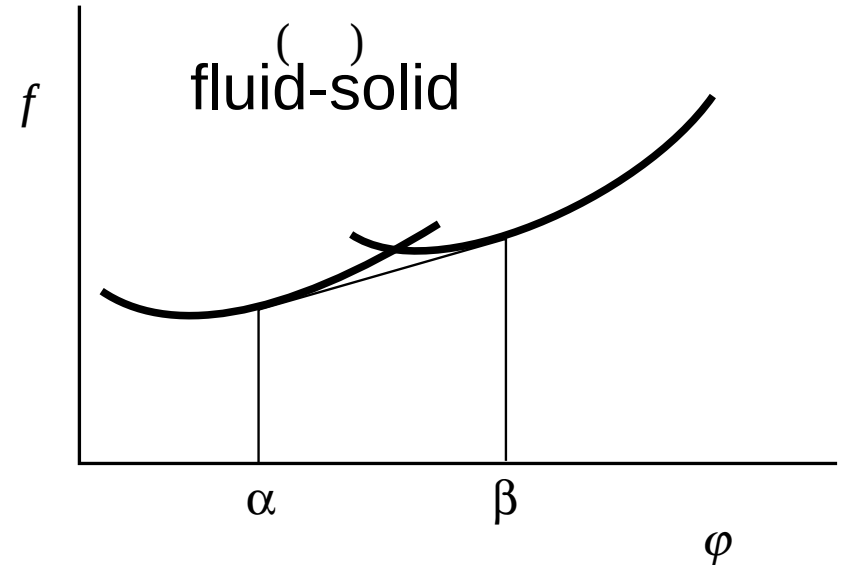
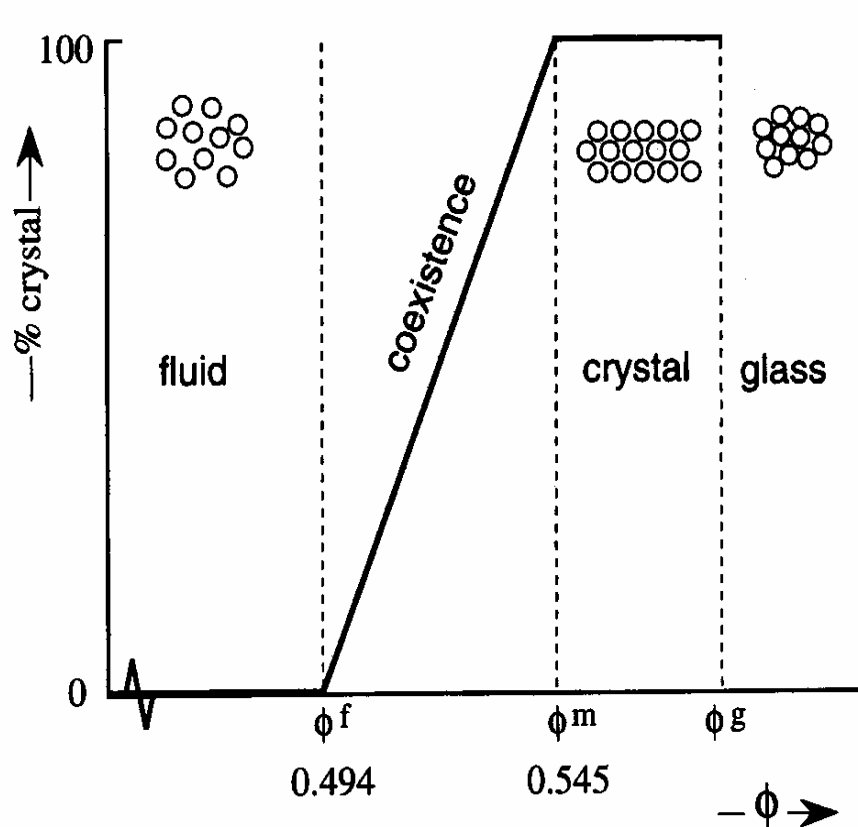
**Thermodynamic Incompatibility and Complex Formation in  
Pectin/Caseinate Mixtures**

Camila F. Redigueri,<sup>†</sup> Osvaldo de Freitas,<sup>†</sup> M. Paul Lettinga,<sup>‡</sup> and Remco Tuinier<sup>\*,‡</sup>

Attractive potential between particles



# Fluid-solid

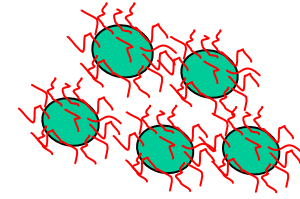


No attraction, so:

- 1) very simple phase diagram.
- 2) use equation of state for fluid and FCC crystal

Nucleation and growth

## Colloidal hard spheres

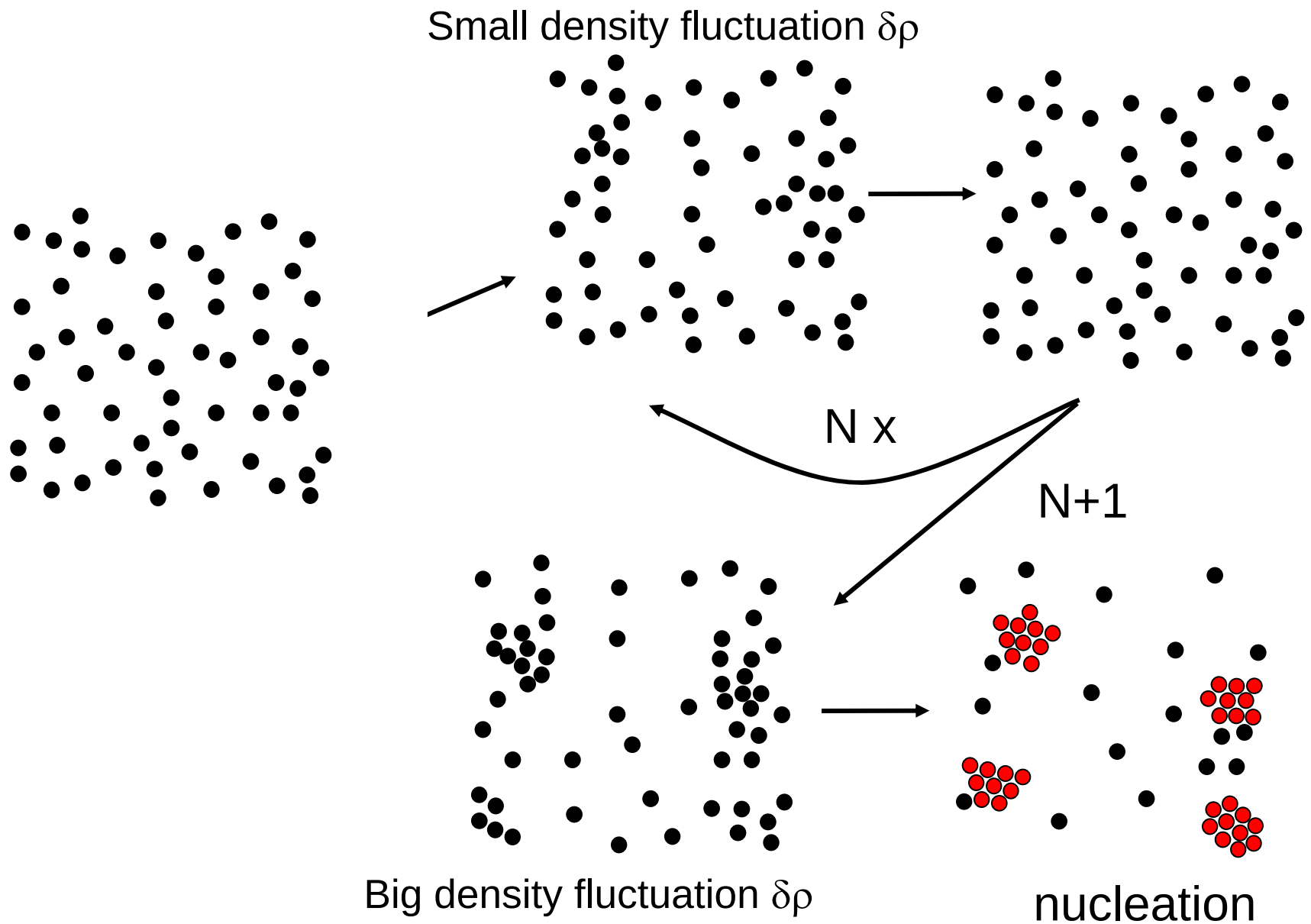


Glass (see Zorn)

heterogeneous

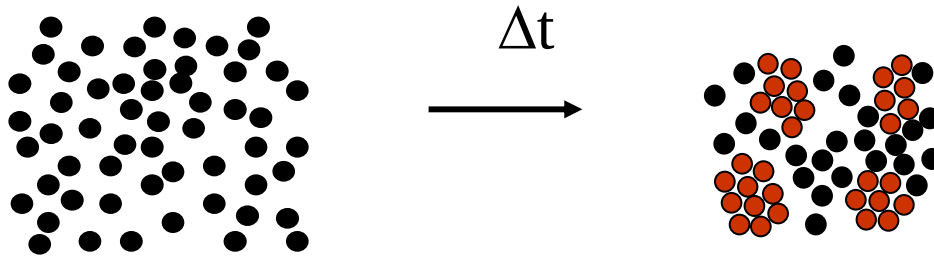
homogeneous

# Shallow quench



Very deep quench

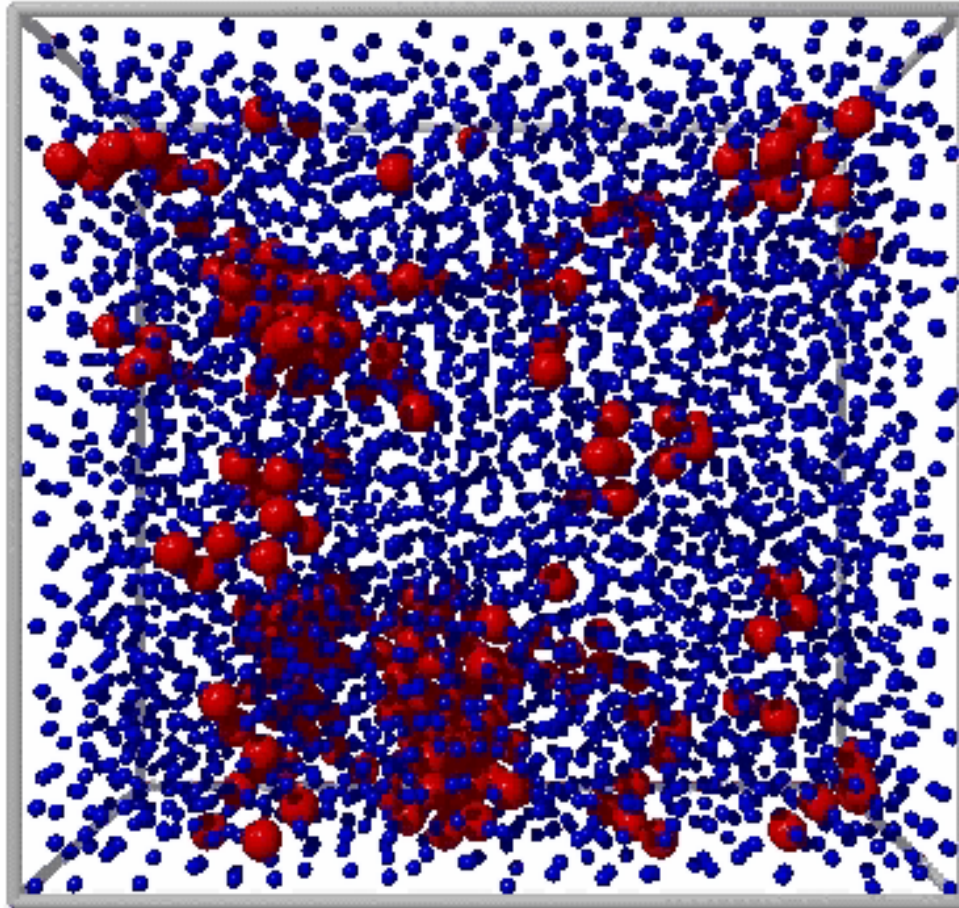
nucleation





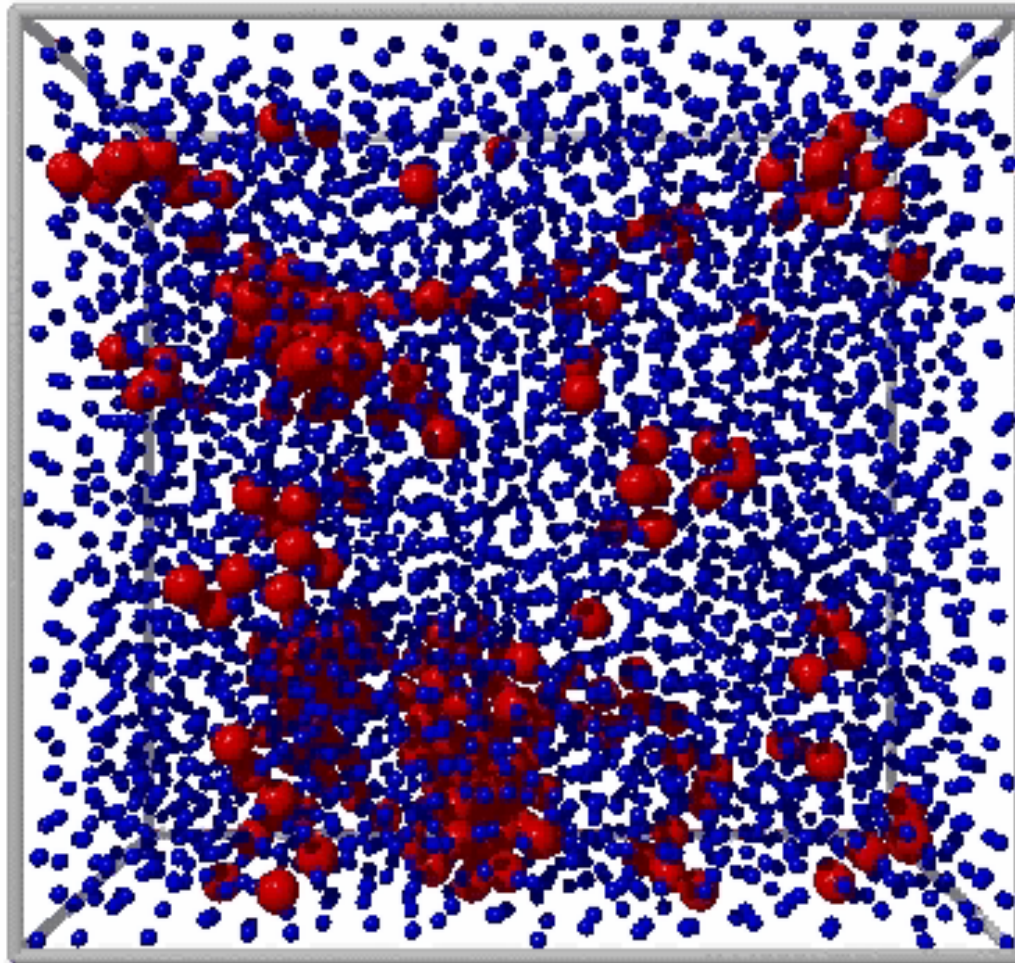
Nucleation and growth

## Concentrated hard spheres, initial stage: *nuclei dissolve*

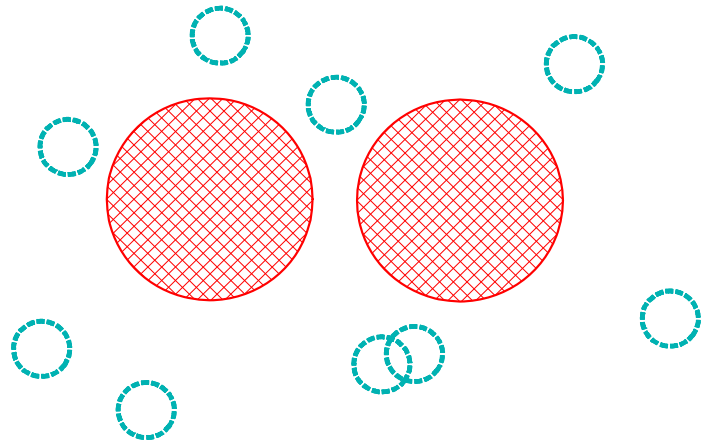


Nucleation and growth

Full picture:



# Hard spheres +freely overlapping spheres



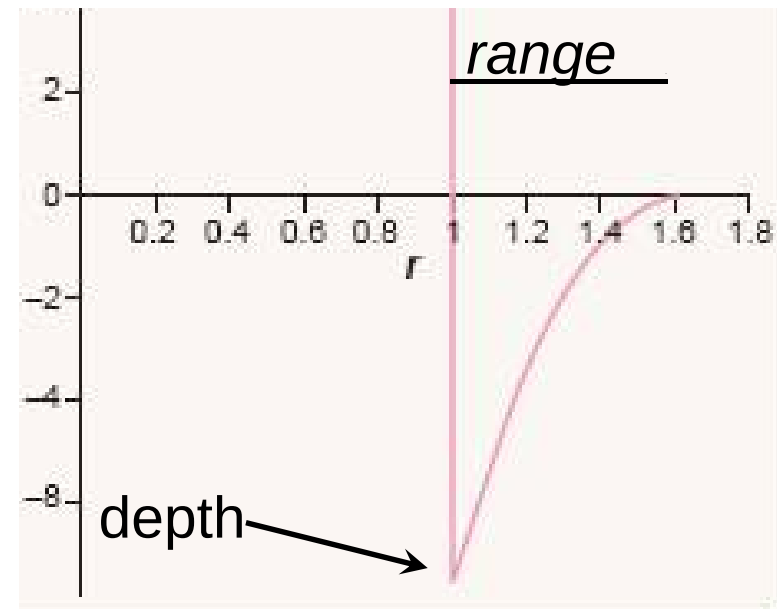
Depletion potential:

$$W_{\text{dep}}(r) = -\Pi V_{\text{overlap}}(r)$$

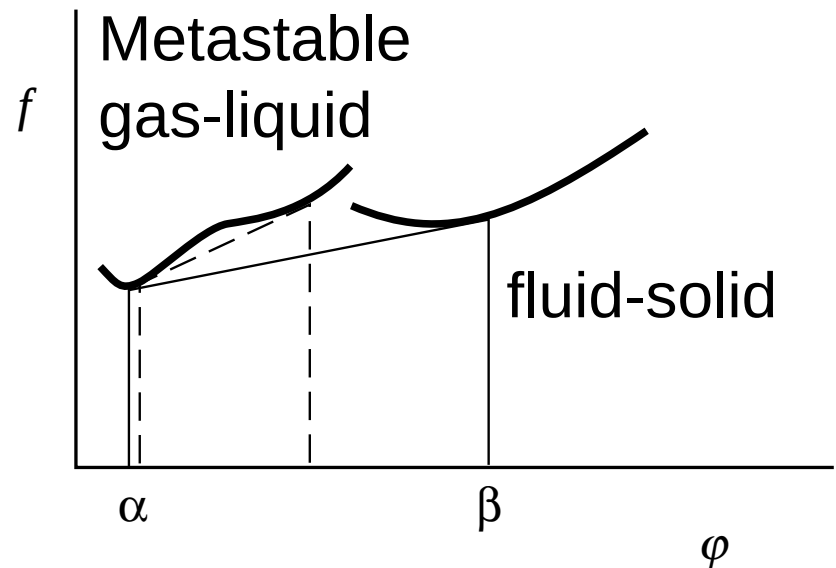
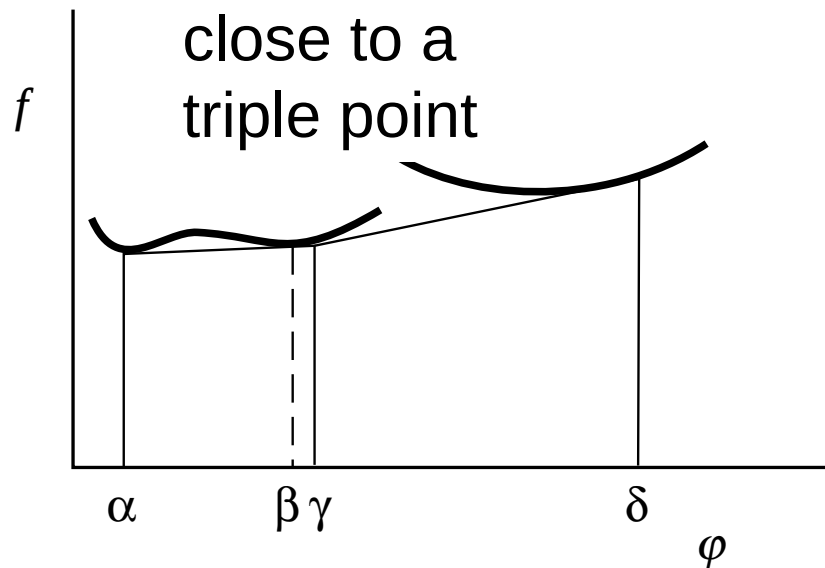
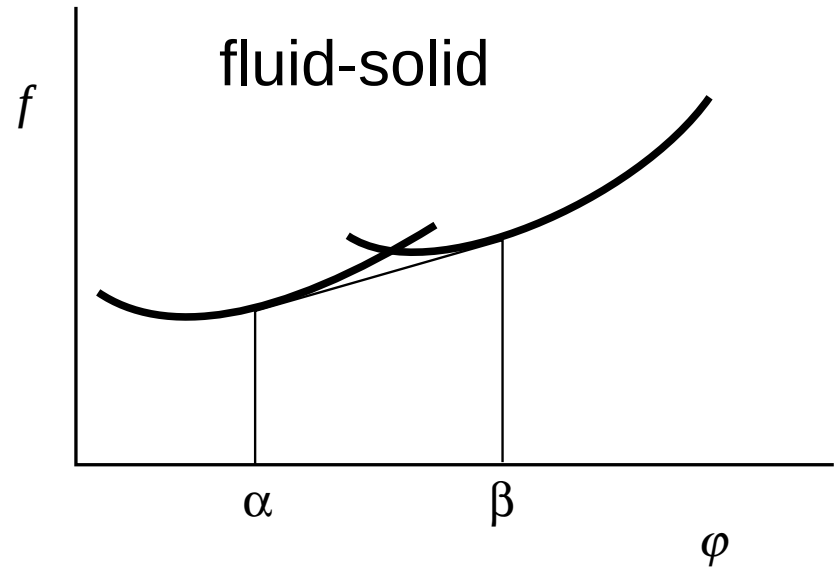
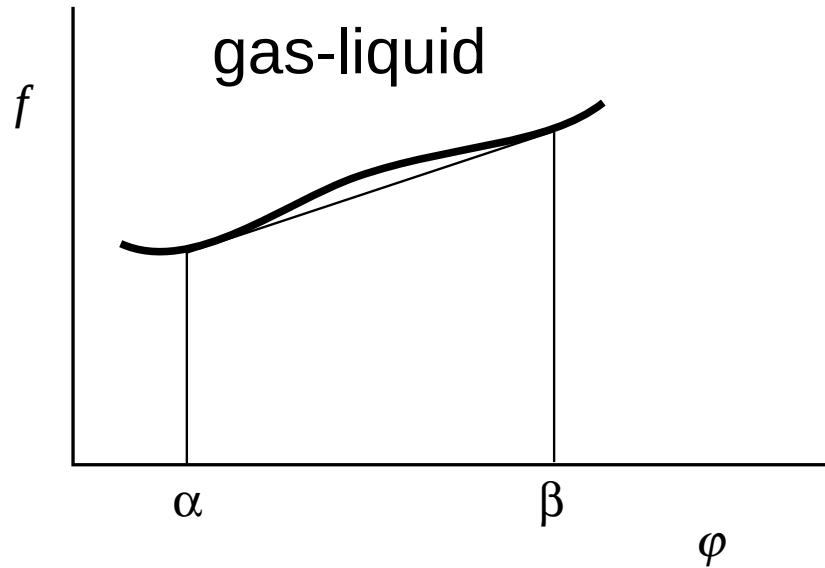
larger chains  $\rightarrow$  longer range of  $W$

more chains  $\rightarrow$  depth,  
more attractive  $W$

at  $c=c^*$ ,  $a_{\text{HS}}/a_{\text{FOS}}=5/3$ ,  $W_{\text{min}} = -2.5 \text{ kT}$

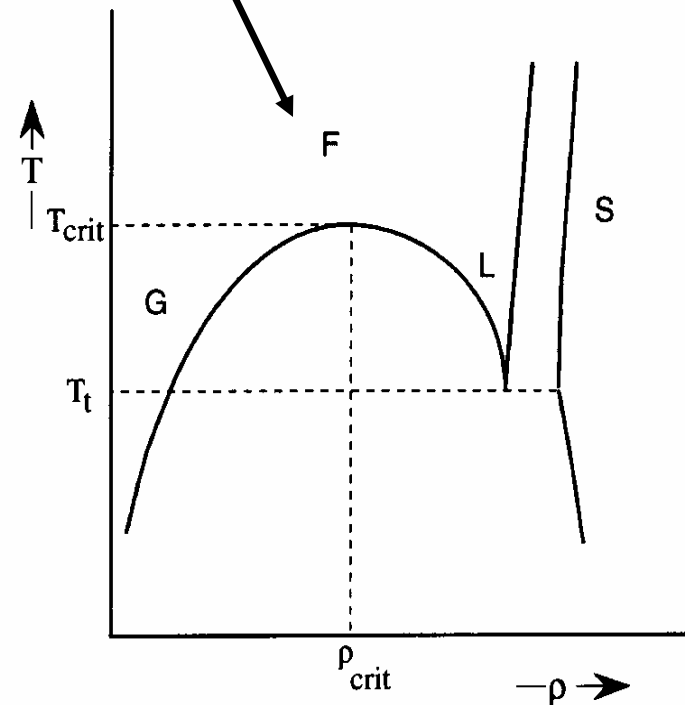
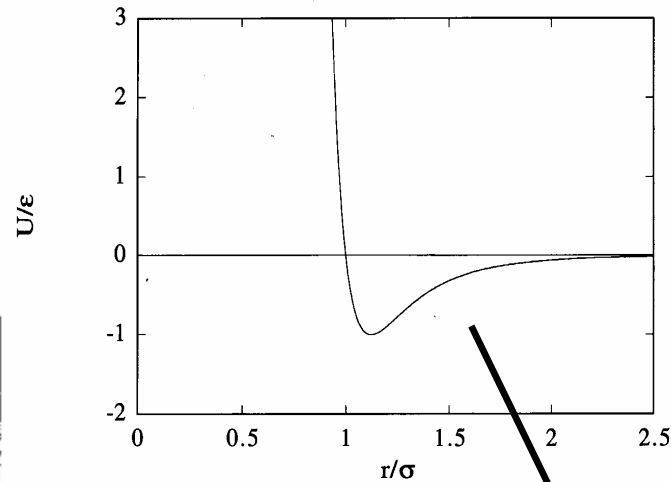
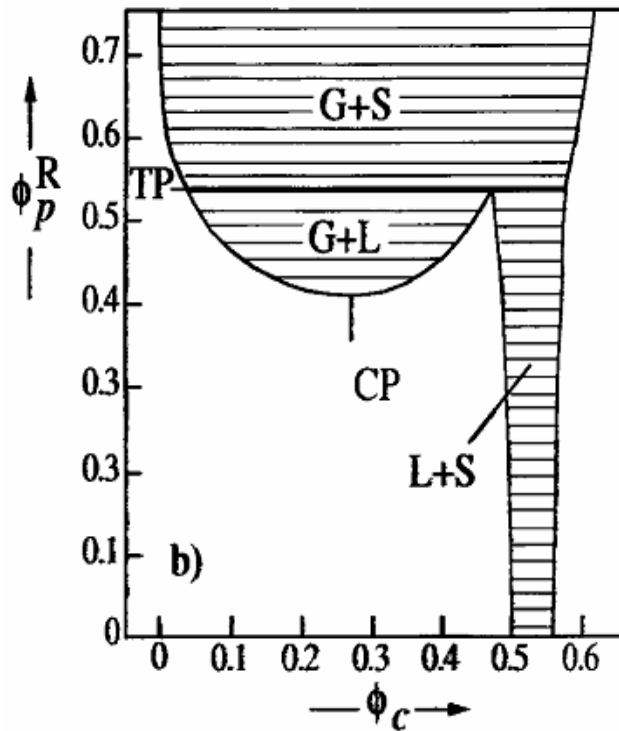


# Phase behaviour from $f$



# Compare to atomic/molecular fluids with Lennard-Jones interaction:

Lennard-Jones intera

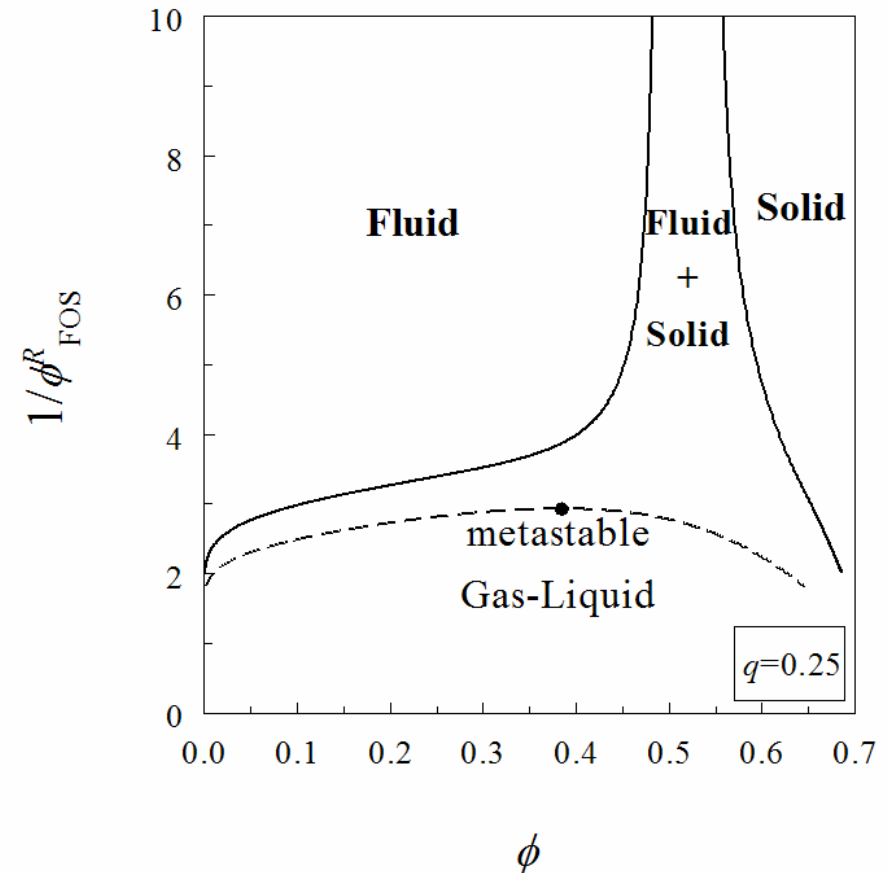
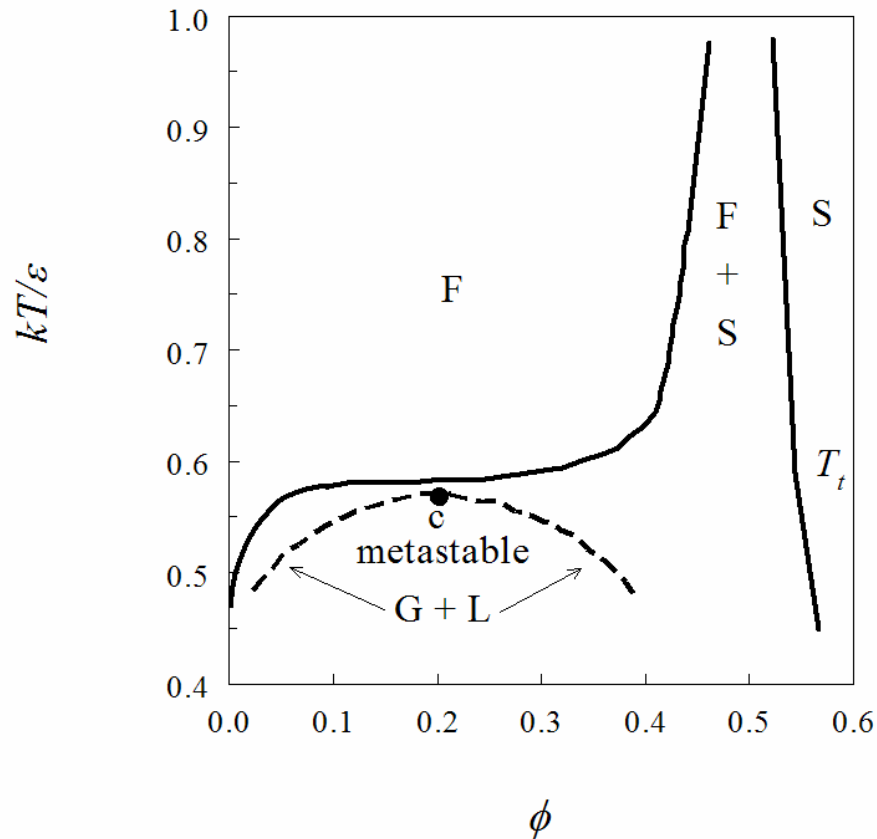


polymer concentration  
 $\approx$  inversed temperature!

# Compare to atomic/molecular fluids with Lennard-Jones interaction:

Lennard-Jones fluid  $n=12$ :  
polymer mixture:

colloid-



Is  $dV \cdot \bar{\rho} \gg 1$  true?

$$\Pi = \frac{cT}{(V - V_{red})} - \alpha(N/V)^2$$