School on Pulsed Neutrons - October 2007 - ICTP Trieste

Complementary accelerator generated probes for materials science

Synchrotron Radiation

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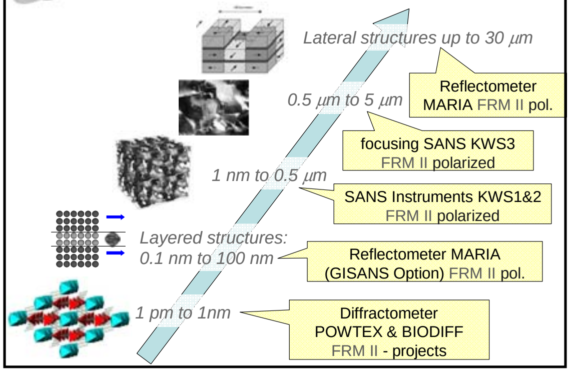
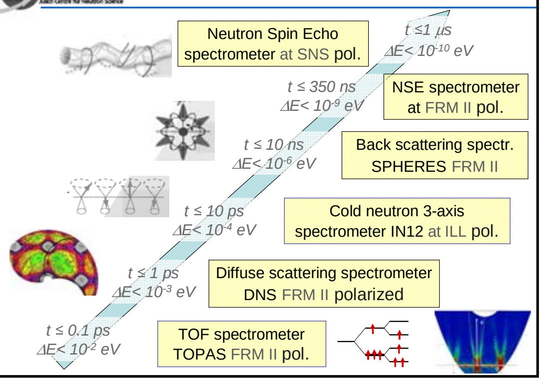
The Jülich Centre for Neutron Science

www.jcns.de

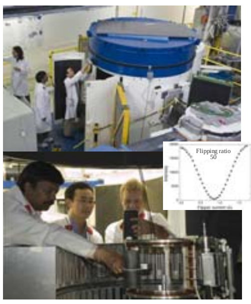
Structures: Length Scales



Dynamics: Time Domain



FRM II Outstation

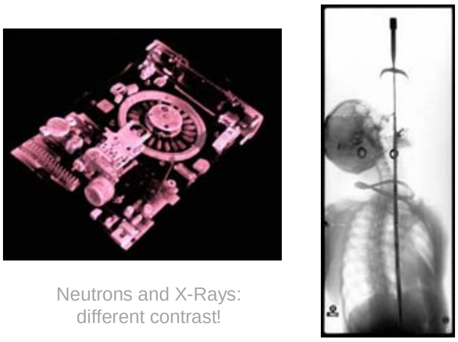


Outside and inside DNS: Polarization Analysis & TOF Cold Neutrons

Outline

- Why x-rays ?
- Laboratory x-ray sources
- Synchrotron radiation sources
- The source layout
- Special theory of relativity
- Properties of Synchrotron Radiation
- Insertion devices and free electron lasers
- Summary

X-Ray and Neutron Penetration



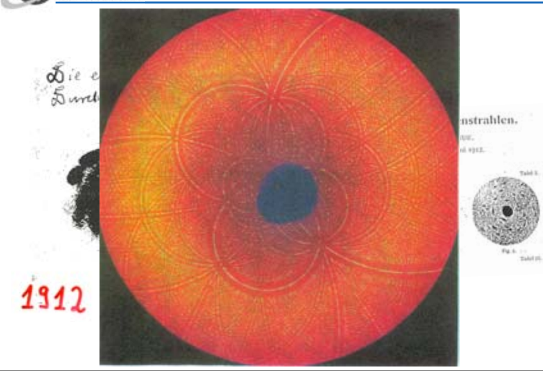
Neutrons and X-Rays:
different contrast!

X-Ray and Neutron Cross Sections

1 barn = 10^{-28}m^2

x-ray: σ_{coh} [barn] x10^{-1}	element (Z)	neutrons: σ_{coh} [barn]
0.66	$_1$H	1.76 (isotopes 1, 2)
24	$_6$C	5.55
416	$_{25}$Mn	1.75
450	$_{26}$Fe	11.22
522	$_{28}$Ni	13.30 (isotopes 58, 60, 62)
1408	$_{46}$Pd	4.39
2986	$_{67}$Ho	8.06
5631	$_{92}$U	8.90

X-Ray Scattering: Structure



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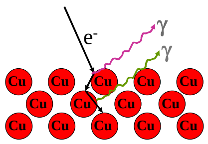
Conventional X-Ray Generators



Bombardement of target (anode) material, e.g. Cu, by high energy electrons

1. Bremsstrahlung

$E_{max} = e \cdot V$ (V: accelerating voltage)

$= \frac{hc}{\lambda_{min}}$

$\rightarrow \lambda_{min}[\AA] \approx \frac{12.4}{V[kV]}$

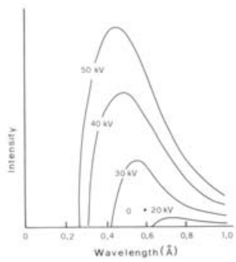
White Radiation Spectrum

Total conversion of e^- energy at λ_{min} unprobable
→ highest intensity at ≈ 1.5 × λ_{min}

Total intensity:

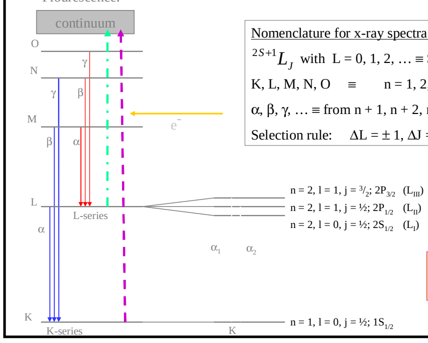
$I_{white} = A \cdot I \cdot Z \cdot V^n$

constant, current, atomic number, accelerating voltage, n ≈ 2



Characteristic Spectrum

Flourescence:



<u>Nomenclature for x-ray spectra (fluorescence):</u>

$^{2S+1}L_J$ with L = 0, 1, 2, … ≡ S, P, D, …

K, L, M, N, O ≡ n = 1, 2, 3, 4, 5, …

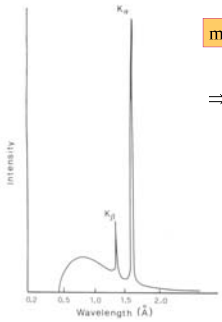
α, β, γ, … ≡ from n + 1, n + 2, n + 3, … shell

Selection rule: $\Delta L = \pm 1$, $\Delta J = 0, \pm 1$

Intensity of K-line:

$I_K = B \cdot I\,(V - V_K)^{1.5}$

Total Spectrum



Example: Cu

max (I_K / I_{white}) for $V \approx 4 \cdot V_K$

$\Rightarrow$ $I_{K\alpha} \approx 90 * I_{white}\,(\lambda_{K\alpha})$

$I_{K\alpha1} \approx 2 \cdot I_{K\alpha2}$

$I_{K\alpha} \approx 5 \cdot I_{K\beta}$

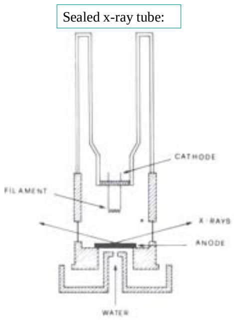
<u>Example:</u>	E [keV]	$K_{\alpha1}$	$K_{\alpha2}$	$K_{\beta1}$
	Cu	8.04778	8.02783	8.90529
	Mo	17.47934	17.3743	19.6083

Sealed-Tube and Rotating-Anode Generators

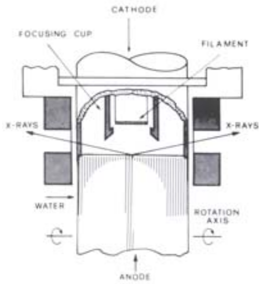
Operation at ≈ $4 \cdot V_K$

Problem: ≈ 0.1 % of power → x-rays; 99.9 %: heat!

Sealed x-ray tube:



Rotating anode:



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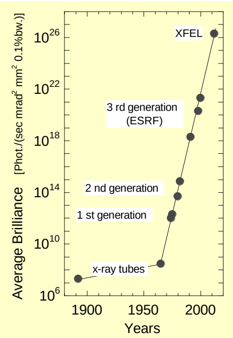
X-Ray Sources



Brilliance increases faster than Moor's Law (packing density in information electronics increases exponentially with time):

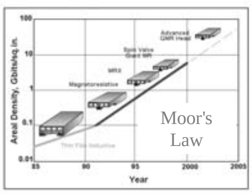


Figure 10. Magnetic head evolution

Synchrotron Radiation Sources World Wide

... their number is increasing even faster ...

LOCATION	RING (INST.)	ELECTRON ENERGY [GeV]	NOTES
ARMENIA			
Yerevan	Candle	3.2	Design/Dedicated
AUSTRALIA			
Melbourne	Australian Synchrotron (Monash Univ.)	3	Dedicated*
BRAZIL			
Campinas	LNLS-1 •	1.35	Dedicated
	LNLS-2	2	Design/Dedicated
CANADA			
Saskatoon	CLS (Canadian Light Source) •	2.9	Dedicated*
CHINA (PRC)			
Beijing	BSRF (Inst. High En. Phys.) •	1.5-2.8	Partly Dedicated
	BLS (Inst. High En. Phys.)	2.2-2.5	Design/Dedicated
Hefei	NSRL (Univ. Sci & Tech of China) •	0.8	Dedicated
Shanghai	SSRF (Inst. Nucl. Res.)	3.5	Design/Dedicated
CHINA (ROC-TAIWAN)			
Hsinchu	SRRC (Synch Rad Res Ctr.) •	1.3-1.5	Dedicated
DENMARK			
Aarhus	ASTRID (ISA) •	0.6	Partly Dedicated
	ASTRID II (ISA)	1.4	Design/Dedicated
FRANCE			
Grenoble	ESRF •	6	Dedicated
Orsay	DCI (LURE)	1.8	Dedicated
	SuperACO (LURE) •	0.8	Dedicated
	SOLEIL (LURE) •	2.5-2.75	Design/Dedicated
GERMANY			
Berlin	BESSY II •	1.7-1.9	Dedicated
Bonn	ELSA (Bonn Univ.)	1.5-3.5	Partly Dedicated
Dortmund	DELTA (Dortmund Univ.)	1.5	Dedicated/FEL Use
Hamburg	DORIS III (HASYLAB/DESY)	4.5	Dedicated
	PETRA II (HASYLAB/DESY)	7-14	Partly Dedicated
Karlsruhe	ANKA (Res. Center Karlsruhe FZK) •	2.5	Dedicated

Synchrotron Radiation Sources World Wide

INDIA			
Indore	INDUS-I (Ctr. Adv. Tech.) ●	0.45	Dedicated
	INDUS-II (Ctr. Adv. Tech.) ●	2	Dedicated*
ITALY			
Frascati	DAFNE (Frascati Nat. Lab.)	0.51	Parasitic
Trieste	ELETTRA (Synch. Trieste) ●	2-2.4	Dedicated
JAPAN			
Hiroshima	HISOR (Hiroshima Univ.)	0.7	Dedicated
Ichihara	NANO-HANA (Japan SOR Inc.)	1.5-2	Design Dedicated
Kashiwa	VSX (Univ. of Tokyo-ISSP)	1-1.6	Design Dedicated
Kusatsu	Rits SR (Ritsumeikan Univ.) ●	0.575	Dedicated
Kyoto	KSR (Kyoto University) ●	0.3	Dedicated
Nishi Harima	SPring-8 (JASRI) ●	8	Dedicated
	NewSUBARU (Himeji Inst. Tech.)	1-1.5	Dedicated
	NIJI III (Sumitomo Electric) ●	0.6	Dedicated
Okasaki	UVSOR (Inst. Mol. Science) ●	0.75	Dedicated
	UVSOR-II (Inst. Mol. Science)	1	Design Dedicated
Rokkasho	MOSLA	2	Design Dedicated
Sendai	TLS (Tohoku Univ.)	1.5	Design Dedicated
Tsukuba	TERAS (ElectroTech. Lab.) ●	0.8	Dedicated
	NIJI II (ElectroTech. Lab.) ●	0.6	Dedicated
	NIJI IV (ElectroTech. Lab.) ●	0.5	Dedicated/FEL Use
	Photon Factory (KEK) ●	2.5	Dedicated
	Accumulator Ring (KEK)	6.5	Planned rebuilding
JORDAN			
Allaan	SESAME	1	Design Dedicated
KOREA			
Pohang	Pohang Light Source ●	2	Dedicated
Seoul	CESS (Seoul Nat. Univ.)	0.1	Dedicated*

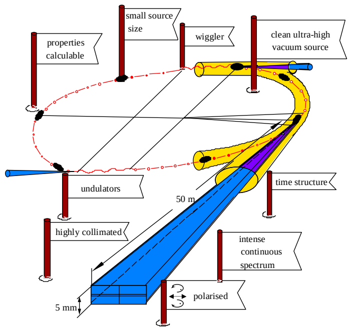
Synchrotron Radiation Sources World Wide

RUSSIA			
Dubna	DELSY	1.2	Dedicated*
Moscow	Siberia I (Kurchatov Inst.) ●	0.45	Dedicated
	Siberia II (Kurchatov Inst.) ●	2.5	Dedicated
Novosibirsk	VEPP-2M (BINP) ●	0.7	Partly Dedicated
	VEPP-3 (BINP) ●	2.2	Partly Dedicated
	VEPP-4M (BINP) ●	5-7	Partly Dedicated
	Siberia-SM (BINP)	0.8	Dedicated*
SINGAPORE			
Singapore	SSLS (National University of Singapore)	0.7	Dedicated
SPAIN			
Barcelona	LLS (Universitat Autonoma de Barcelona) ●	2.5	Dedicated*
SWEDEN			
Lund	MAX I (Univ. of Lund) ●	0.55	Dedicated
	MAX II (Univ. of Lund) ●	1.5	Dedicated
	MAX III (Univ. of Lund) ●	0.7	Dedicated*
	MAX IV (Univ. of Lund)	1.5/3	Design Dedicated
SWITZERLAND			
Villigen	SLS (Paul Scherrer Inst.) ●	2.4	Dedicated
THAILAND			
Nakhon Ratchasima	SIAM (NSRC, Suranaree Univ. of Tech.)	1	Dedicated
UK			
Daresbury	SRS (Daresbury) ●	2	Dedicated
Oxfordshire	DIAMOND (Rutherford Acc. Lab.) ●	3	Dedicated*
UKRAINE			
Kharkov	Pulse Stretcher/Synch. Rad.	0.75-2	Partly Dedicated
Kiev	ISI-800 (UNSC)	0.7-1.0	Design Dedicated

Synchrotron Radiation Sources World Wide

USA			
Argonne, IL	APS (Argonne Nat. Lab.) ●	7	Dedicated
Baton Rouge, LA	CAMD (Louisiana State Univ.) ●	1.3-1.5	Dedicated
Berkeley, CA	ALS (Lawrence Berkeley Lab.) ●	1.5-1.9	Dedicated
Durham, NC	DFELL (Duke University) ●	1-1.3	Dedicated/FEL Use
Gaithersburg, MD	SURF III (NIST) ●	0.386	Dedicated
Ithaca, NY	CESR (CHESS/Cornell Univ.)	5.5	Partly Dedicated
Stanford, CA	SPEAR2 (SSRL/SLAC) ●	3	Dedicated (Until 3/2003)
	SPEAR3 (SSRL/SLAC)	3	Dedicated*
Stoughton, WI	Aladdin (Synch. Rad. Center) ●	0.8-1	Dedicated
Upton, NY	NSLS I (Brookhaven Nat.Lab.) ●	0.8	Dedicated
	NSLS II (Brookhaven Nat. Lab.) ●	2.5-2.8	Dedicated

Synchrotron X-Ray Sources



3rd Generation X-Ray Sources

ESRF: 6 GeV

3rd Generation X-Ray Sources

APS: 7 GeV

3rd Generation X-Ray Sources

SPRING8:
8 GeV

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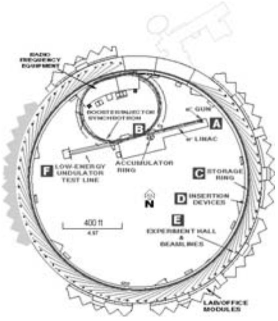
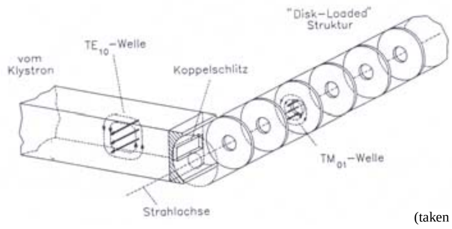
General Source Layout

Example: APS



LINAC

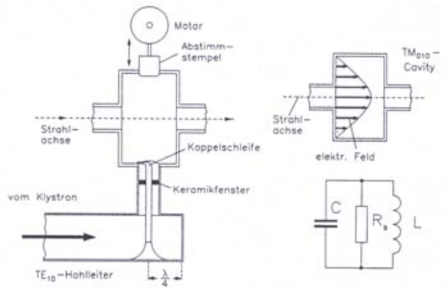
- Electrostatic acceleration:
  - limited to a few MeV by discharges
- Solution: high frequency cavities fed from clystrons
  " e^- are surf-riding on the electric HF-field (e. g. 500 MHz)"
- LINAC: Disk-loaded structures with defined wavelength



(taken from Wille)

LINAC

- Single cell cavity in storage ring:

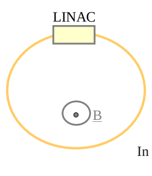


⇒ bunch structure!

(taken from Wille)

Synchrotron

Idea: Avoid long LINAC by "beam-recycling"



$$\underline{F}_{Lorentz} = e\ \underline{v} \times \underline{B} = m \frac{v^2}{R} \cdot \underline{n} = \underline{F}_{centripedal}$$

$$\Rightarrow R = \frac{mv}{e \cdot B} = \frac{vmc^2}{ec^2 \cdot B} \xrightarrow{v \to c} R = \frac{E}{ecB}$$

In practical units: $R[m] = \frac{E[GeV]}{0.3 B[T]}$

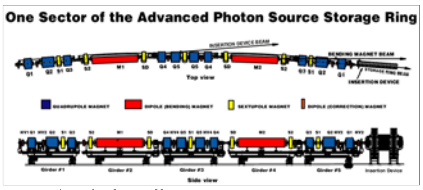
Example: APS: E = 7 GeV; B = 0.6 T → R = 39 m

Synchrotron: R fix → increase B synchronously with E

(• compare: cyclotron)

Storage Ring

= "synchrotron with constant beam energy"



Layout of one Sector at APS

Beam orbit = ideal particle track;

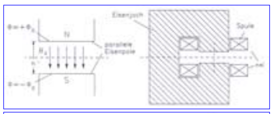
vertical field component transverse to orbits:

$B_z(x) =$	a	+	bx	+	cx^2	+	dx^3	+	...
	Dipole		Quadrupole		Sextupol		Octupole		

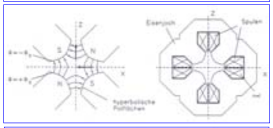
Dipole:	bending of orbit
Quadrupole:	focusing
Sextupole:	correction of chromaticity ($\Delta p/p$)

Magnets

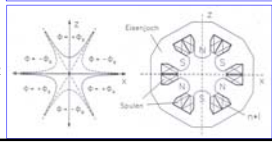
Dipole



Quadrupole



Sextupole



(taken from Wille)

Beta-Function

Orbit = idealised, stable trajectory

particle movement (fluctuation) is a

- "transverse" Δx (Betatron-oscillation)

and

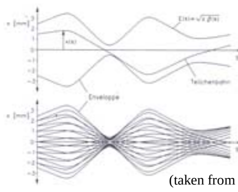
- "longitudinal" $\Delta p \rightarrow \Delta x$ (Synchrotron-oscillation)

oscillation around stable orbit:

$$x''(s) - b(s) \cdot x(s) = 0$$

Envelope = max. amplitude of all particles described by Betafunction

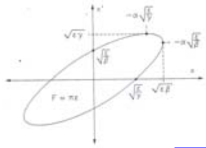
$$E(s) = \sqrt{\varepsilon \cdot \beta(s)}$$



(taken from Wille)

Emittance

All particle trajectories lie within a phase space ellipse, e. g. along x:



$$\alpha(s) = -\beta'(s)/2$$

$$\gamma(s) = \frac{1+\alpha^2(s)}{\beta(s)}$$

Liouville's Theorem: constant area $F = \pi \cdot \varepsilon$

ε = Emittance; characterises particle beam quality

third generation sources: small ε !

Significance of beta-function:

Wiggler / imaging BL:	small source size desirable	→ small β
undulator / high resol. scattering BL:	small divergence desirable	→ small β' → high β

Some Parameters (APS):

Electron Gun

LINAC:

200 MeV electron linac

$e^- \rightarrow$ W-target foil $\rightarrow e^+$

(Bremsstrahlung pair production; low efficiency!)

450 MeV positron linac

Booster-Synchrotron:

368 m circumference

650 MeV → 7 GeV

Parameters APS Storage Ring

• Vacuum pressure	≈ 10^{-9} mbar
• Beam life time	≈ 70 h
(gas scattering	≈ 80 h
Touschek	≈ 190 h)
• Filling time	≈ 1 min
• Circumference	1104 m
(Diameter	≈ 350 m)
• Revolution time	≈ 3.7 µs
(speed of light	(≈ $3 \cdot 10^8$ m / s)
	≙ 271 000 times / s)
• Nominal energy	7 GeV
• Dipole bending radius	≈ 39 m
• Dipole field	0.6 T
• Critical energy (dipole)	19.5 keV
• Nominal current (multibunch)	100 mA
• Bunch length (rms, natural):	5.3 mm
(⇒ FWHM)	≈ 35 ps
• Beam size (rms)	≈ 300 µm (H) x 90 µm (V)

Parameters APS Storage Ring

• Beam divergence (rms)	≈ 23 µrad (H) x 9 µrad (V)
• Beam emittance:	7.5 nmrad (H) x 0.75 nmrad (V)
(compare: ESRF	≈ 3 nmrad
DORIS III	≈ 415 nmrad)
• Max. insertion device length:	5.2 m
• Insertion device vacuum chamber aperture:	12 mm
• Number of sectors:	40
• Max. number of insertion device and BM beamlines:	35
• Energy loss per turn:	
bending magnet	5.45 MeV
insertion devices	1.25 MeV
total	6.9 MeV
• Source power (@7 GeV, 100 mA):	1.3 MW
• Radio frequency	352 MHz

Highly Relativistic!

Quiz

Lifetime:

which distance do the electrons / positrons cover before they collide with a gas molecule?

• Lifetime: ESRF ≈ 50h → $s=v\cdot t = 3\cdot10^{8}$m/s · 50·3600s = $5.4\cdot10^{10}$ km

Distance Earth – Sun: $1.5\cdot10^{8}$ km

⇒ *during the beam life time the electrons cover a distance of about 400 times the distance to the sun without collisions with gas molecules!*

Mass:

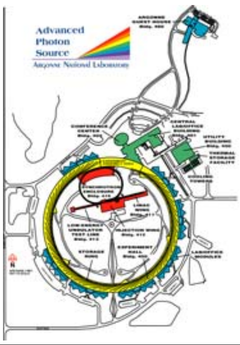
how heavy are the electrons / positrons circulating in the ring?

• e^- -mass: ESRF 6 GeV → $\gamma = 6\cdot10^{9}$eV / $511\cdot10^{3}$eV ≈ 12000 ⇒ $m=\gamma\cdot m_0$

Proton mass: $m_p=1836\cdot m_e$; ⇒ m_e(6GeV) ≈ 6.5 m_p(0GeV)

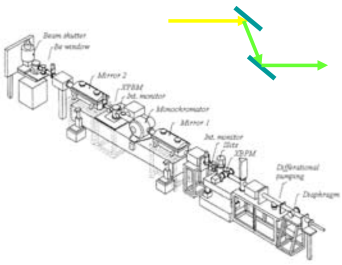
⇒ *the moving electron is as heavy as a Li atom!*

Inside the APS Storage Ring

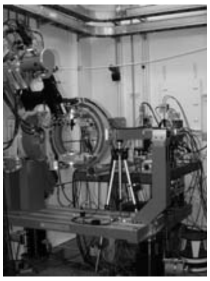


Undulator & Front End

Beamlines

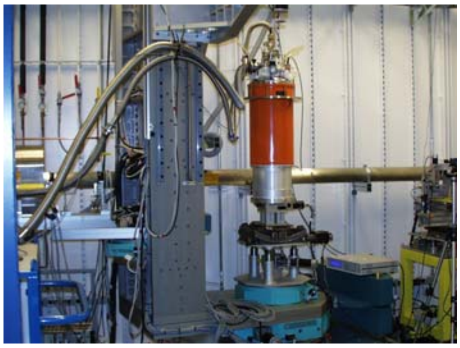


ID 20 @ ESRF



MuCAT @ APS

The Juelich Station at APS



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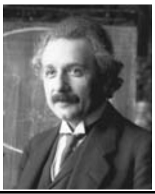
Excursion: Special Theory of Relativity

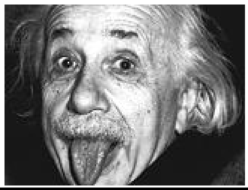
Postulates:

1. The same physical laws hold in all reference frames with uniform relative motion (There is no way to determine velocities on an absolute scale, movements are "relative").

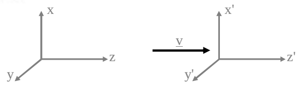
2. The vacuum speed of light has the same isotropic value c in all uniformly moving reference frames = inertial reference frames (Michelson-experiment)



Albert Einstein
1879 - 1955



Relativistic Kinematics



Galilei-transformation: $t' = t$, $\underline{r}' = \underline{r} - \underline{v}t$

contradicts postulate 2: $\dot{\underline{r}}' = \dot{\underline{r}} - \underline{v} \Rightarrow \underline{c}' = \underline{c} - \underline{v}$

Lorentz-transformation:

$$x' = x$$
$$y' = y$$
$$z' = \gamma \cdot (z + \beta c t)$$
$$t' = \gamma \cdot (t + \frac{\beta}{c} z)$$

with $\beta = \frac{v}{c}$

and $\gamma = (1 - \beta^2)^{-1/2}$

Important consequences:

Length contraction: $\Delta z' = \Delta z / \gamma$

" a moving ruler appears shorter than a stationary one"

Time dilatation: $\Delta t' = \gamma \cdot \Delta t$

" a moving clock runs slower than a stationary clock"

Relativistic Dynamics

Newtons form can be kept for the spatial components, if a speed dependent mass m is introduced (m_o = rest mass): $m = \gamma m_o = m_o / \sqrt{1 - \beta^2}$

and $\underline{p} = m \cdot \underline{v}$; $\underline{F} = \frac{d\,\underline{p}}{dt}$

$\Rightarrow$ kinetic energy of a relativistic particle:

$$E = mc^2 = \gamma m_o c^2 \quad ; \quad E^2 = p^2 c^2 + m_o{}^2 c^4$$

$\Rightarrow$ in addition to pure energy due to movement there is a constant rest energy $m_o c^2$:

$$E \xrightarrow[\beta \to 0]{} m_o c^2 + \frac{1}{2} m_o v^2$$

Relativistic Electrodynamics

This formulation of classical electrodynamics is not invariant against Lorentz-transformation!

Example:

stationary frame: static, spatially varying B-field

$\Rightarrow$ moving frame: in addition E-field (law of induction)

Form-invariant formulation of Maxwell equations via the relativistic field tensor $\underline{\underline{F}}_{\nu\mu}$, which's components are the components of $\underline{E}$ and $\underline{B}$.

special case: Lorentz-transformation along z:

$$E_1' = \gamma (E_1 - \beta B_2) \qquad B_1' = \gamma (B_1 - \beta E_2)$$
$$E_2' = \gamma (E_2 - \beta B_1) \qquad B_2' = \gamma (B_2 - \beta E_1)$$
$$E_3' = E_3 \qquad B_3' = B_3$$

Outline

- Why x-rays ?
- Laboratory x-ray sources
- Synchrotron radiation sources
- The source layout
- Special theory of relativity
- Properties of Synchrotron Radiation
- Insertion devices and free electron lasers
- Summary

Emitted Power

The Pointingvector $\underline{S} = \frac{1}{\mu_0} \underline{E} \times \underline{B}$

determines the flow of energy through a unit area per unit time at the position of the observer

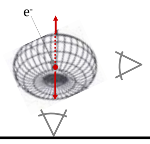
Result from classical Electrodynamics (see e.g. Jackson):

Total power emitted by particle of charge e and mass m_0: $P_s = \frac{e^2}{6\pi\varepsilon_o m_o^2 c^3} \left(\frac{d\underline{p}}{dt} \right)^2$

Galilei: emitted power independent of uniform motion; only accelerated movement "shakes off" the field!

Azimutal angular distribution: Hertz Dipole (radio-antennas):

$$\frac{dP_s}{d\Omega} = \frac{e^2}{16\pi^2 \varepsilon_o m_o^2 c^3} \left(\frac{d\underline{p}}{dt} \right)^2 \sin^2 \Psi$$



Radiation of Accelerated Relativistic Charged Particles

$$P_s = \frac{e^2}{6\pi\varepsilon_o m_o^2 c^3} \left(\frac{d\underline{p}}{dt} \right)^2$$

Classical formula not relativistic invariant: change of reference frame changes dt!

→ relativistic form-invariant generalisation:

$$dt \rightarrow d\tau = \frac{1}{\gamma} dt; \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{E}{m_o c^2}$$

$$\underline{p} \rightarrow p_\mu = \left(mv_1, mv_2, mv_3, \frac{1}{c} E \right); m = \gamma m_o; E = mc^2$$

$$\Rightarrow P_s = \frac{e^2 c}{6\pi\varepsilon_o} \frac{1}{(m_o c^2)^2} \left[\left(\frac{d\underline{p}}{d\tau} \right)^2 - \frac{1}{c^2} \left(\frac{dE}{d\tau} \right)^2 \right]$$

- same prefactor as classical formula
- However, for relativistic case emission depends on direction of acceleration and on direction of movement!

Radiation of Accelerated Relativistic Charged Particles

limiting cases:

$$P_s = \frac{e^2 c}{6\pi\varepsilon_o} \frac{1}{(m_o c^2)^2} \left[\left(\frac{d\underline{p}}{d\tau} \right)^2 - \frac{1}{c^2} \left(\frac{dE}{d\tau} \right)^2 \right]$$

- linear acceleration: E increases with p

→ partial compensation of terms

⇒ radiation losses are not relevant for LINACS

- circular acceleration: $\frac{dE}{d\tau} = 0$

→ in the rest frame of the particle, the emission is identical to the classical case

Observation in laboratory system of moving particle →

- increase of mass of inertia
- time dilatation

For circular movement: $\frac{dp}{d\tau} = \gamma \frac{dp}{dt} = \gamma \frac{p d\alpha}{dt} = \gamma p \frac{v}{R}$

$$\underset{v \approx c}{\approx} \gamma mc \frac{c}{R} = \gamma \frac{E}{R} = \frac{E}{m_o c^2} \cdot \frac{E}{R}$$

Radiation of Accelerated Relativistic Charged Particles

Emitted power for circular movement

$$\Rightarrow P_s = \frac{e^2 c}{6\pi\varepsilon_o} \frac{1}{(m_o c^2)^4} \frac{E^4}{R^2}$$

For electrons (positrons), the energy loss per turn amounts in practical units to:

$$\Delta E[keV] = \oint P_s dt = 88.5 \frac{E^4[GeV]}{R[m]}$$

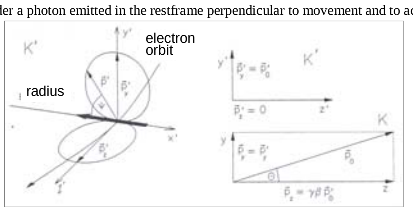
Note:

- in relativistic case: strong energy dependence of emitted power (~ E^4)
- only e^- and e^+ are effective for production of SR (compare: proton synchrotron COSY @ FZJ):

$$\frac{P_{s,e}}{P_{s,p}} = \left(\frac{m_p c^2}{m_e c^2} \right)^4 = \left(\frac{938\ MeV}{0.511\ MeV} \right)^4 \approx 1 \cdot 10^{13}$$

Width of Angular Distribution

- consider a photon emitted in the restframe perpendicular to movement and to acceleration:



in restframe:

$$\rightarrow \quad E_s = \hbar\omega_s = \hbar c k_s = c \cdot p_s \qquad p_{\mu,s} = (0, p_s, 0, \frac{1}{c} E_s)$$

Lorentztransformation into laboratory frame:

$$p_{\mu,l} = \left(0, p_s, \gamma\beta \frac{E_s}{c}, \gamma \frac{E_s}{c} \right) =: \left(p_{x,l}, p_{y,l}, p_{z,l}, \frac{E_l}{c} \right)$$

Width of Angular Distribution

⇒ consequences of optical Doppler effect:

- photon has additional momentum along direction of movement in laboratory frame; larger by factor $\gamma\beta$ compared to perpendicular component

⇒ angle of emittance in laboratory frame:

$$\tan\Theta = \frac{p_s}{\gamma\beta p_s} \xrightarrow{\beta \to 1} \Theta \approx \frac{1}{\gamma} = \frac{m_o c^2}{E}$$

⇒ opening angle of $1/\gamma$

e. g. E = 4.5 GeV; with m_0c^2 = 511 keV: γ = 8806

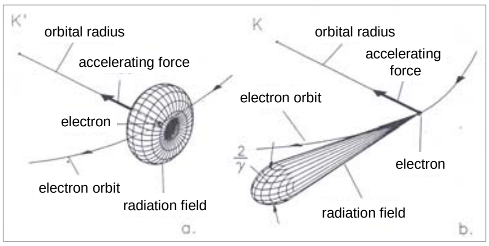
⇒ $\Theta \approx$ 0.1 mrad ≈ 0.007° ≈ 23 ''

→ in 10 m distance 1.1 mm width!

(but: convolution with e^--beam divergence!)

Width of Angular Distribution

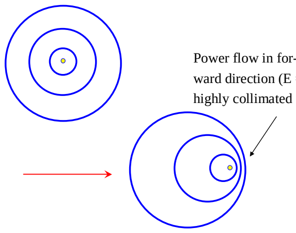
⇒ frequency shift by factor γ to higher frequencies:
propagation of light wave with c in both reference frames (in contrast to acoustic Doppler-effect), but frequency shift!



in momentary rest frame of electron — in laboratory frame

Doppler Effect:

Rest frame:
emittance for one frequency



Power flow in forward direction (E = hν) highly collimated

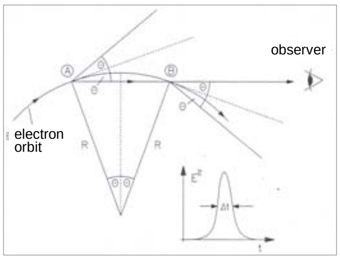
Laboratory frame:

Optical Doppler effect due to time dilatation in addition to "classical" Doppler effect: frequency change by factor γ!

Time Structure

light flash due to sharp collimation $\theta = \frac{1}{\gamma}$:

$$\Delta t = t_e - t_\gamma = \frac{2R\theta}{c\beta} - \frac{2R\sin\theta}{c} \approx \frac{4R}{3c\gamma^3}$$



⇒ typical frequency:

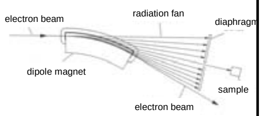
$$\omega_{typ} = \frac{2\pi}{\Delta t} = \frac{3\pi c\gamma^3}{2R} = \frac{3\pi}{2}\omega_0\gamma^3$$

⇒ line spectrum from fundamental ω_0 up to ≈ ω_{typ}. Smeared due to Betatron oscillations

Flux, Brightness and Brilliance

for radiation from a Dipole magnet (time-averaged)

Characterization of a source of radiation:



Spectral Flux — F(E) — $\left[\frac{Photons}{s \cdot mrad \cdot 0.1\% \frac{\Delta E}{E}}\right]$

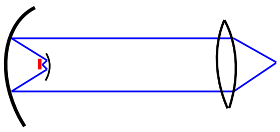
Brilliance — F(E, Ψ) — $\left[\frac{Photons}{s \cdot mrad^2 \cdot 0.1\% \frac{\Delta E}{E}}\right]$ — $F(E) = \int_{vert.distr.} F(E,\psi)d\psi$

D↔E

Brightness — F(E, Ψ, x, z) — $\left[\frac{Photons}{s \cdot mrad^2 \cdot mm^2 \cdot 0.1\% \frac{\Delta E}{E}}\right]$

Liouville's theorem

The flux at the sample position is in general determined by the Brightness, which is conserved in an optical system according to Liouville's theorem.



Spectral Distribution

see e. g. Jackson:

Universal curves, identical for all storage rings, if normalised:

Ordinate: on e^--energy and e^--current
Abszissa: on "critical wavelength"

$$\lambda_c = \frac{4\pi}{3}\frac{R}{\gamma^3}$$

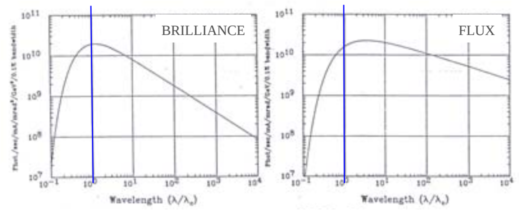
; in practical units: $\lambda_c[\mathring{A}] = 5.6\frac{R[m]}{E^3[GeV^3]}$

with E [keV] = $\frac{12.4}{\lambda[\mathring{A}]}$

$$E_c[keV] = 2.218 E^3[GeV^3]/R[m]$$

Spectrum: Universal Curves

for a bending magnet:



critical wavelength: equal emitted power on both sides of spectrum:

$$\int_0^{\lambda c} P_s(\lambda)d\lambda = \int_{\lambda c}^{\infty} P_s(\lambda)d\lambda$$

Critical Wavelength

λ_c determines, whether bending magnet radiation from a storage ring is usable for x-ray experiments or not:

hard x-ray:

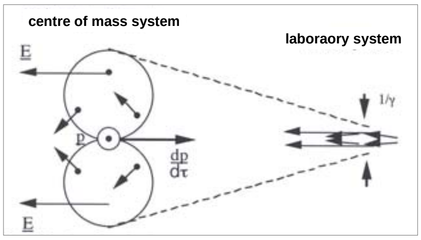
APS:	R = 39 m	E = 7 GeV	→	E_c = 19.5 keV

soft x-ray:

BESSY II:	R = 4.4 m	E = 1.7 GeV	→	E_c = 2.5 keV

Polarization

Transformation of Hertz' Dipole radiation into laboratory system:

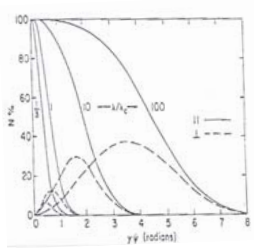


"relative compression of vertical components by factor 1/γ"

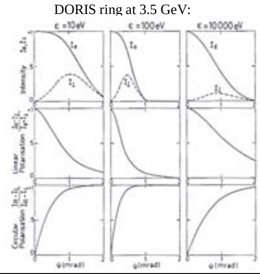
Polarization

Parallel component: Gaussian function with width σ_p: $\gamma\sigma_p = \begin{cases}(\lambda/\lambda_c)^{1/3} & \lambda \gg \lambda_c \\ 0.565(\lambda/\lambda_c)^{0.425} & \lambda \approx \lambda_c \\ (\lambda/3\lambda_c)^{1/2} & \lambda \le \lambda_c\end{cases}$

Universal functions for the vertical intensity distribution for both polarisation states:



Degree of polarisation for the DORIS ring at 3.5 GeV:



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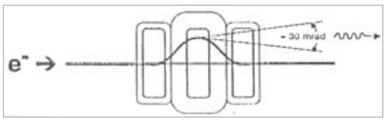
Insertion Devices

Magnet arrangement in straight section with $\int Bds = 0$ → orbit not disturbed

1. Wavelength shifter:

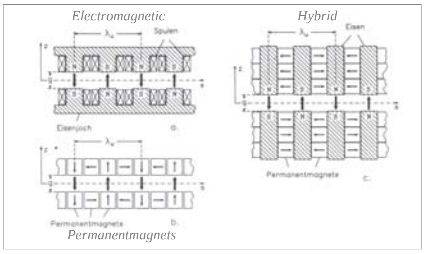
$$E_c[keV] = 0.665 E^2[GeV^2] \cdot B[T]$$



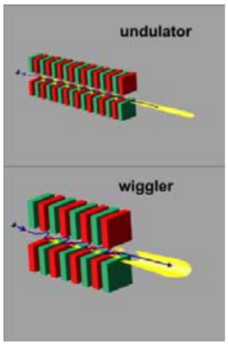
Example BESSY II:	E_c = 1.7 keV	soft x-ray range
with 7 T Wiggler:	E_c = 10 keV	hard x-rays

Wiggler & Undulators

Array of (permanent) magnets of alternating polarity in straight section:



Wiggler & Undulator



Properties of radiation determined by **"K"- or Undulatorparameter**

$$K = \alpha \cdot \gamma \approx 0.934 \cdot \lambda_0[cm] \cdot B_0[T]$$

- α: max. angle of deviation from ideal orbit
- γ: 1/θ natural opening angle
- λ_0: undulator-period (N-S-N)
- B_0: field amplitude

(α determined from equations of movement in external field)

Wiggler

K >> 1 → α >> 1/θ (typically K = 10)

→ incoherent superposition of radiation from 2 N dipole magnets

→ spectrum & polarization ≈ dipole (B_0)

$I_{Wiggler} \approx 2\,N \cdot I_{Dipole}$

→ horizontal opening angle

$2\,\alpha = 2\,K / \gamma$

Undulator

K ≈ 1 → same magnet structure as wiggler, smaller field strength by e. g. larger gap opening

→ coherent superposition of radiation from all poles for a certain wavelength

- Intensity $I \propto N^2$
- spectral width $\frac{\Delta\lambda}{\lambda} \sim \frac{1}{N}$
- angular width (diffraction limited): $\sigma = \sqrt{\lambda / L} = \sqrt{\frac{1}{N} \cdot \frac{\lambda}{\lambda_0}}$

Interference Condition:

in moving frame: period of magnet-structure is Lorentz contracted: $\lambda_0' = \lambda_0/\gamma$

→ harmonic oszillations with frequency

$$\omega' = \frac{2\pi c}{\lambda_0'}$$

in laboratory frame: frequency shift due to optical Doppler-effect:

$$\omega = \gamma \cdot \omega'$$

$$\Rightarrow \lambda = \lambda_0 / \gamma^2$$

Undulator: Interference Condition

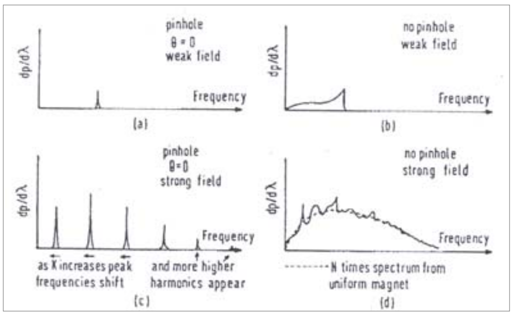
Angular dependence: $\lambda = \frac{\lambda_0}{2\gamma^2}\left(1 + K^2/2 + \gamma^2\theta^2\right)$

→

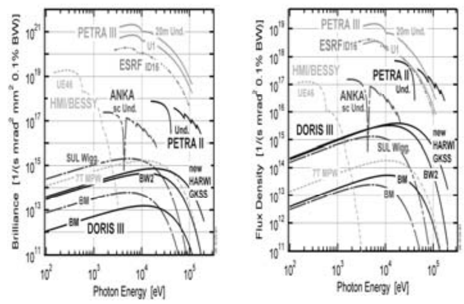
- monochromatic radiation in foreward direction (pinhole)
- tunable via gap size → B_0 → K
  (note: larger gap → smaller field → shorter wavelength or higher energy**!**)
- spectral „tail“ to longer wavelength for finite slit
- stronger field → longitudinal oscillations (e^- performs movement „ 8" in reference frame moving on orbit) → higher harmonics
- on axis: only odd harmonics (1, 3, 5, ...)
  off axis: also even harmonics (2, 4, ...)

Undulator Spectrum

$$\lambda = \frac{\lambda_0}{2\gamma^2}\left(1+K^2/2+\gamma^2\theta^2\right)$$

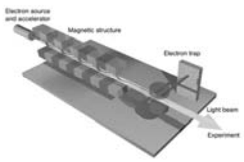


BM-, Wiggler-, Undulator- Spectra

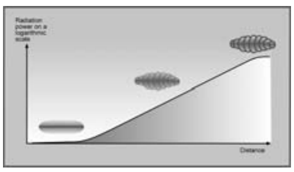


XFEL: X-Ray Free Electron Laser

schematics:



SASE-Principle:

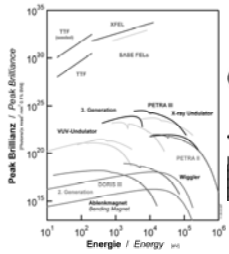


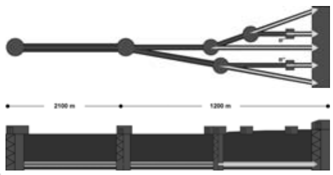
→interaction e-m-field ↔ e^--bunches

⇒ microbunching ⇒ coherent radiation from e^- within microbunch ~ n_{el}^2 !!!

(compare undulator: ~N_{pole}^2 ; only radiation between poles is coherent, not between electrons)

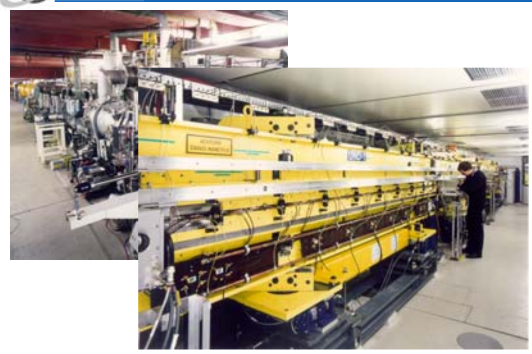
TESLA XFEL





Realisation

Realisation



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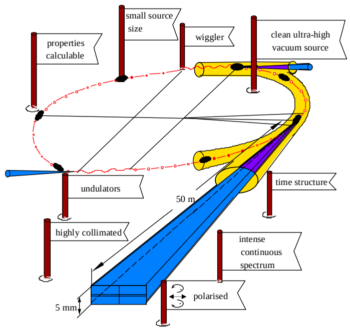
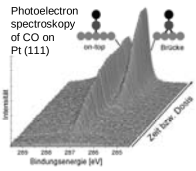
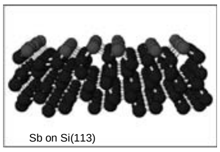
Synchrotron X-Ray Sources



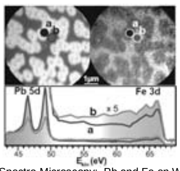
Synchrotron Radiation Applications

Surfaces:
- chemical reactions on surfaces
- imaging of nanostructures on surfaces
- structures of surfaces and absorbed layers





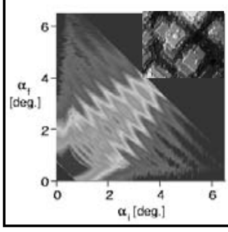
Sb on Si(113)



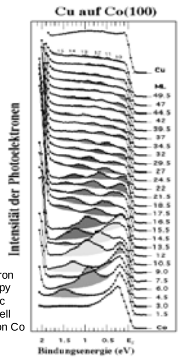
Spectro-Microscopy: Pb and Fe on W

Synchrotron Radiation Applications

Thin films:
- layer structure
- interface morphology
- electronic quantum well states



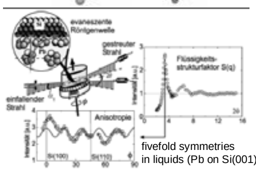
anomalous x-ray reflectometry from a Fe/Cr/Fe GMR trilayer



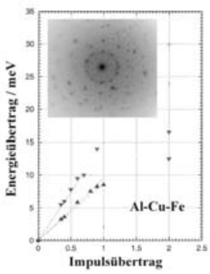
Photoelectron spectroscopy of electronic quantum well states Cu on Co

Synchrotron Radiation Applications

Condensed Matter research:
- chemical structure
- phonon dispersions
- structure of liquids
- magnetism



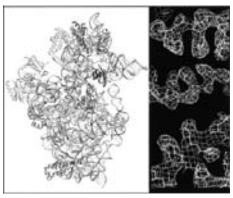
fivefold symmetries in liquids (Pb on Si(001)



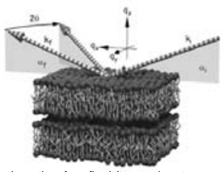
Longitudinal and transversal phonon dispersion in quasi-crystalline Al-Cu-Fe alloys

Synchrotron Radiation Applications

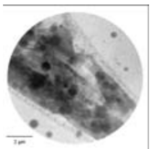
life science:
- structure of macromolecules (proteins)
- x-ray microscopy
- structure of biomembranes



structure of a subunit of the Ribosom



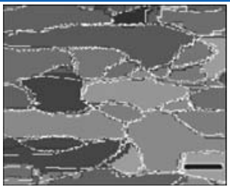
schematics of a reflectivity experiment on lipid membranes



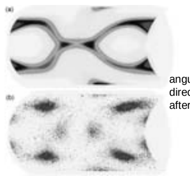
x-ray microscopy of the worm *Caenorhabditis elegans*

Synchrotron Radiation Applications

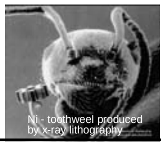
engineering / material sciences:
- nano fabrication using x-ray lithography
- 3d x-ray microscopy of grain structures
- stress / strain /textures



grainstructure of a polycrystalline material determined by high energy x-ray scattering / 3d microscopy



angular distribution of 111 directions in two Ni sheets after two different treatments



Ni - toothweel produced by x-ray lithography