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Our experience with the IBM Quantum Experience

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Nomenclature

- Single-qubit state:

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle; \qquad |a_0|^2 + |a_1|^2 = 1; \qquad a_0, a_1 \in \mathbb{C}$$

- Singlet state = maximally entangled two-qubit state:

$$|\psi\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$$

- State of a 5-qubit quantum computer:

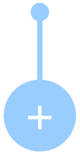
$$|\psi\rangle = a_0|0_4 0_3 0_2 0_1 0_0\rangle + a_1|0_4 0_3 0_2 0_1 1_0\rangle + \cdots + a_{31}|1_4 1_3 1_2 1_1 1_0\rangle$$

Nomenclature

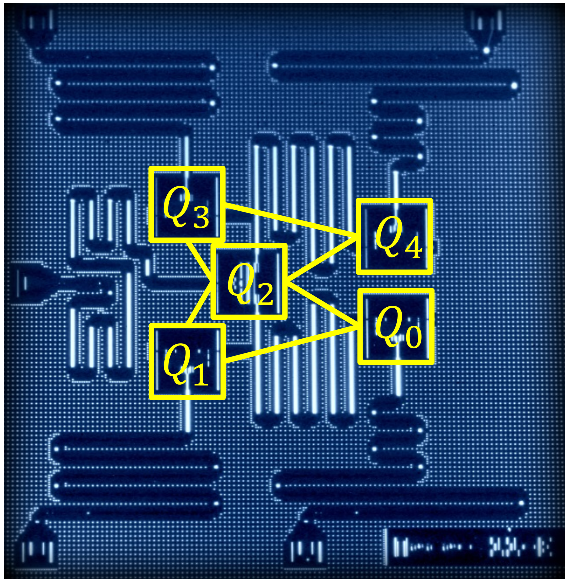
- X-gate
  $\mathrm{X}|0\rangle = |1\rangle; \quad \mathrm{X}|1\rangle = |0\rangle$

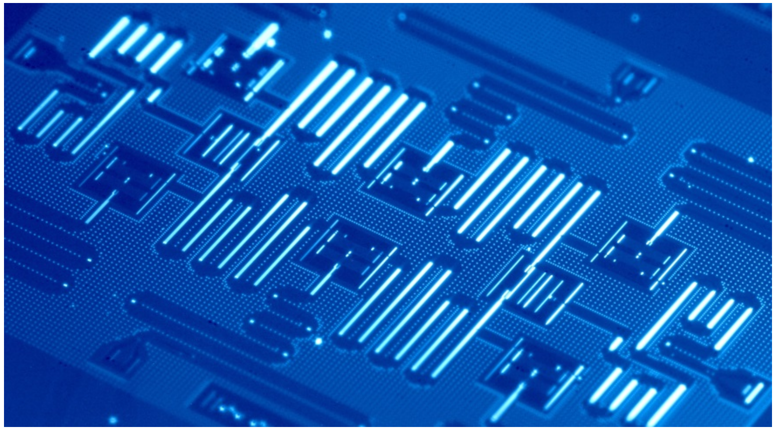
- Hadamard gate
  $H|0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}; \qquad H|1\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$

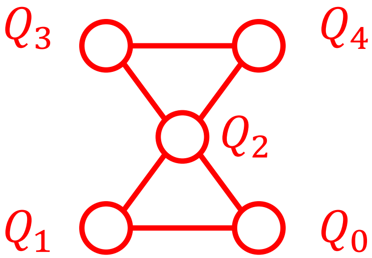
- CNOT gate
  $\mathrm{CNOT}_{\mathrm{Control-Target}}|Q_j Q_i\rangle \equiv \mathrm{C}_{\mathrm{CT}}|Q_j Q_i\rangle; \; j \neq i \in \{0,1,2,3,4\}$
  $\mathrm{C}_{01}|0_1 0_0\rangle = |0_1 0_0\rangle; \mathrm{C}_{01}|0_1 1_0\rangle = |1_1 1_0\rangle$
  $\mathrm{C}_{01}|1_1 0_0\rangle = |1_1 0_0\rangle; \mathrm{C}_{01}|1_1 1_0\rangle = |0_1 1_0\rangle$

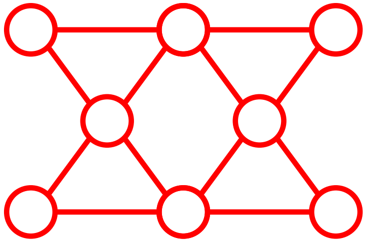


Hardware





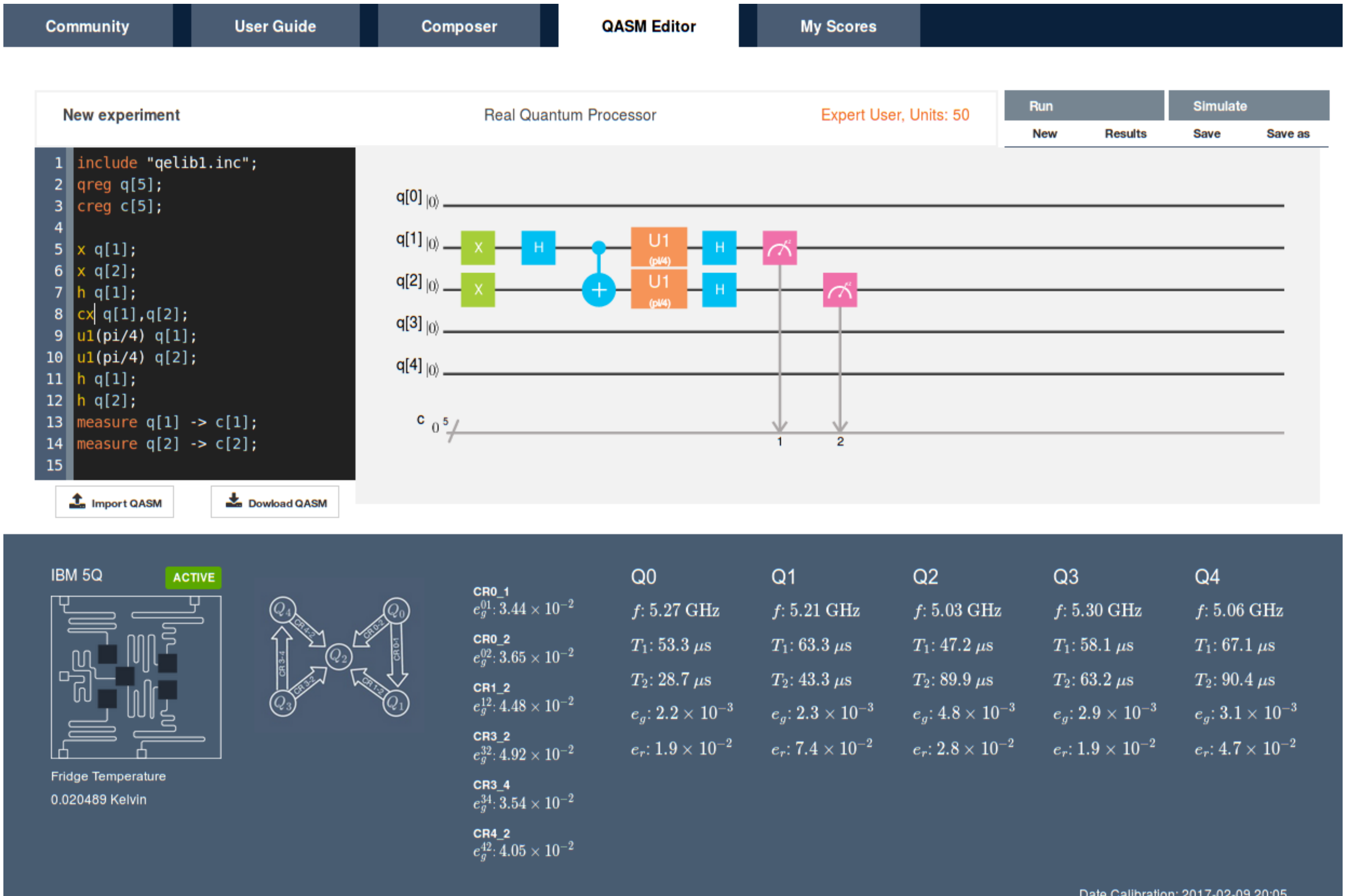


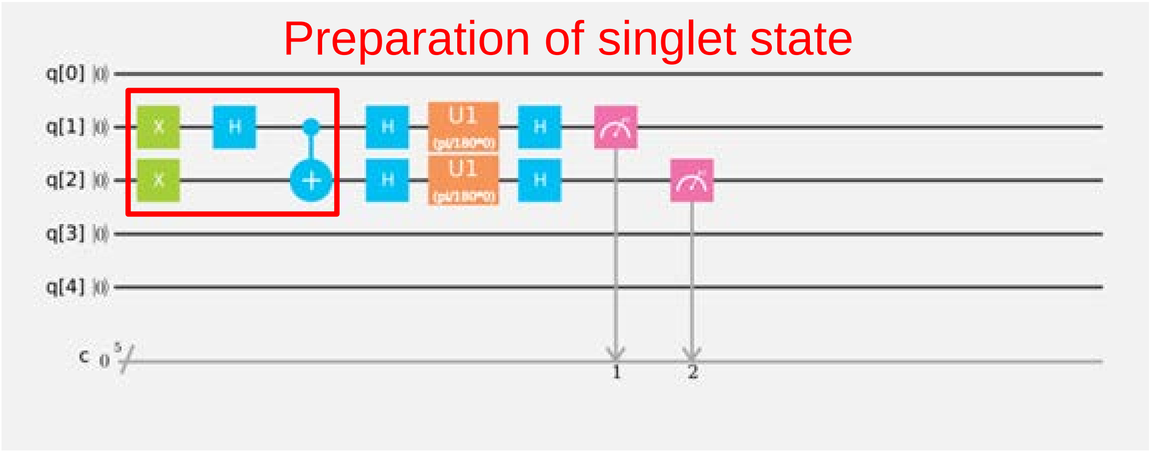


Device parameters

- Size: 7.5 mm x 9.5 mm
- Temperature: ≈ 15 mK
- Coherence time of a single qubit: ≈ 100 μs
- Gate errors: 10^{-2} – 10^{-3}
- Duration of gate operations:
  - X-gate: 130 ns
  - Hadamard: 130 ns
  - CNOT gate: 650 ns

The IBM Quantum Experience





MEASURING THE SINGLET STATE

Measuring the singlet state

According to quantum theory:

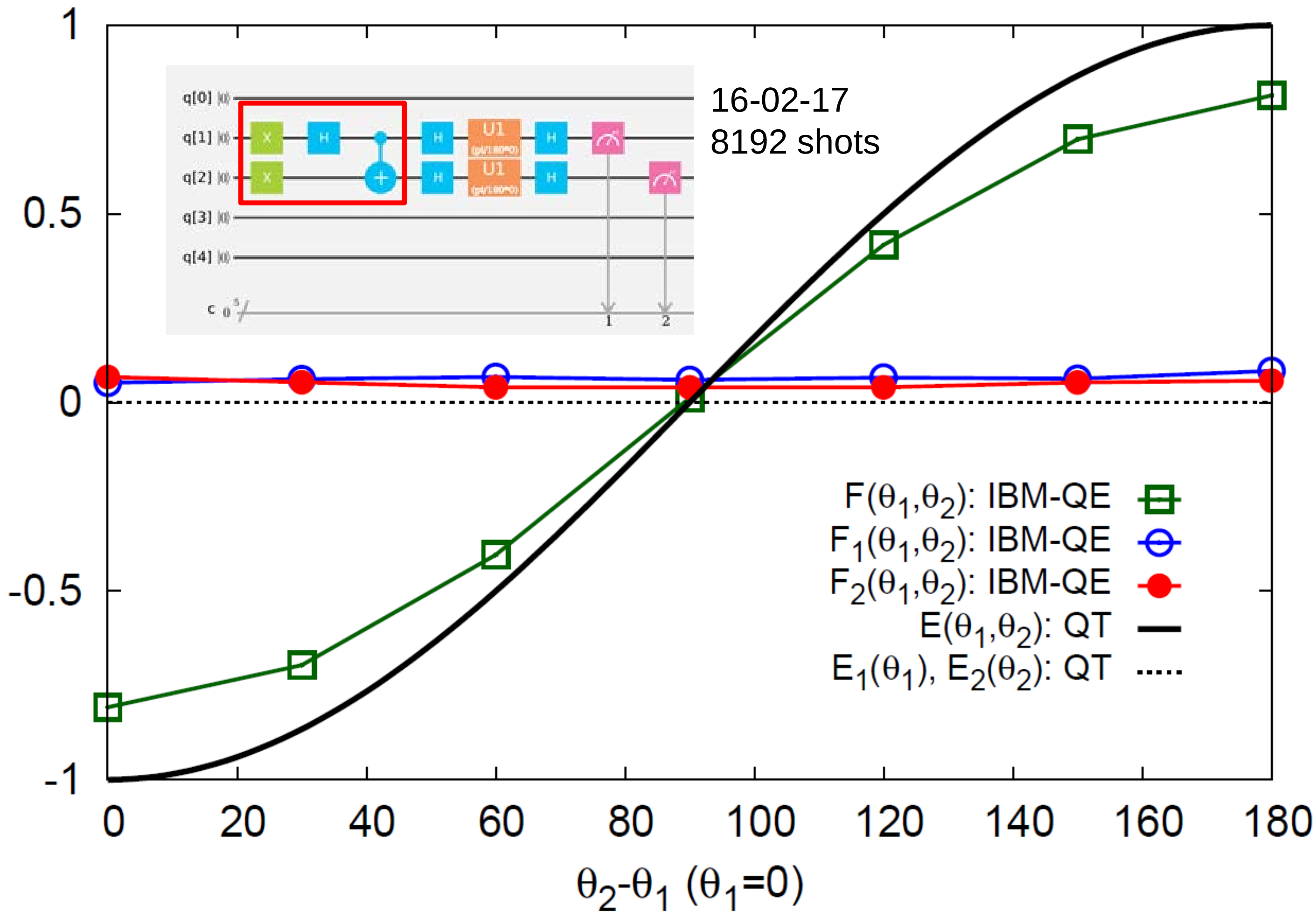
The singlet state $|\psi\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ is fully determined by

$$E_i(\mathbf{a}_i) = \langle\psi|\boldsymbol{\sigma}_i \cdot \mathbf{a}_i|\psi\rangle = \langle\psi|\boldsymbol{\sigma}_i|\psi\rangle \cdot \mathbf{a}_i = 0, \quad i = 1,2$$

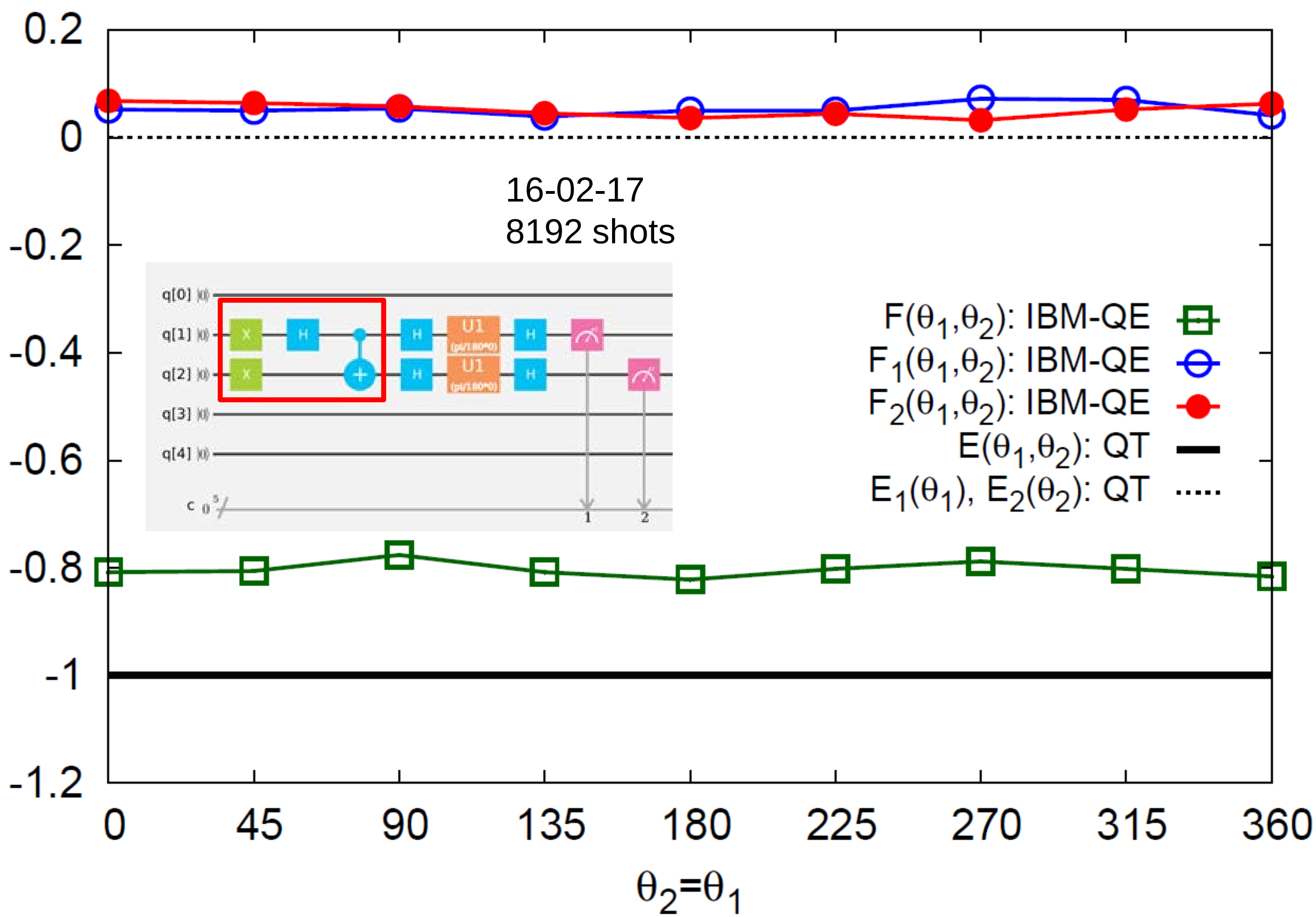
$$E(\mathbf{a}_1, \mathbf{a}_2) = \langle\psi|\boldsymbol{\sigma}_1 \cdot \mathbf{a}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{a}_2|\psi\rangle = \mathbf{a}_1 \cdot \langle\psi|\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2|\psi\rangle \cdot \mathbf{a}_2 = -\mathbf{a}_1 \cdot \mathbf{a}_2$$

We choose $\mathbf{a}_i = (0, -\sin\theta_i, \cos\theta_i)$ so $-\mathbf{a}_1 \cdot \mathbf{a}_2 = -\cos(\theta_2 - \theta_1)$

Measuring the singlet state



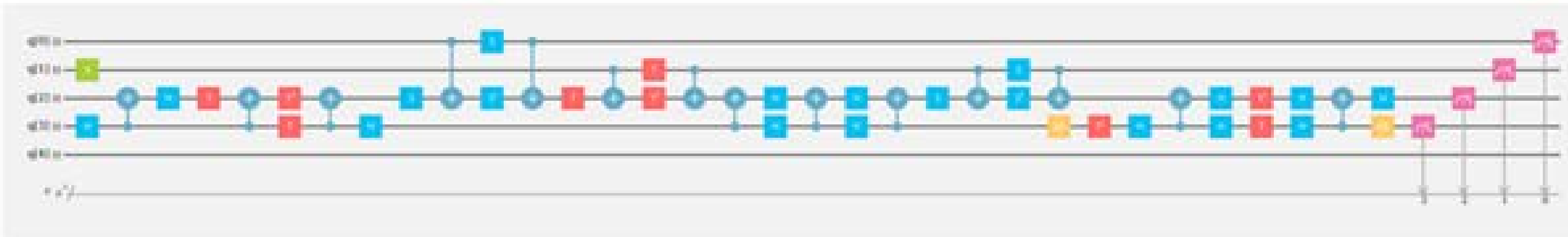
Measuring the singlet state

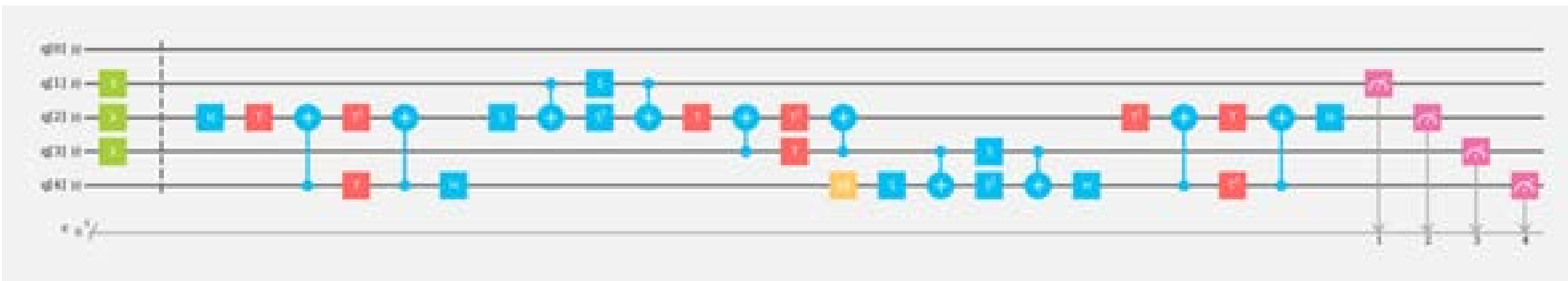


Measuring the singlet state: Conclusions

- For this extremely simple system
  - Qualitatively: The results show the features expected from quantum theory
  - Quantitatively:
    - $E_i(\mathbf{a}_i) \neq 0\ , \qquad i = 1,2$
    - $E(\mathbf{a}_1, \mathbf{a}_2)$ *: cosine with reduced amplitude*

Error ≠ statistical error





2+2 QUBIT ADDER

S.J. Devitt, Performing quantum computing experiments in the cloud, Phys. Rev. A94, 032329 (2016) + Supplementary Information

T.G. Draper, Addition on a quantum computer, arXiv:quant-ph/0008033 (2000)

2+2 qubit adder

- Modulo-4 addition of two two-bit integers

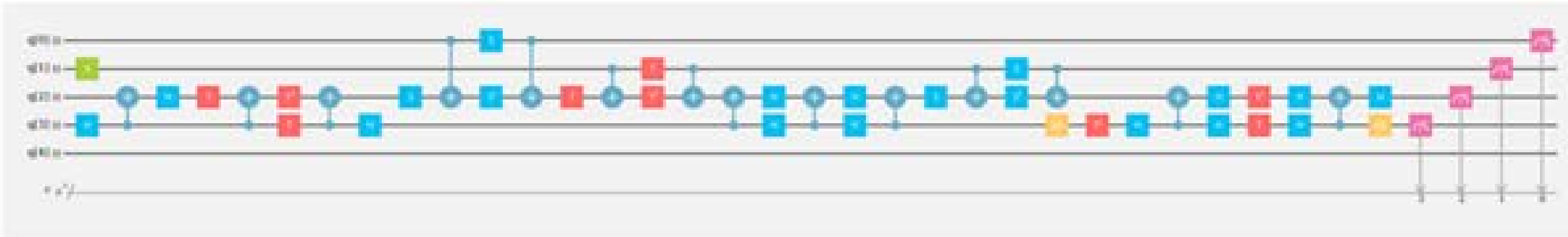
+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

2+2 qubit adder

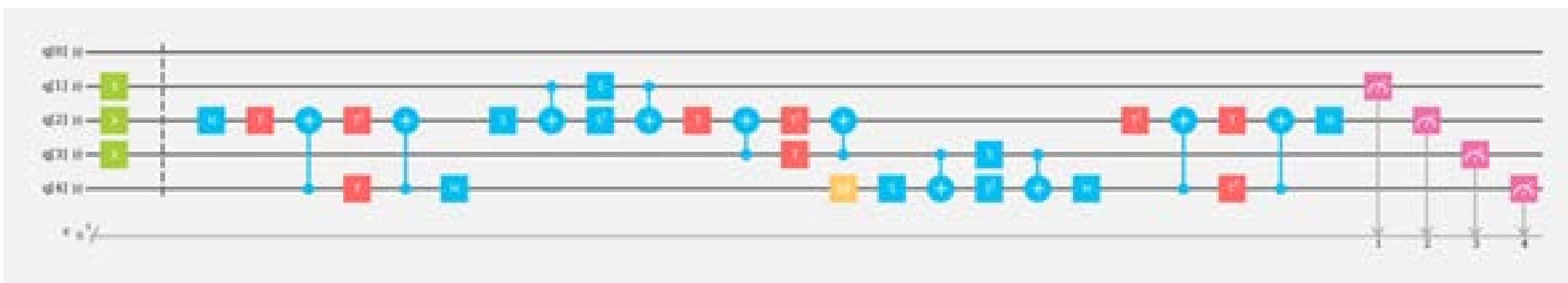
- How to judge the outcome?
  - Rule: the state with the largest frequency is regarded as the result of the computation
  - Compare to the output states and their probability from quantum theory

Correct output state(s) based on largest frequency

Wrong output state based on largest frequency



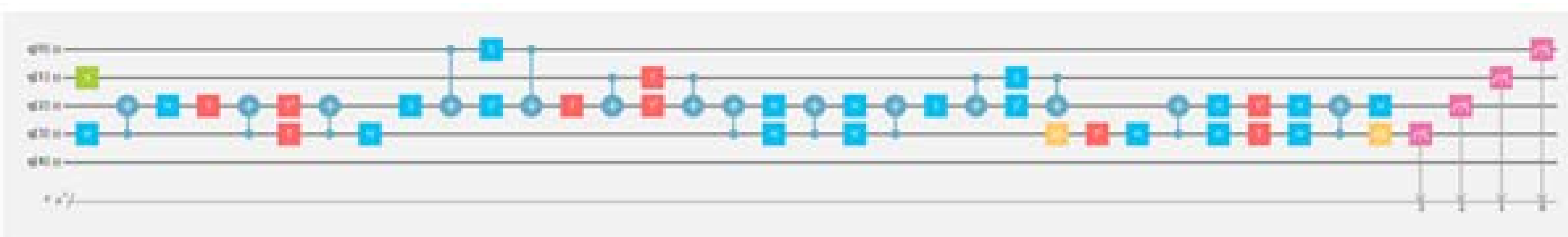
Operation $\frac{|Q_0Q_1\rangle + |Q_2Q_3\rangle}{|Q_3Q_2Q_0Q_1\rangle}$	Output QT $|state\rangle$ (prob.)	Output IBM-QE $|state\rangle$ (freq.) Date, # shots = 8192
$2+1=3$	$|\mathbf{1_3 1_2 0_1 1_0}\rangle$ **(1.000)**	$|\mathbf{1_3 1_2 0_1 1_0}\rangle$ **(0.275)** ; $|0_3 0_2 0_1 1_0\rangle$ (0.160) 22-01-2017



Operation $\frac{|Q_1Q_3\rangle + |Q_2Q_4\rangle}{|Q_2Q_4Q_1Q_3\rangle}$	Output QT $|state\rangle$ (prob.)	Output IBM-QE $|state\rangle$ (freq.) Date, # shots = 8192
$2+1=3$	$|\mathbf{1_4 0_3 1_2 1_1}\rangle$ **(1.000)**	$|\mathbf{1_4 0_3 0_2 1_1}\rangle$ **(0.342)** ; $|\mathbf{1_4 0_3 1_2 1_1}\rangle$ **(0.341)** 25-01-2017

■ Correct output state(s) based on largest frequency

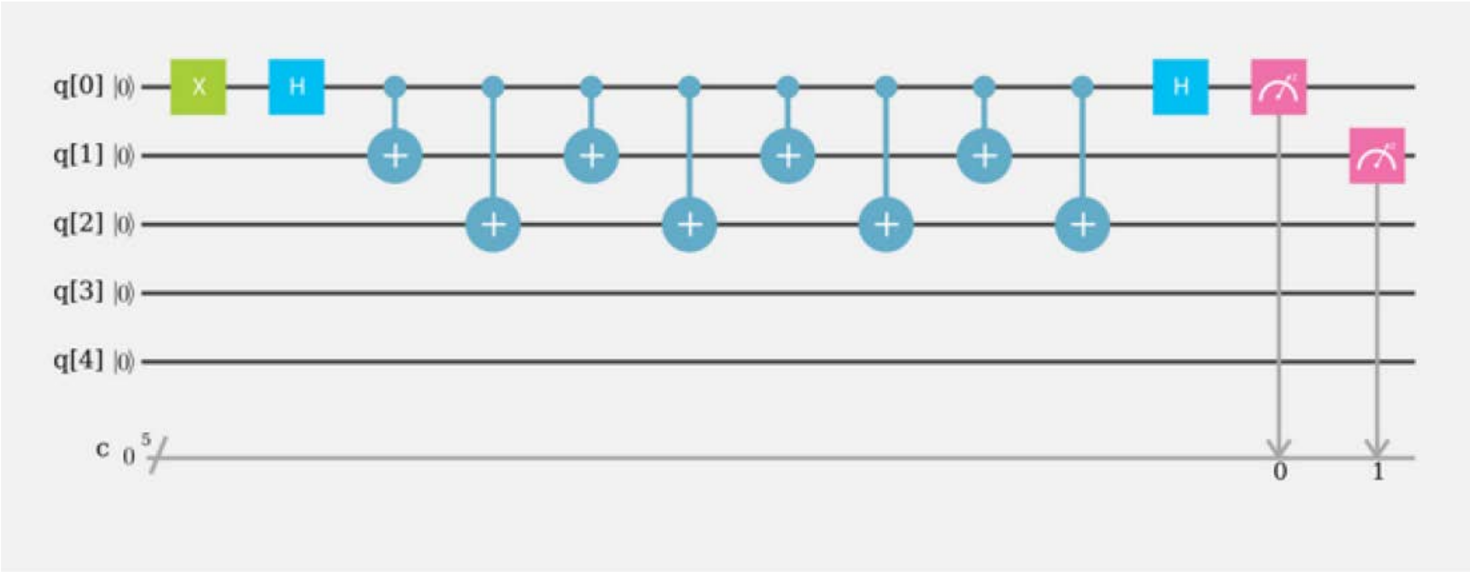
Operation $\begin{array}{r} |Q_0Q_1\rangle \\ + \; |Q_2Q_3\rangle \\ \hline |Q_3Q_2Q_0Q_1\rangle \end{array}$	**Output QT** $|state\rangle$ **(prob.)**	**Output IBM-QE** $|state\rangle$ **(freq.)** **Date, # shots = 8192**
$0+0=0$ $1+0=1$	$|\mathbf{0_3 0_2 0_1 0_0}\rangle$; $|\mathbf{0_3 1_2 1_1 0_0}\rangle$ **(0.500)**	$|\mathbf{0_3 0_2 0_1 0_0}\rangle$ **(0.354)** ; $|\mathbf{0_3 1_2 1_1 0_0}\rangle$ **(0.311)** $|1_3 0_2 0_1 0_0\rangle$ (0.066) ; $|0_3 1_2 0_1 0_0\rangle$ (0.062) 17-01-2017
$1+0=1$ $1+3=0$	$|\mathbf{0_3 1_2 1_1 0_0}\rangle$; $|\mathbf{0_3 0_2 1_1 0_0}\rangle$ **(0.500)**	$|\mathbf{0_3 1_2 1_1 0_0}\rangle$ **(0.314)** ; $|\mathbf{0_3 0_2 1_1 0_0}\rangle$ **(0.262)** $|1_3 1_2 1_1 0_0\rangle$ (0.098) ; $|1_3 0_2 1_1 0_0\rangle$ (0.085) 17-01-2017
$1+0=1$ $1+3=0$ $3+0=3$ $3+3=2$	$|\mathbf{0_3 1_2 1_1 0_0}\rangle$; $|\mathbf{1_3 1_2 1_1 1_0}\rangle$ **(0.250)** $|\mathbf{0_3 0_2 1_1 0_0}\rangle$; $|\mathbf{1_3 0_2 1_1 1_0}\rangle$ **(0.250)**	$|\mathbf{0_3 1_2 1_1 0_0}\rangle$ **(0.185)** ; $|\mathbf{1_3 1_2 1_1 1_0}\rangle$ **(0.152)** $|\mathbf{0_3 0_2 1_1 0_0}\rangle$ **(0.147)** ; $|\mathbf{1_3 0_2 1_1 1_0}\rangle$ **(0.097)** 17-01-2017



2+2 qubit adder: Conclusions

- The rule "largest frequency ➔ result" gave the correct answer with exceptions
- The frequencies deviate strongly from the probability given by quantum theory
- The results and their frequencies vary strongly between different device calibrations

Error ≠ statistical error

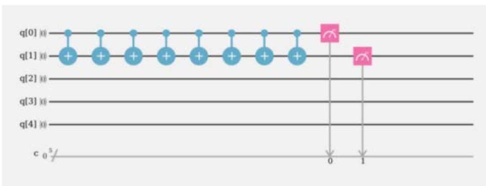
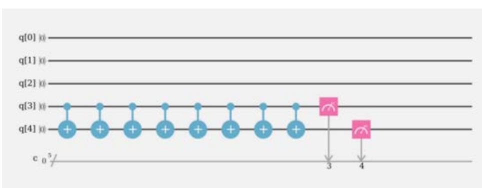
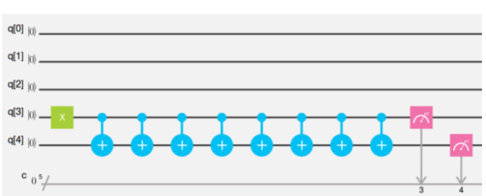


IDENTITY OPERATIONS

H. De Raedt, K. Michielsen, A.H. Hams, S. Miyashita and K. Saito,
Quantum Spin Dynamics as a Model for Quantum Computer Operation, Eur. Phys. J. B 27, 15 - 28 (2002)

Correct output state based on largest frequency (green)

Wrong output state based on largest frequency (red)

Operation	Input	Output QT $\|state\rangle$ (prob.)	Output IBM-QE $\|state\rangle$ (freq.) Date, # shots = 8192
$(C_{01})^8$ 	$\|0_1 0_0\rangle$	$\|0_1 0_0\rangle$ **(1.000)**	$\|0_1 0_0\rangle$ (0.661) ; $\|1_1 0_0\rangle$ (0.299) 16-01-2017 $\|0_1 0_0\rangle$ (0.700) ; $\|1_1 0_0\rangle$ (0.198) 18-01-2017 $\|0_1 0_0\rangle$ (0.642) ; $\|1_1 0_0\rangle$ (0.289) 19-01-2017 $\|0_1 0_0\rangle$ (0.580) ; $\|1_1 0_0\rangle$ (0.335) 23-01-2017 $\|0_1 0_0\rangle$ (0.628) ; $\|1_1 0_0\rangle$ (0.256) 23-01-2017
$(C_{34})^8$ 	$\|0_4 0_3\rangle$	$\|0_4 0_3\rangle$ **(1.000)**	$\|1_4 0_3\rangle$ (0.512) ; $\|0_4 0_3\rangle$ (0.372) 15-01-2017 $\|1_4 0_3\rangle$ (0.567) ; $\|0_4 0_3\rangle$ (0.318) 16-01-2017 $\|1_4 0_3\rangle$ (0.548) ; $\|0_4 0_3\rangle$ (0.363) 18-01-2017 $\|1_4 0_3\rangle$ (0.616) ; $\|0_4 0_3\rangle$ (0.275) 19-01-2017 $\|1_4 0_3\rangle$ (0.590) ; $\|0_4 0_3\rangle$ (0.323) 22-01-2017 $\|1_4 0_3\rangle$ (0.618) ; $\|0_4 0_3\rangle$ (0.321) 23-01-2017
$(C_{34})^8$ 	$\|0_4 1_3\rangle$	$\|0_4 1_3\rangle$ **(1.000)**	$\|0_4 1_3\rangle$ (0.794) ; $\|0_4 0_3\rangle$ (0.084) 16-01-2017 $\|0_4 1_3\rangle$ (0.797) ; $\|0_4 0_3\rangle$ (0.088) 18-01-2017 $\|0_4 1_3\rangle$ (0.853) ; $\|0_4 0_3\rangle$ (0.077) 23-01-2017 $\|0_4 1_3\rangle$ (0.849) ; $\|0_4 0_3\rangle$ (0.068) 23-01-2017

Correct output state based on largest frequency

Wrong output state based on largest frequency

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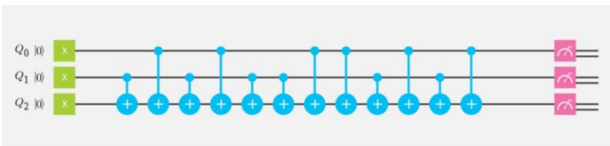
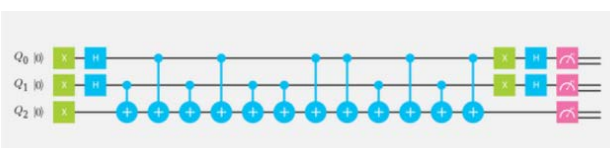
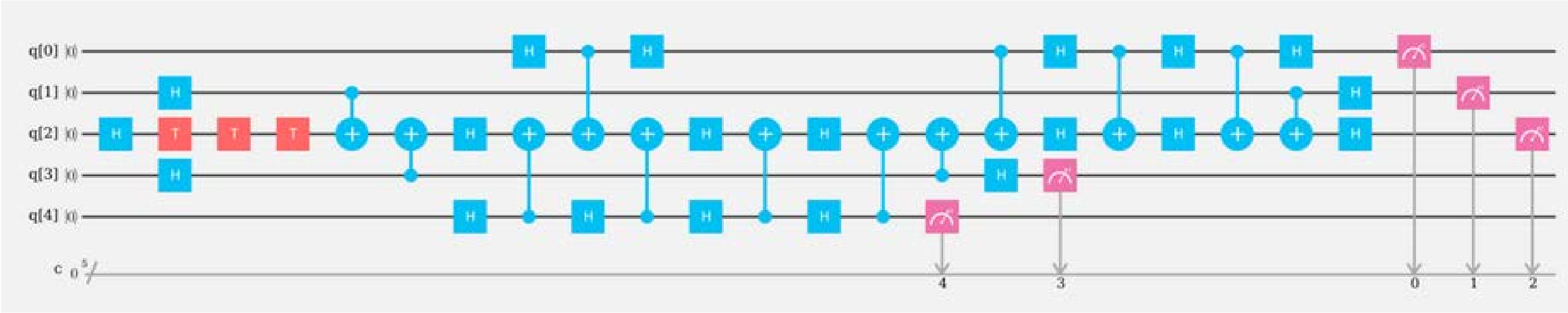
Operation	Input	Output QT $\|state\rangle$ (prob.)	Output IBM-QE $\|state\rangle$ (freq.) Date, # shots = 8192
$(C_{02}C_{12})^2(C_{02})^2(C_{12})^2(C_{02}C_{12})^2$ 	$\|1_2 1_1 1_0\rangle$	$\|\mathbf{1_2 1_1 1_0}\rangle$ **(1.000)**	$\|\mathbf{1_2 1_1 1_0}\rangle$ **(0.355)** ; $\|1_2 1_1 0_0\rangle$ (0.304) 10-01-2017 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.262)** ; $\|1_2 1_1 1_0\rangle$ (0.238) 12-01-2017 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.250)** ; $\|1_2 1_1 1_0\rangle$ (0.237) 20-01-2017 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.374)** ; $\|0_2 1_1 0_0\rangle$ (0.151) 27-01-2017 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.347)** ; $\|0_2 0_1 1_0\rangle$ (0.164) 27-01-2017 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.368)** ; $\|0_2 0_1 1_0\rangle$ (0.161) 27-01-2017 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.358)** ; $\|0_2 0_1 1_0\rangle$ (0.166) 27-01-2017
$H_0 H_1 X_0 X_1 (C_{02}C_{12})^2(C_{02})^2(C_{12})^2$ $(C_{02}C_{12})^2 H_0 H_1$ 	$\|1_2 1_1 1_0\rangle$	$\|\mathbf{1_2 1_1 1_0}\rangle$ **(1.000)**	$\|\mathbf{1_2 1_1 1_0}\rangle$ **(0.304)** ; $\|0_2 1_1 1_0\rangle$ (0.157) 05-05-2016 $\|\mathbf{1_2 1_1 1_0}\rangle$ **(0.298)** ; $\|0_2 1_1 1_0\rangle$ (0.192) 09-11-2016 $\|\mathbf{1_2 1_1 1_0}\rangle$ **(0.223)** ; $\|1_2 1_1 0_0\rangle$ (0.156) 20-11-2016 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.202)** ; $\|0_2 0_1 0_0\rangle$ (0.170) 27-01-2016 $\|\mathbf{0_2 1_1 1_0}\rangle$ **(0.232)** ; $\|0_2 0_1 0_0\rangle$ (0.154) 27-01-2016

Identity operations: Conclusions

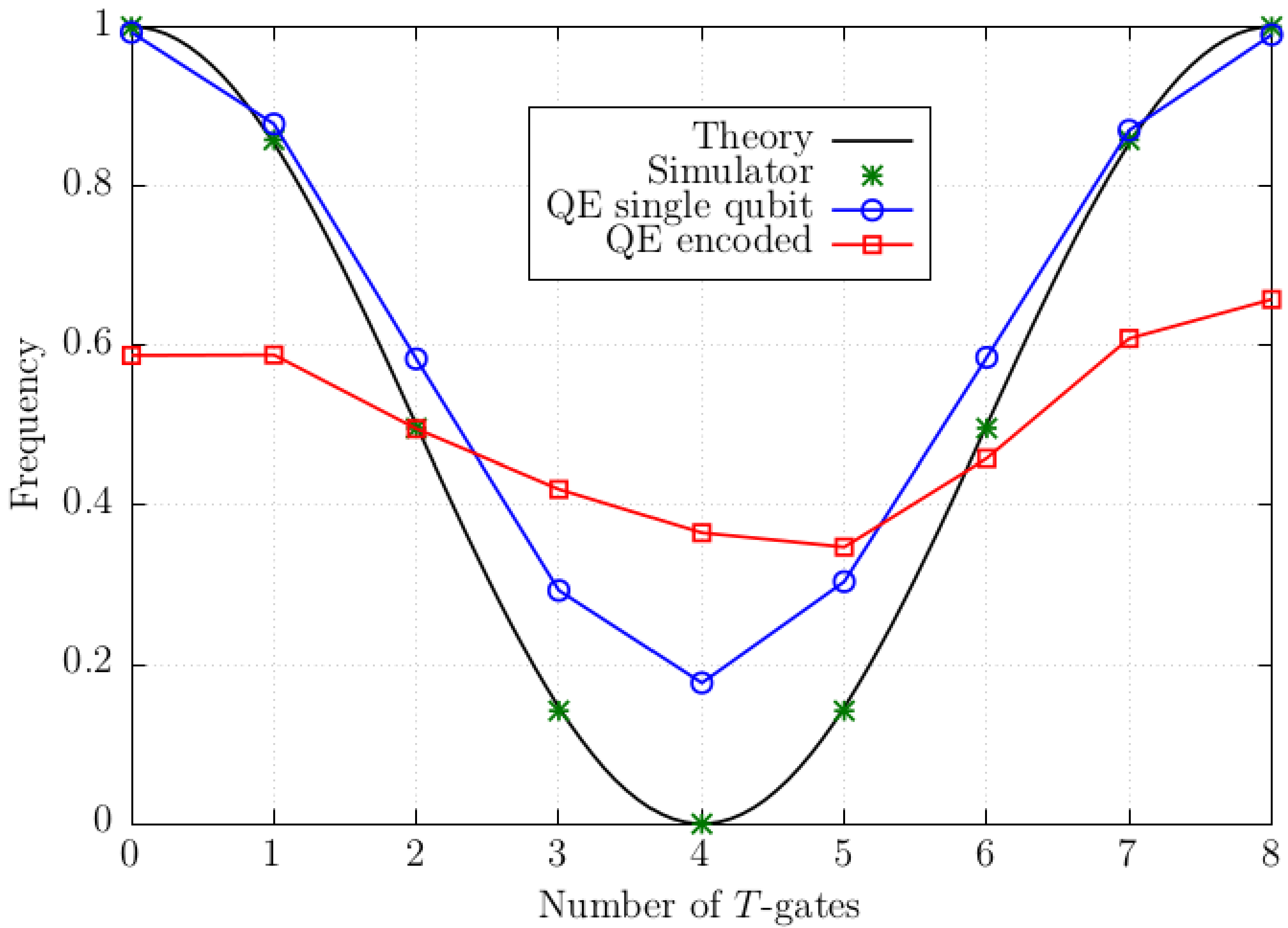
- Very simple, scalable but sensitive quantum algorithms to validate the operation of quantum computer devices
- The outcome is sometimes correct, sometimes wrong
- Results seem to be systematic
  - Similar results for different device calibrations
- Results that were correct on the old device (< Jan. 11, 2017) turn out to be incorrect on the new device (≥ Jan. 11, 2017)

Error = systematic error



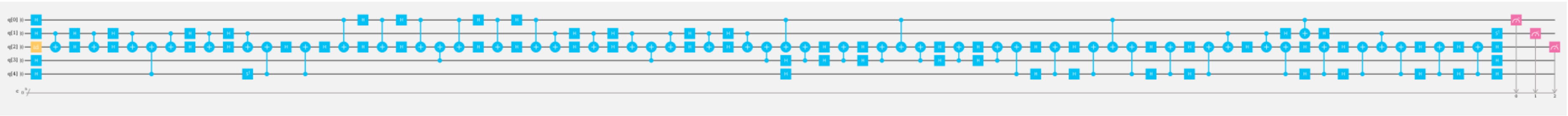
ERROR CORRECTION

Error correction ("distance-two surface code")

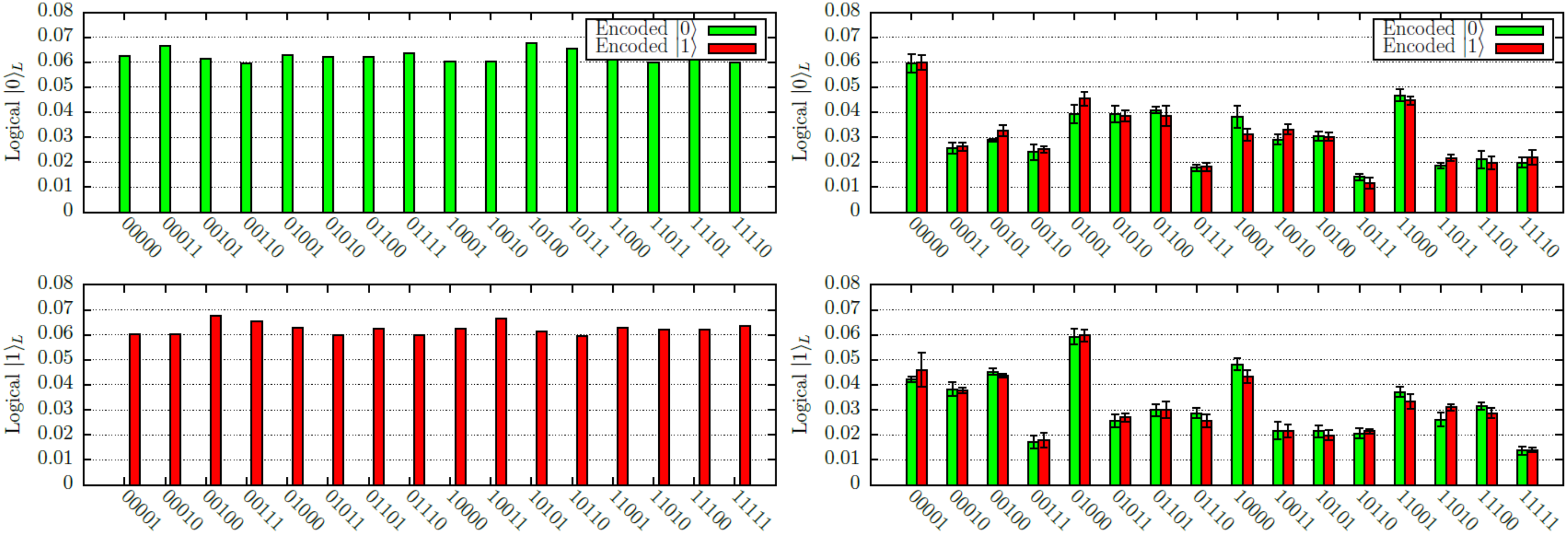


S.J. Devitt, Performing quantum computing experiments in the cloud,
Phys. Rev. A94, 032329 (2016) + Supplementary Information

Error correction (“distance-three 5-qubit code”)



Simulator | Real Device



R. Laflamme, C. Miquel, J. P. Paz, and W. H. Zurek,
Perfect quantum error correcting code, Phys. Rev. Lett. 77, 198 (1996)

Error correction: Conclusions

- The device can be used to test small error correction codes
- Error correction makes the outcome worse
- The encoding results obtained with the IBM-QE device are completely different from those given by quantum theory

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Summary

- Very simple algorithms have been used to test the IBM-QE from a user perspective
- In some cases we could observe qualitative agreement with quantum theory for qubit systems
- Errors cannot be identified by the user and they cannot be attributed to the specified gate errors
- There are strong differences between calibrations
- The current device does not qualify as a computer
- A theoretician can perform laboratory experiments

THANK YOU